PHYSICS

Unit- 1. Electric Charges and Field

Class - 12th

BoardGuru

1. ELECTRIC CHARGE AND FIELD

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1. ELECTRIC CHARGE AND FIELD

Electric Charge -

The property of material by which it exerts or experience electric and magnetic effects is called the electric charge.

Examples -

(i) Experience of seeing a spark or hearing a crackle, when we take of our synthetic cloths.

(ii) Lightning in the sky during thunderstorms due to electric discharge.

Electrostatics -

The branch of Physics, which deals with the study of charges at rest (static charges), the forces, fields and potential due to these charges is called Electrostatics or static Electricity.

Important Facts about charge -

(1.) There are only two types of electric charge.

(2.) Like charges repel and unlike charges attracts each other.

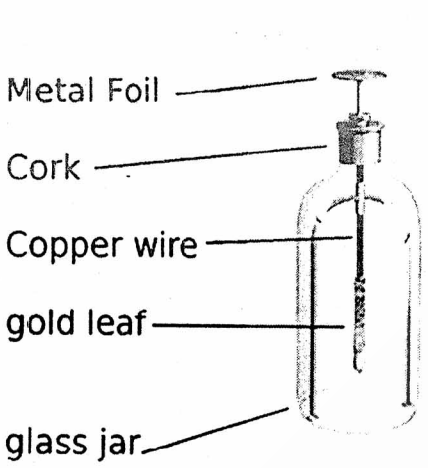
(3.) The unlike charges nullify (Cancel) each other's effect when they come in contact. Therefore the charges are named as positive and negative by the American scientist 'Benjamin Frenklin'.

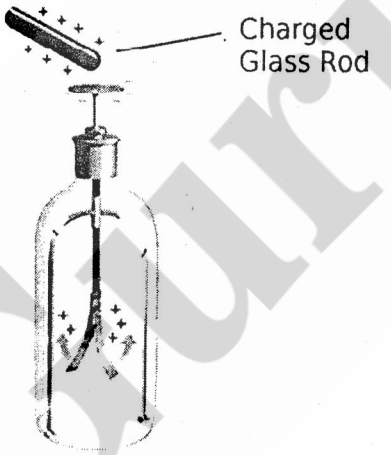
(4.) Normally the materials are electrically neutral, they do not contain charge because their charges (Protons and electrons) are exactly balanced.

(5.) The electron of the outermost orbit of an atom are far from the nucleus so these electrons are Loosely bounded with the nucleus and can be separated easily from the orbit by giving some energy. These electron are then called free electrons.

(6.) The body or atom can be charged positively by loosing some of its electrons and can be charged negatively by gaining electrons.

(7.) The apparatus used to detect charge on a body is "Gold leaf Electroscope".





(8.) An electric charge can creates electric field (E), magnetic field ($\vec{B}$) and electromagnetic radiations

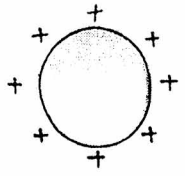
$V = 0$ ⊕ → Only Electric field ($\vec{E}$)

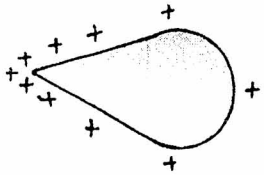
$V = Const.$ ⊕ → Electric Field ($\vec{E}$) & Magnetic Field ($\vec{B}$)

$V \neq Const.$ ⊕ → $\vec{E}$, $\vec{B}$ and Electromagnetic Radiations

(9.) Electric charge uniformly distribute on a uniform surface but the charge density on a non-uniform surface is maximum on that points where the radius of curve is minimum.

Charge density $\boxed{\sigma \propto \frac{1}{R}}$





Point Charge - It is a point where all the charge of a body is concentrated.

Conductor, Insulator & Semiconductor -

Conductor - The material through which the electric charge and current can flow easily are called Conductors.

Examples - All Metals, Human body, earth etc.

Insulator - The materials which resist the flow of electric charge and current are called insulators.

Examples - Wood, Porcelene, plastic etc.

Semiconductors - The materials which resist the flow of electric charge and current but the value of their resistance is in between the conductor and insulator are called semiconductors.

Examples - Silicon (Si), Germenium (Ge)
Carbon (C) → Excellent thermal Conductivity.

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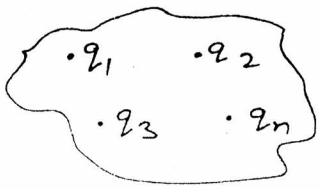
Properties of Electric Charge -

(1) Additivity of Charges:-

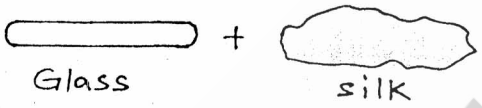
Electric charge is a scalar quantity so the total charge of the system is obtained simply by adding the different charges algebrically with proper sign according to their charge.



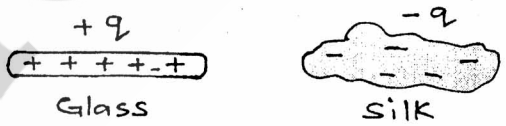
$$q = q_1 + q_2 + \dots q_n$$

(2) Charge is conserved:-

Charge can not be either created or distroyed. It can only be transferred from one body to another. Hence the total charge of the isolated system is always conserved.



Total charge = 0+0 = 0



Total charge = +q - q = 0

(3) Quantisation of Charge :-

The electric charge on a body is always an integral multiple of e (charge on 1 electron). This is called quantisation of charge.

$$Q = \pm ne$$

$n = 0, 1, 2 \dots \infty$

$e = 1.602 \times 10^{-19}$ C

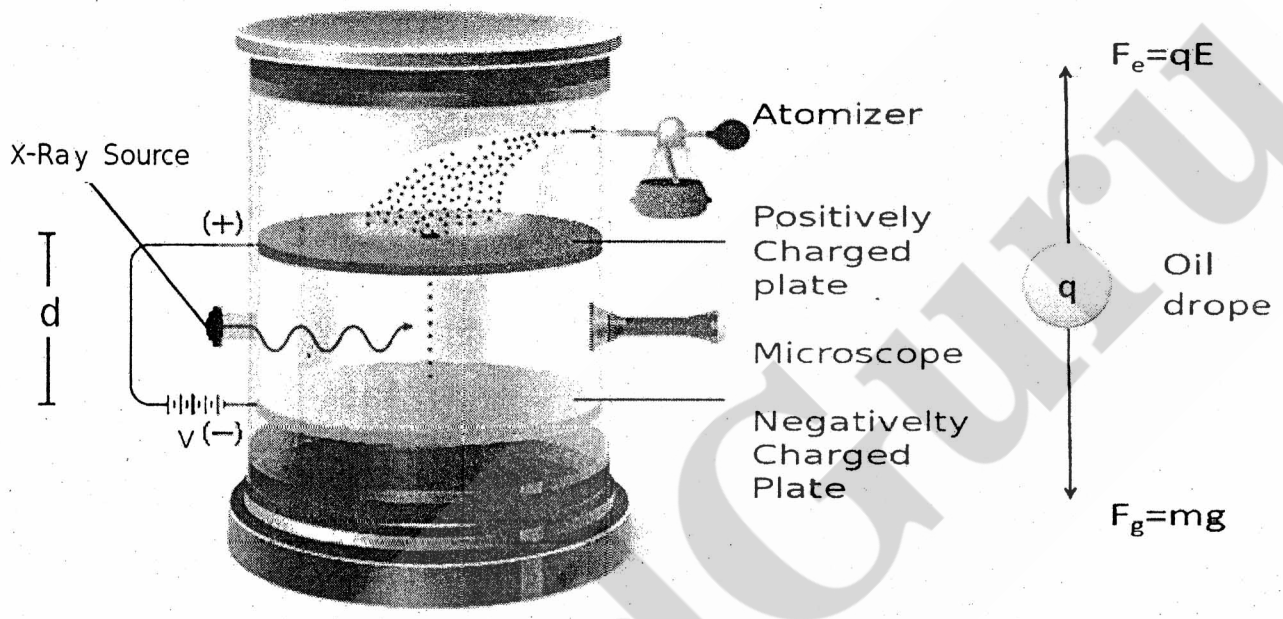
* The unit of electric charge is coulomb.

1 Coulomb -

The total charge on 6.24×10^{18} electrons is equal to 1 coulomb.

Millikan's Oil drop Experiment –

The charge on one electron was found by this oil drop experiment by Millikan in 1912.



When the oil drop remain stationary in the experiment then the forces on oil drop

Electric Force (F_e) = Gravitational Force (F_g)

$$qE = mg$$

or $$\boxed{q = \frac{mg}{E} = \frac{mgd}{V}}$$ where $E = \frac{V}{d}$

We Know that – mass = density x volume

$$\boxed{m = d \times \frac{4}{3}\pi r^3}$$

where $r \rightarrow$ radius of oil drop (0.5 mm)

* Mass of Electron (m_e)	$= 9.11 \times 10^{-31}$ Kg.
* Mass of Proton (m_p)	$= 1.67 \times 10^{-27}$ Kg.

Q. An oil drop of 12 excess electrons is held stationary under a constant electric field of 2.55×10^4 N/C in Millikan's oil drop experiment. The density of the oil is 1.26 gm/cm^3. Then estimate the radius of the oil drop.

Sol. Given that -

Charge $q = ne = 12 \times 1.6 \times 10^{-19}$ C

Density $d = 1.26 \times \frac{10^{-3}}{10^{-6}} = 1.26 \times 10^3$ kg/m^3

When the oil drop is held stationary then

$$q = \frac{mg}{E}$$

We Know $m = d \times \frac{4}{3}\pi r^3$

so that $$q = \frac{d \times \frac{4}{3}\pi r^3 \times g}{E}$$

or $$r^3 = \frac{qE}{d \times \frac{4}{3}\pi \times g}$$

$$r^3 = \frac{12 \times 1.6 \times 10^{-19} \times 2.55 \times 10^4}{1.26 \times 10^3 \times \frac{4}{3} \times 3.14 \times 9.8} = 9.47 \times 10^{-18} \text{ m}^3$$

$$r = (9.47 \times 10^{-18})^{1/3} = 9.81 \times 10^{-7} \text{ m}.$$

Q. What is the total charge on 75 Kg of electrons?

Sol. Number of electrons in 75 Kg.

$$n = \frac{\text{total mass}}{\text{mass of 1 electron}} = \frac{75}{9 \times 10^{-31}}$$

$$n = 8.3 \times 10^{31}$$

So Total charge $Q = ne$

$$Q = 8.3 \times 10^{31} \times (-1.6 \times 10^{-19})$$

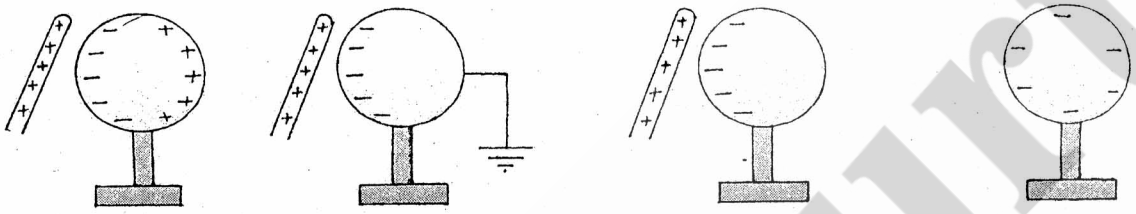
$$Q = -1.33 \times 10^{13} \text{ C}$$

Charging by Induction -

Method 1. (Using a ground Connection)

(i) A positively charged rod is brought near a neutral metal sphere and polarising it.

(ii) The sphere is grounded allowing electrons to be attracted from the earth.



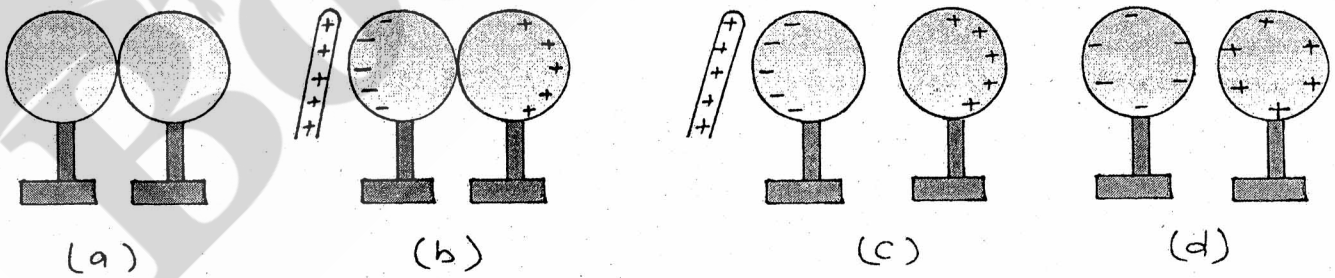
(iii) The ground connection is broken.

(iv) The positive rod is removed leaving the sphere with an induced negative charge.

Method 2.

(i) Two uncharged metal spheres are in contact with each other but insulated from rest of the world.

(ii) A positively charged rod is brought near the sphere, attracting negative charge and leaving the other sphere positively charged.



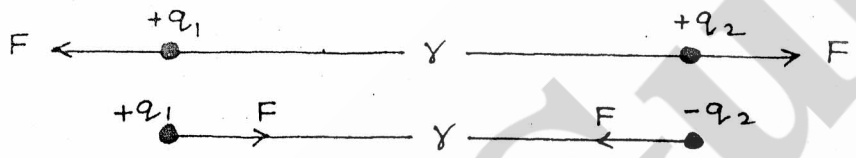
(iii) Now the spheres are separated before the rod is removed and thus separating positive & negative charge.

(iv) Remove the rod. The charges on spheres will rearrange themselves and uniformly distributed over the spheres.

Coulomb's Law -

The force of interaction between any two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them, and acts along the line joining the two charges.

Suppose two point charges q_1 and q_2 are separated in vacuum by a distance r.



According to Coulomb's Law,

$$F \propto q_1 q_2 \quad \text{and} \quad F \propto \frac{1}{r^2}$$

or $$\boxed{F = \frac{K q_1 q_2}{r^2}}$$ K = Electrostatic Constant

* Electrostatic constant K depends on the nature of medium separating the charges and on the system of units.

In cgs system, $K = 1$

In SI system, $K = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \ Nm^2/C^2$

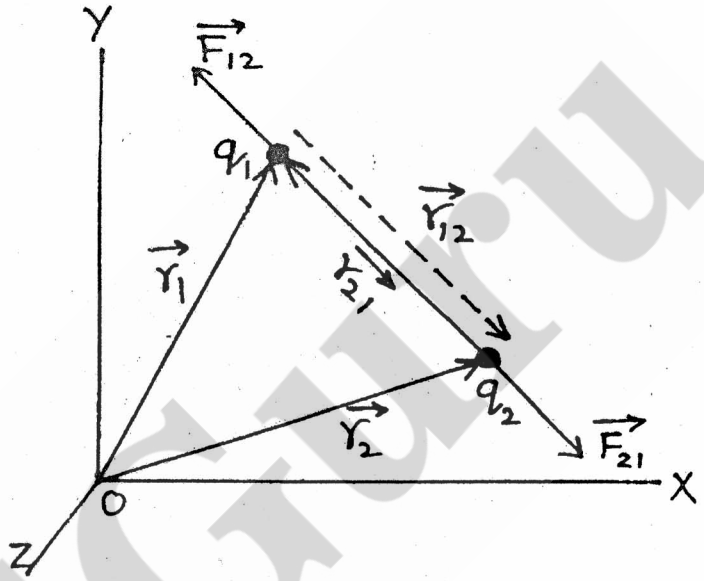
* ϵ_0 electrical permittivity of free space

$$\epsilon_0 = 8.85 \times 10^{-12} \ C^2/Nm^2$$

Coulomb's Law in vector form -

Let the position vector of charges q_1 and q_2 be $\vec{r}_1$ and $\vec{r}_2$ respectively.

We denote the force on q_1 due to q_2 by $\vec{F}_{12}$ and the force on q_2 due to q_1 by $\vec{F}_{21}$.



$$\boxed{F_{12} = F_{21}}$$

Here $\vec{r}_1 + \vec{r}_{12} = \vec{r}_2$

$$\boxed{\vec{r}_{12} = \vec{r}_2 - \vec{r}_1}$$

also $\vec{r}_2 + \vec{r}_{21} = \vec{r}_1$

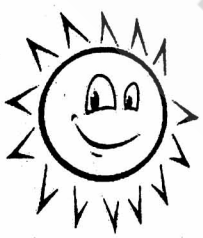
or $\boxed{\vec{r}_{21} = \vec{r}_1 - \vec{r}_2}$ and $\boxed{\vec{r}_{12} = -\vec{r}_{21}}$

The magnitude of vectors $\vec{r}_{12}$ and $\vec{r}_{21}$ is denoted by $|\vec{r}_{12}|$ and $|\vec{r}_{21}|$ and their unit vectors are given as -

$$\hat{r}_{12} = \frac{\vec{r}_{12}}{|\vec{r}_{12}|} \quad \text{and} \quad \hat{r}_{21} = \frac{\vec{r}_{21}}{|\vec{r}_{21}|}$$

So the force $\vec{F}_{12}$ can be given as -

$$\boxed{\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\vec{r}_{21}|^2} \hat{r}_{21}}$$



(i) If q_1 and q_2 are of same sign ; $q_1 q_2 > 0$ then $\vec{F}_{12}$ is along $\vec{r}_{21}$, which denotes repulsion

(ii) If q_1 and q_2 are of opposite sign ; $q_1 q_2 < 0$ then $\vec{F}_{12}$ is along $-\vec{r}_{21}$ or $\vec{r}_{12}$, which denotes attraction

Q. Two point charges of $+2\,\mu C$ and $+6\,\mu C$ repel each other with a force of 12 N. Now if each is given an additional charge of $-4\,\mu C$, what will be the new force?

Sol. Here $q_1 = +2\,\mu C$, $q_2 = +6\,\mu C$, $F = 12\,N$

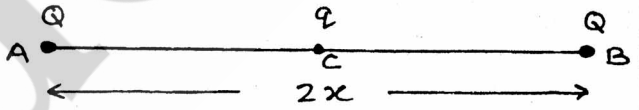
and $q_1' = +2-4 = -2\,\mu C$, $q_2' = +6-4 = 2\,\mu C$

So $$\frac{F'}{F} = \frac{q_1' q_2'}{q_1 q_2} = \frac{(-2)(2)}{(2)(6)} = -\frac{1}{3}$$

$$F' = -\frac{F}{3} = -\frac{12}{3} = -4\,N \text{ (Attractive)}$$

Q. A charge q is placed at the centre of the line joining two equal charges Q. Show that the system of three charges will be in equilibrium if $q = -Q/4$.

Sol.



Let two equal charges Q each, be held at A and B, where $AB = 2x$.

Here C is the centre of AB, where charge q is held. Net force on q is zero. So q is already in equilibrium.

For the three charges to be in equilibrium, net force on each charge must be zero.

Now, total force on Q at B is -

$$\frac{KQq}{x^2} + \frac{KQ \cdot Q}{(2x)^2} = 0$$

$$\frac{KQq}{x^2} = -\frac{KQ^2}{4x^2}$$

So $$\boxed{q = -\frac{Q}{4}}$$

Hence Proved.

Dielectric Constant (K) –

force in a medium $(F_m) = \frac{1}{4\pi\epsilon} \times \frac{q_1 q_2}{r^2}$ — (1)

where ϵ - electrical permittivity of the medium.

force in vacuum $(F_0) = \frac{1}{4\pi\epsilon_0} \times \frac{q_1 q_2}{r^2}$ — (2)

Dividing eq. (2) by (1), we get -

$$\frac{F_0}{F_m} = \frac{\epsilon}{\epsilon_0} = \epsilon_r \text{ or } K$$

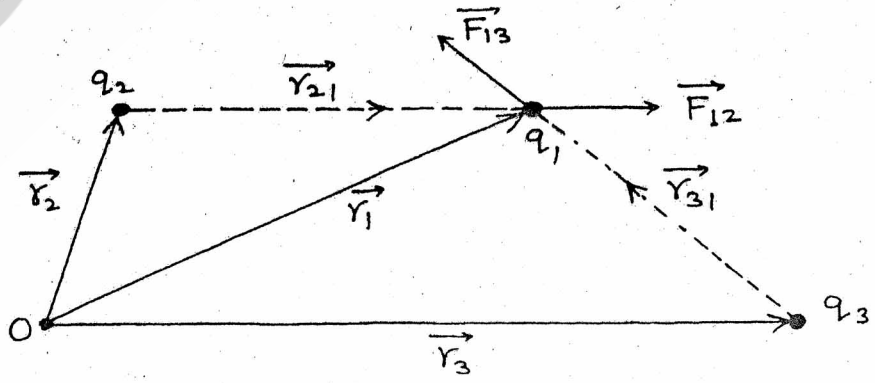
where ϵ_r or K is called dielectric constant or relative electrical permittivity of the medium.

* Dielectric constant of a medium is the ratio of absolute electrical permittivity of the medium to the electrical permittivity of free space.
* Thus force between two given charges in a medium (K) is only $\frac{1}{K}$ times of the force between them in air/ vacuum.

$$F_m = \frac{F_0}{K}$$

Force between Multiple Charges –

Total force on any charge due to a number of charges at rest is the vector sum of all the forces on that charge due to other charges.



Let the force on charge q_1 due to two other charges q_2 and q_3 are $\vec{F}_{12}$ and $\vec{F}_{13}$ respectively.

The position vectors of charges q_1, q_2 and q_3 are $\vec{r}_1$, $\vec{r}_2$ and $\vec{r}_3$.

Here $\boxed{\vec{F}_{12} = \frac{Kq_1q_2}{|\vec{r}_{21}|^2}\hat{r}_{21}}$ and $\boxed{\vec{F}_{13} = \frac{Kq_1q_2}{|\vec{r}_{31}|^2}\hat{r}_{31}}$

Thus the resultant force $\vec{F}_1$ on charge q_1 is obtained by the vector sum of the forces $\vec{F}_{12}$ and $\vec{F}_{13}$ as-

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13}$$

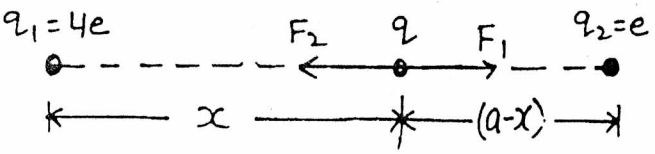
Similarly if a system have n charges $q_1, q_2, \ldots q_n$ then the force on q_1 due to $q_2, q_3, \ldots q_n$ charges can be calculated by the vector sum of forces $\vec{F}_{12}$, $\vec{F}_{13}$, $\ldots$ $\vec{F}_{1n}$ as-

$$\boxed{\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \ldots \vec{F}_{1n}} \quad \text{or} \quad \boxed{\vec{F}_1 = Kq_1 \sum_{i=2}^{n} \frac{q_i}{|\vec{r}_{i1}|^2}\hat{r}_{i1}}$$

Q. Two fixed point charges $+4e$ and $+e$ units are separated by a distance 'a'. where should the third point charge be placed for it to be in equilibrium.

Sol: For the charge $+q$ to be in equilibrium

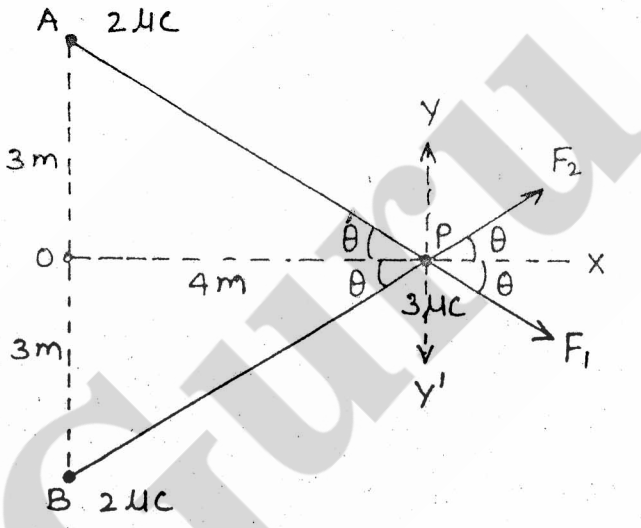
$F_1 = F_2$



$$\frac{Kq \cdot 4e}{x^2} = \frac{Kq \cdot e}{(a-x)^2} \Rightarrow \frac{4}{x^2} = \frac{1}{(a-x)^2}$$

on solving - $x = 2a/3$ — Ans.

Q. Two equal positive charges each of 2 μC interact with a third positive charge of 3 μC situated as shown in fig. Calculate the magnitude and direction of the force on the 3 μC charge.



Sol.

Here $OA = OB = 3\,m$

and $OP = 4\,m$

So $AP = BP = \sqrt{3^2 + 4^2}$

$= 5\,m.$

According to Coulomb's Law

Force on charge at P due to charge at A and B are equal in magnitude so-

$$F_1 = F_2 = \frac{Kq_1q_2}{r^2} = \frac{9 \times 10^9 \times 2 \times 10^{-6} \times 3 \times 10^{-6}}{5^2}$$

So $F_1 = F_2 = 2{\cdot}16 \times 10^{-3}\ N$

Here the components of F_1 and F_2 along Y and Y' axis are equal in magnitude ($F_1 \sin\theta = F_2 \sin\theta$) but opposite in direction so they cancel each other

The Components along PX add up. So total force on charge 3 μC at P is -

$$F = 2F_1 \cos\theta = 2 \times 2{\cdot}16 \times 10^{-3} \times \frac{4}{5}$$

So $F = 3{\cdot}5 \times 10^{-3}\ N$ — Ans.

Electric Field -

An electric field is the region of space surrounding electrically charged particles where a force is exerted on other electrically charged objects.

* The direction of the field is taken to be the direction of the force it would exert on a positive test charge.

* The electric field is radially outword from a positive charge and radially inward towards a negative charge.

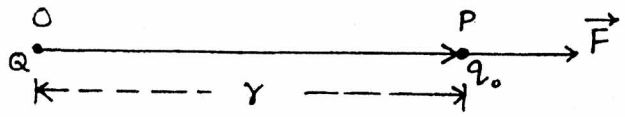
* Concept of Field was introduced first of all by "Faraday".

Electric Field Intensity ($\vec{E}$)

The electric field intensity at any point is the strength of electric field at that point. It is defined as the force experienced by unit Positive charge placed at that Point.

According to coulomb's Law force on test charge q_0 due to a point charge Q is

$$F = \frac{1}{4\pi\epsilon_0}\frac{Q \cdot q_0}{r^2}$$



by defintion $\boxed{\vec{E} = \frac{\vec{F}}{q_0}}$ so $\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0}\cdot\frac{Q}{r^2}\hat{r}}$

where $\hat{r}$ is unit vector directed from Q towards q_0.

SI Unit of $E = \frac{N}{C}$

Important Points about Electric field -

(1) The magnitude of electric field $\vec{E}$ is same at equal distances from the charge Q, thus it has a spherical symmetry.

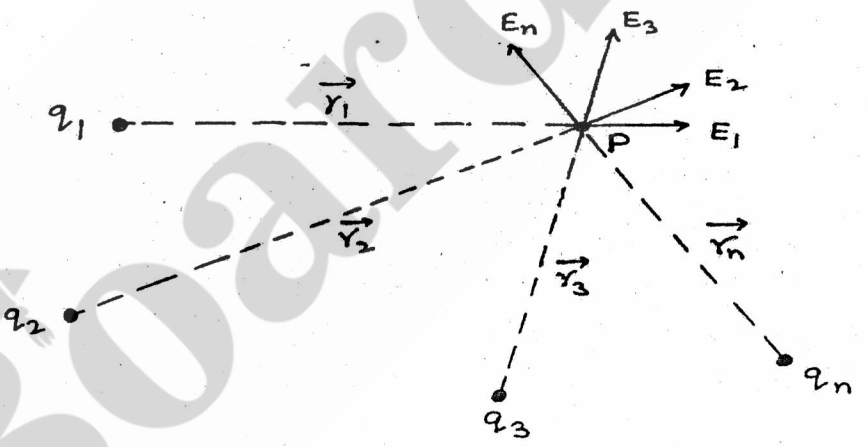
(2) The magnitude of the electric field $\vec{E}$ depends only on the charge Q and the distance r.

(3) The test charge q_0 may have its own electric field, it may modify the electric field of the source charge. Therefore to minimize this effect we rewrite electric field intensity at $\vec{r}$ as -

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$$

Electric Field due to a system of charges -

Consider a system of charges $q_1, q_2, \ldots q_n$ with position vectors $\vec{r}_1, \vec{r}_2, \ldots \vec{r}_n$.



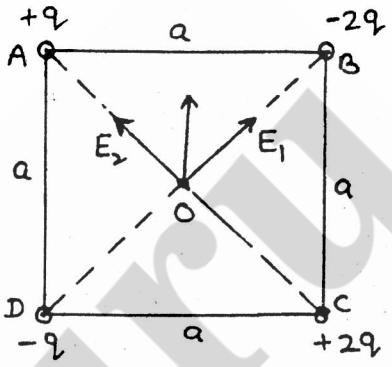
The electric field intensity at a point P due to a system of charges is the vector sum of the electric fields at the point due to individual charges.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots \vec{E}_n \qquad \vec{E} = \frac{Kq_1}{|\vec{r}_1|^2}\hat{r}_1 + \frac{Kq_2}{|\vec{r}_2|^2}\hat{r}_2 + \ldots \frac{Kq_n}{|\vec{r}_n|^2}\hat{r}_n$$

or

$$\vec{E} = K\sum_{i=1}^{n} \frac{q_i}{|\vec{r}_i|^2}\hat{r}_i$$

Q. What are the magnitude and direction of the electric field at centre of the square in figure. If $q = 10^{-8}C$ and $a = 5$ cm.



Sol. Here

$OA = OB = OC = OD = r$

$r = \frac{5 \times 10^{-2}}{\sqrt{2}}$ m.

also here $E_1 = E_2 = \frac{q}{4\pi\epsilon_0 r^2}$

or $E_1 = E_2 = \frac{10^{-8} \times 9 \times 10^9}{(5 \times 10^{-2}/\sqrt{2})^2} = 7.2 \times 10^4$ N/C

In square ABCD the $\angle AOB = 90°$

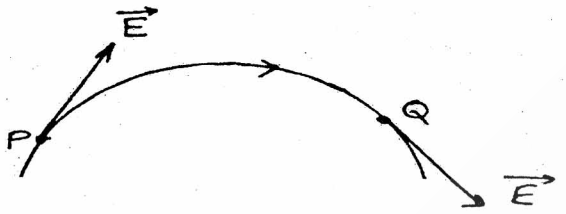
So the Net Electric field at O $= \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos 90°}$

$E = \sqrt{E_1^2 + E_2^2} = E_1\sqrt{2}$

$E = 7.2\sqrt{2} \times 10^4$ N/C at 45° to OA.

Electric Field Lines –

An electric field line is a curve drawn in such a way that the tangent to it at each point is in the direction of the net field at that point.

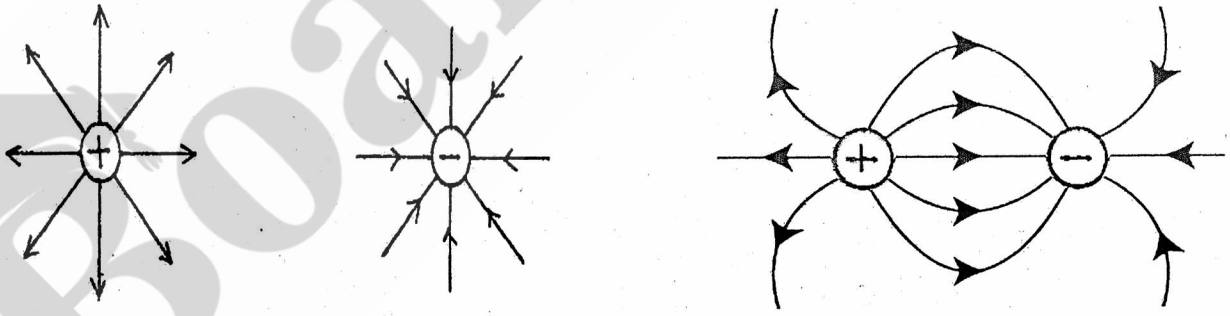


The magnitude of the field is represented by the density of field lines.

Hence Electric field intensity at a point is equal to number of field lines crossing normally a unit area around that point.

Properties of Electric Field Lines –

(1) Electric field lines are continuous curves. They start from a positively charged body and end at a negatively charged body.
If there is a single charge then the field lines may start or end at infinity.

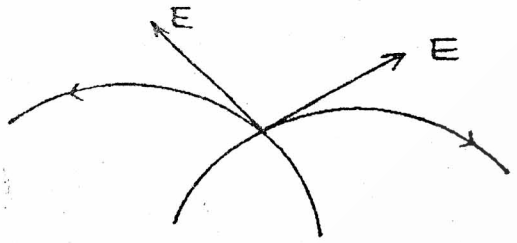


(2) No electric field lines exist inside the charged body. Thus electrostatic field lines do not form continuous closed loops.

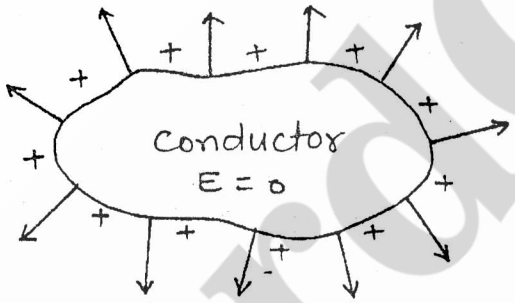
★ The magnetic field lines are continuous (or endless) closed loops as against the electric field lines.

(3) Tangent to the electric field line at any point gives the direction of electric field intensity at that point.

(4) No two electric field lines of force can intersect each other. This is because at the point of intersection P, we can draw two tangents PA and PB. This would mean two directions of electric field intensity at the same point, which is not possible.

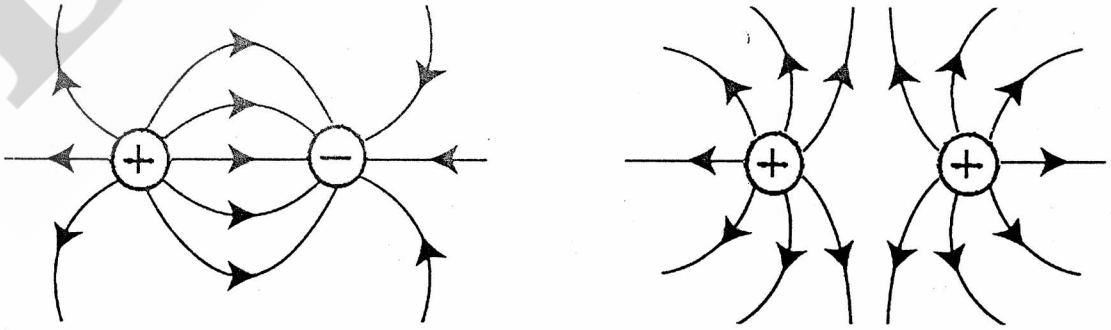


(5) The electric field lines are always normal to the surface of a conductor. So there is no component of electric field intensity parallel to the surface of the conductor.



(6) The field lines around a system of two positive charges (q, q) gives a pictorial representation of their mutual repulsion.

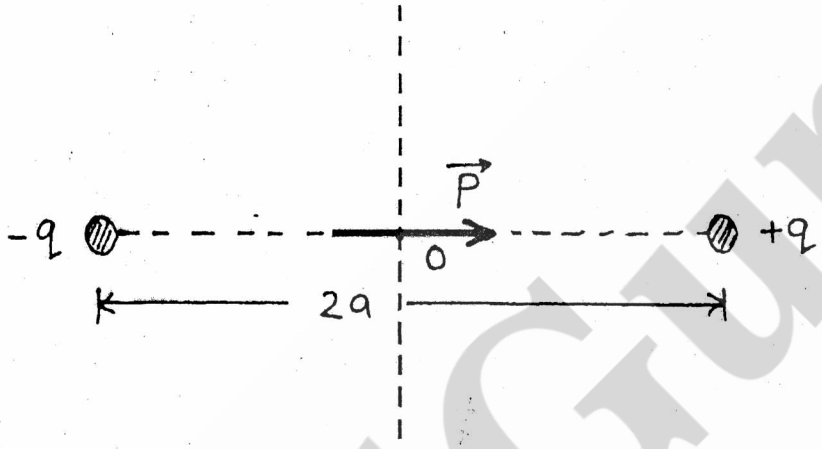
While around the configuration of two equal and opposite charges (q, -q), a dipole, shows the mutual attraction between the charges.

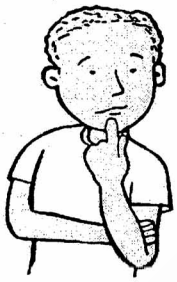


Electric Dipole -

An electric dipole is a pair of equal and opposite point charges separated by a short distance.

Let q and $-q$ are two equal and opposite point charges separated by a small distance '$2a$'.





* The total charge of the electric dipole is zero, but this does not mean that the field of the electric dipole is zero.
* But at distances much larger than the separation of the two charges forming a dipole ($r >> 2a$) then the field due to q and $-q$ nearly cancel out.

Dipole Moment ($\vec{P}$) -

Dipole moment is a measure of the strength of electric dipole. It is a vector quantity whose direction is from negative charge to positive charge.

Dipole moment is equal to the product of the magnitude of either charge and the distance between them.

$$\vec{P} = q \times 2a \, \hat{p}$$

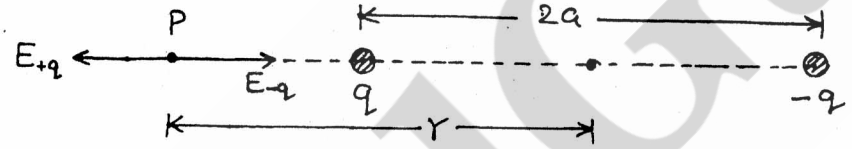
Dipole Field -

It is the space around the dipole in which the electric effect of the dipole can be experienced.

The electric field at any point P is obtained by adding the electric fields E_{-q} due to charge $-q$ and E_{+q} due to the charge q by the parallelogram Law of vectors.

(i) Field Intensity on Axial Line of Electric Dipole -

Let the point P be at distance r from the centre of the dipole on the side of the charge q, then



Here $E_{-q} = \frac{Kq}{(r+a)^2}$ and $E_{+q} = \frac{kq}{(r-a)^2}$

So the total Electric field at point P is -

$$E = E_{+q} - E_{-q} \qquad (E_{+q} > E_{-q})$$

$$E = Kq\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]$$

$$E = Kq \cdot \frac{4ar}{(r^2-a^2)^2}$$

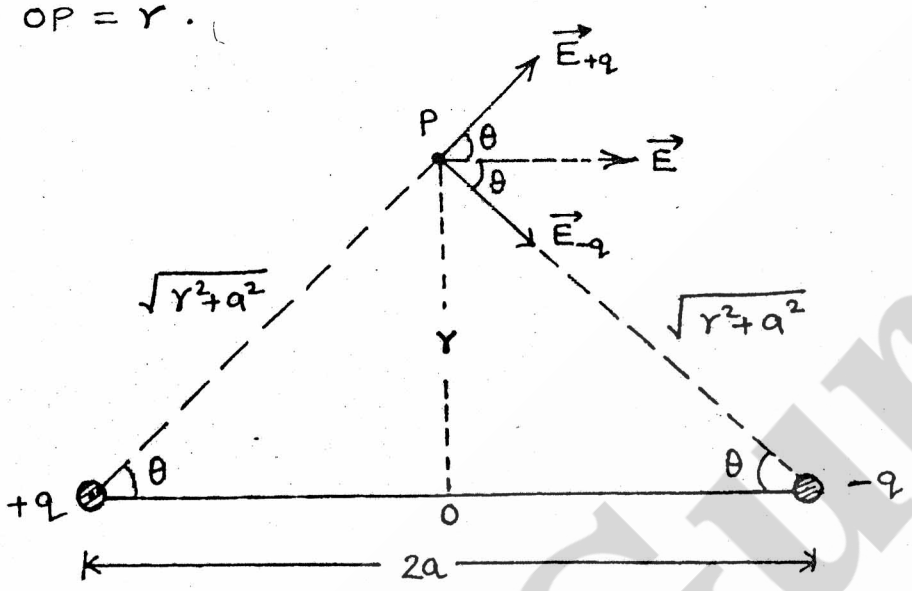
For $(r >> a)$ $E = \frac{4Kqa}{r^3}$ $\qquad \boxed{\vec{E} = \frac{2K\vec{P}}{r^3}}$

The direction of $\vec{E}$ is along $\vec{P}$ i.e from $-q$ to $+q$.

* $\boxed{E \propto \frac{1}{r^3}}$

(ii) Field Intensity on Equatorial Line of Electric Dipole -

Let the point P is on the equatorial line, where OP = r.



So the magnitudes of the electric fields due to the two charges q and -q are given as -

$$E_{+q} = \frac{Kq}{(r^2+a^2)} \quad \text{and} \quad E_{-q} = \frac{Kq}{(r^2+a^2)}$$

Here the components of E_{+q} and E_{-q} normal to the dipole axis cancel each other and the components along the dipole axis add up.

So the total electric field is in opposite direction to $\hat{P}$.

$$E = E_{+q}\cos\theta + E_{-q}\cos\theta$$

$$\text{or} \quad E = 2E_{+q}\cos\theta \quad \Rightarrow \quad E = \frac{K \cdot 2qa}{(r^2+a^2)^{3/2}}$$

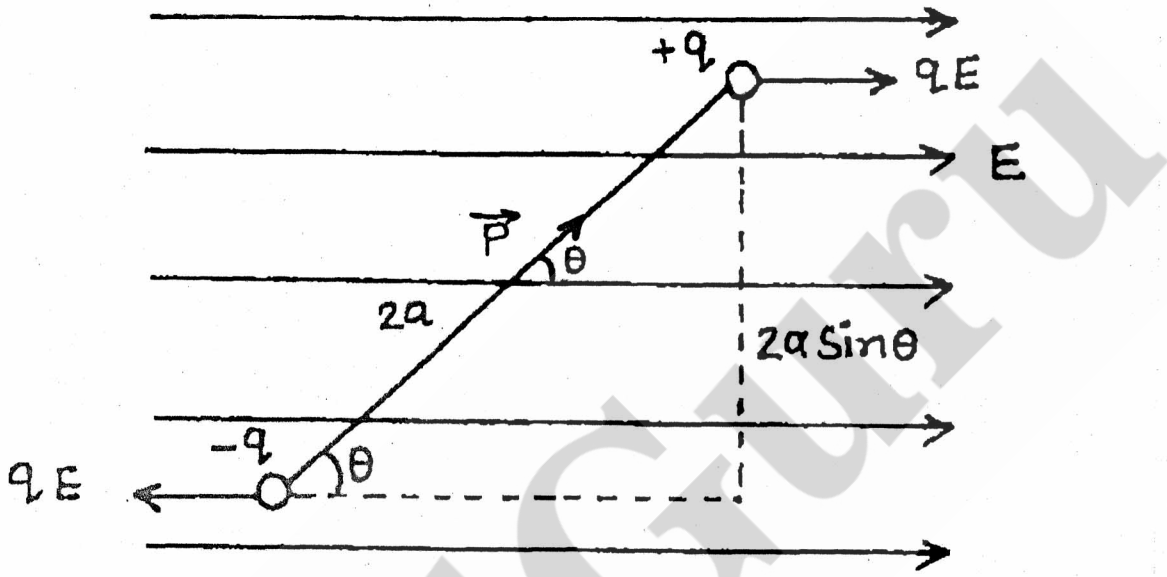
For $(r >> a)$ $\quad E = \dfrac{K \cdot 2qa}{r^3}$ $\qquad$ $\boxed{\vec{E} = \dfrac{-K\vec{P}}{r^3}}$

* Hence the dipole field at large distances falls of not as $1/r^2$ but as $1/r^3$.

* Due to an electric dipole $\boxed{E_{axial} = 2\,E_{equatorial}}$

Dipole in a uniform Electric Field -

Let a dipole be held in a uniform external electric field $\vec{E}$ at an angle θ with the direction of $\vec{E}$.



Net force on the dipole = zero

The forces on charge +q and -q resulting in a torque on the dipole which rotates the dipole.

Thus the torque tend to align the dipole axis along the direction of field $\vec{E}$.

The magnitude of torque is equals the magnitude of each force (qE) multiplied by the perpendicular distance between the two antiparallel forces.

$$\text{Torque} \quad \tau = qE \times 2a\sin\theta$$

$$\tau = pE\sin\theta$$

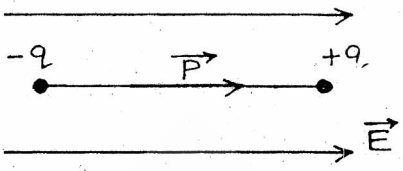
or $$\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$$

The direction of $\vec{\tau}$ is given by right handed screw rule and is perpendicular to $\vec{p}$ and $\vec{E}$.

* When $\vec{P}$ is along $\vec{E}$, $\theta = 0^\circ$

$\tau = PE\sin 0^\circ = 0$

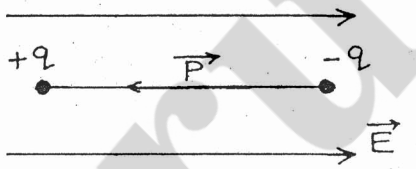
The dipole is in stable equilibrium.



* When $\vec{P}$ is along $\vec{E}$, $\theta = 180^\circ$

$\tau = PE\sin 180^\circ = 0$

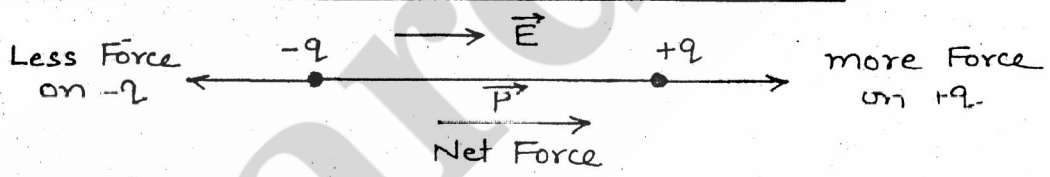
The dipole is in unstable equilibrium.



Dipole in a non-uniform Electric Field -

In either case, when $\vec{P}$ is parallel to $\vec{E}$ or antiparallel to $\vec{E}$, the net torque is zero, but there is a net force on the dipole if $\vec{E}$ is not uniform.

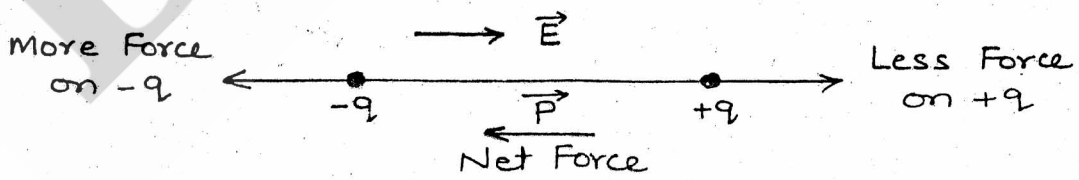
(a) When Field is increasing along $\vec{P}$



Net Force on dipole is along $\vec{E}$.

Net Torque $= PE\sin\theta = 0$ (as $\theta = 0^\circ$)

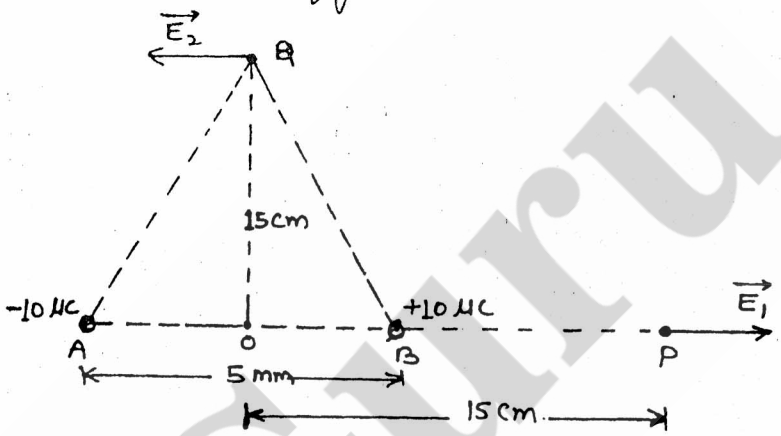
(b) When Field is decreasing along $\vec{P}$



Net Force on dipole is apposite to $\vec{E}$.

Net Torque $= PE\sin\theta = 0$ as $\theta = 0^\circ$

Q. Two charges ±10 μC are placed 5 mm apart. Determine the electric field at (a) a point P on the axis of dipole 15 cm. away from its centre on the side of the positive charge. (b) a point Q, 15 cm away from O on a line passing through O and normal to the axis of the dipole as shown in figure.



Sol:

Here $|\vec{P}| = q \times 2a$

$|\vec{P}| = 10 \times 10^{-6} \times 5 \times 10^{-3}$

$= 5 \times 10^{-8}$ C.m.

Also $2a << r$.

(a) As point P lies on axial line of dipole so -

$$E_1 = \frac{2KP}{r^3} = \frac{2 \times 9 \times 10^9 \times 5 \times 10^{-8}}{(0.15)^3}$$

so $E_1 = 2.67 \times 10^5$ N/C (along the direction of $\vec{P}$)

(b) As point Q lies on equatorial line of dipole so -

$$E_2 = \frac{KP}{r^3} = \frac{9 \times 10^9 \times 5 \times 10^{-8}}{(0.15)^3}$$

$E_2 = 1.33 \times 10^5$ N/C (opposite to the direction of $\vec{P}$)

Q. A system has two charges $q_A = 2.5 \times 10^{-7}$ C and $q_B = -2.5 \times 10^{-7}$ C located at points A: (0, 0, -15 cm) and B: (0, 0, 15 cm) respectively. What are the total charge and electric dipole moment of the system?

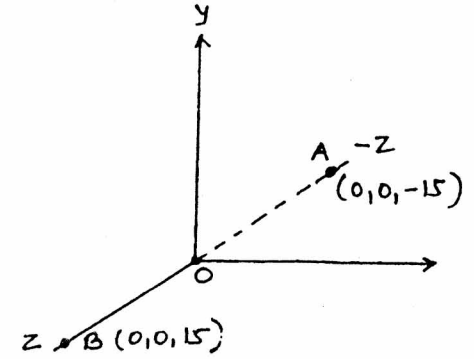
Sol:

Total charge of the system

$q = q_A + q_B$

$q = 2.5 \times 10^{-7}$ C $- 2.5 \times 10^{-7}$ C

$q = 0$



Electric dipole moment of the system is given by-

$$|\vec{P}| = q \times 2a = 2.5 \times 10^{-7} \times 0.30$$

so $|\vec{P}| = 7.5 \times 10^{-8}$ C.m (along Positive Z-axis).

Q. In a certain region of space, electric field is along the z-direction. The magnitude of electric field is not constant and increases uniformly along positive z-axis, at the rate of 10^5 N/c per metre. Calculate the Force and torque experienced by a system having a total dipole moment 10^{-7} C.m. in the negative z direction?

Sol. Dipole moment of the system $|\vec{P}| = q \times dL$

$$|\vec{P}| = q \times dL = -10^{-7} \text{ C.m.}$$

Rate of increase of electric Field per unit length,

$$\frac{dE}{dL} = 10^5 \text{ N/c per meter}$$

Force experienced by the system is given by

$$F = qE$$

or $$F = q \times dL \times \frac{dE}{dL}$$

$$F = -10^{-7} \times 10^{-5} = -10^{-2} \text{ N (along -Z axis)}$$

Here the angle between electric field and dipole moment is 180°.

So Torque $\tau = PE\sin\theta$

$$\tau = PE \sin 180^\circ = 0$$

BoardGuru

Charge Distribution -

There are two types of distribution of charge on a body-

(1) Discontinuous Charge Distribution -

In this distribution the charges are distributed randomly (non uniformly) in a body at different places.

(2) Continuous charge distribution -

In this distribution the charges are equally distributed in a body or object.

There are three types of continuous charge distribution -

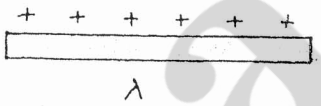
(a) Linear Charge Distribution -

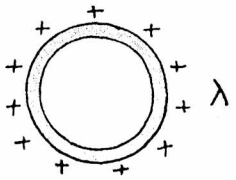
It is represented by linear charge density (λ).

$$\text{Linear charge density } (\lambda) = \frac{\text{Charge}}{\text{Length}} = \frac{\Delta Q}{\Delta l}$$

* SI. Unit → C/m.

Examples - Charged straight wire, Charged circular ring etc.





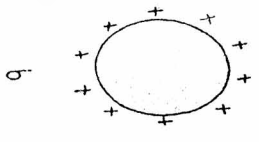
(b) Surface charge Distribution

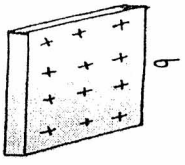
It is represented by surface charge density σ.

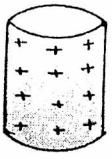
$$\text{Surface charge density } (\sigma) = \frac{\text{Charge}}{\text{Area}} = \frac{\Delta Q}{\Delta S}$$

SI Unit → C/m^2.

Example - Charged Plane sheet, Charge conducting sphare, cylinder etc.





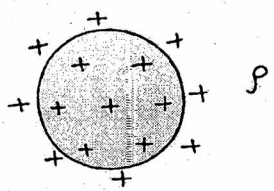


(c) Volume Charge Distribution

$$\text{Volume charge density } (\rho) = \frac{\text{Charge}}{\text{Volume}} = \frac{\Delta Q}{\Delta V}$$

SI Unit → C/m^3.

Examples – Charged insulating sphere etc.

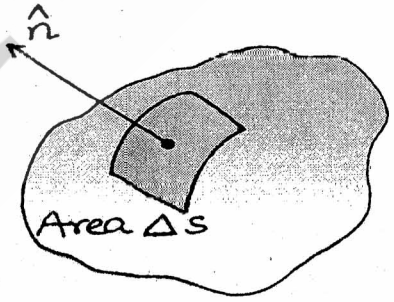


Area Vector –

By convention, the vector associated with every area element of a closed surface is taken to be in the direction of the outward normal.

$$\Delta\vec{S} = (\Delta S)\hat{n}$$

Here ΔS is magnitude of the area element and $\hat{n}$ is a unit vector in the direction of outward normal at that point.



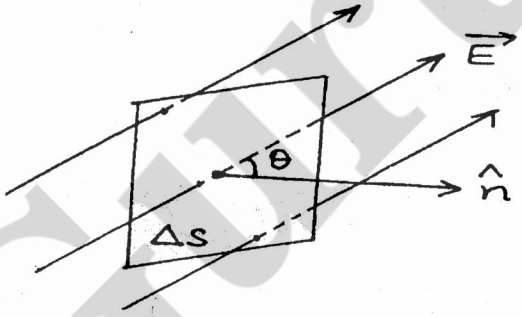
Electric Flux (ϕ) -

Electric flux over an area in an electric field represents the total number of electric field lines crossing this area normally.

The electric flux $\Delta\phi$ through an area element $\Delta\vec{S}$ in an electric field $\vec{E}$ is defined as -

$$\Delta\phi = \vec{E}\cdot\Delta\vec{S} = E(\Delta S)\cos\theta$$

where θ is the angle between $\vec{E}$ and $\Delta\vec{S}$



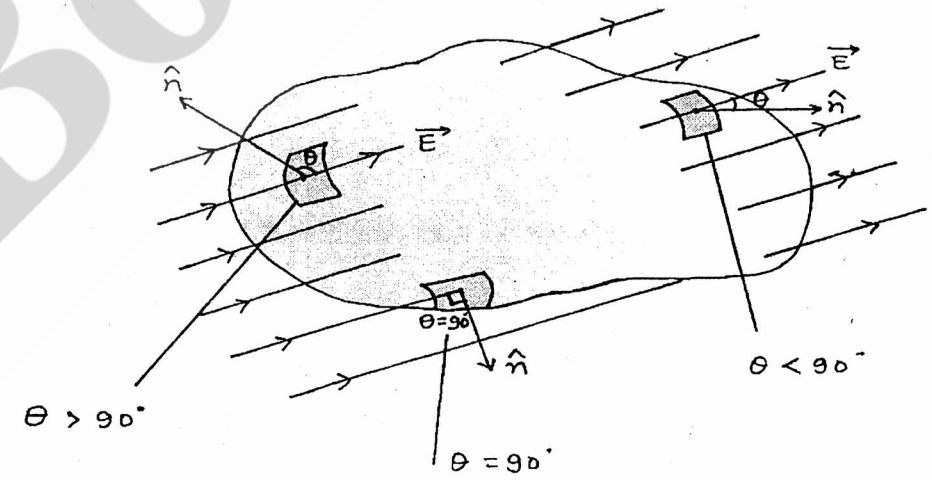
For a non uniform surface total electric flux is -

$$\phi = \oint \vec{E}\cdot\vec{ds} = \oint E\,ds\cos\theta$$

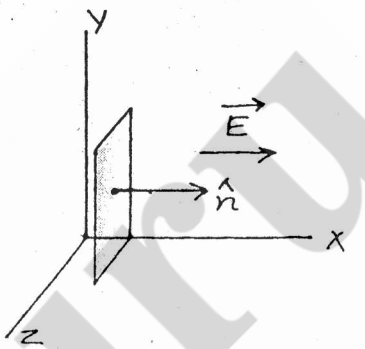
The circle on the integral sign indicates that the surface of integration is a closed surface.

★ Electric flux is a scalar quantity.

★ For a closed surface the flux comes outside from the surface is taken positive and the flux going inside to the surface is taken negative.



Q. Consider a uniform electric field $\vec{E} = 3\times10^3\hat{i}$ N/c (a) What is the flux of this field through a square of 10 cm. on a side whose plane is parallel to the YZ plane? (b) What is the flux through the same square if the normal to its plane makes a 60° angle with the x-axis?



Sol: Here Electric Field $\vec{E} = 3\times10^3\hat{i}$ N/c

Side of the square = 0.1 m.

Area of the square (A) = $(0.1)^2$

So $A = 0.01\ m^2$

(a)

When the plane of the square is parallel to the Y-Z plane then the angle between the area vector and electric field is $\theta = 0°$.

So Electric Flux $\phi = EA\cos\theta$

$\phi = 3\times10^3 \times 0.01 \times \cos 0° = 30\ Nm^2/c$

(b) When plane makes an angle of 60° with the x-axis then $\theta = 60°$.

So Electric flux $\phi = EA\cos\theta$

$= 3\times10^3 \times 0.01 \times \cos 60°$

$\phi = 15\ Nm^2/c$

Q. The electric field in a certain region of space is $(5\hat{i} + 4\hat{j} - 4\hat{k})\times10^5$ N/c. Calculate electric flux due to this field over an area of $(2\hat{i} - \hat{j})\times10^{-2}\ m^2$.

Sol: We Know that Electric Flux $\phi = \vec{E}\cdot\vec{ds}$

$\phi = (5\hat{i} + 4\hat{j} - 4\hat{k})\times10^5 \cdot (2\hat{i} - \hat{j})\times10^{-2}$

$\phi = (10 - 4)\times10^3$

$\phi = 6\times10^3\ Nm^2/c$

$\left[\begin{array}{l}\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1 \\ \hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0\end{array}\right.$

Gauss's Law -

The total electric flux over the closed surface in vacuum is $1/\epsilon_0$ times the total charge Q contained inside surface.

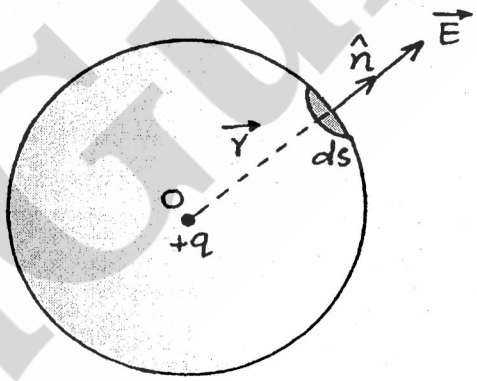
$$\phi = \oint_s \vec{E} \cdot \vec{ds} = \frac{Q}{\epsilon_0}$$

Proof of Gauss's Law -

Suppose an isolated positive point charge q is situated at the centre O of a sphere of radius r.

The electric field intensity at any point P on the surface of the sphere is

$$\vec{E} = \frac{Kq}{r^2}\hat{r}$$



where $\hat{r}$ is unit vector directed from O to P.

So the flux over the area element is -

$$d\phi = \vec{E} \cdot \vec{ds} = \frac{Kq}{r^2} \times ds \cos 0^\circ$$

or $$d\phi = \frac{Kq}{r^2} ds$$

So Total electric flux over the entire surface of sphere

$$\phi = \oint_s \vec{E} \cdot \vec{ds} = \frac{Kq}{r^2} \oint_s ds$$

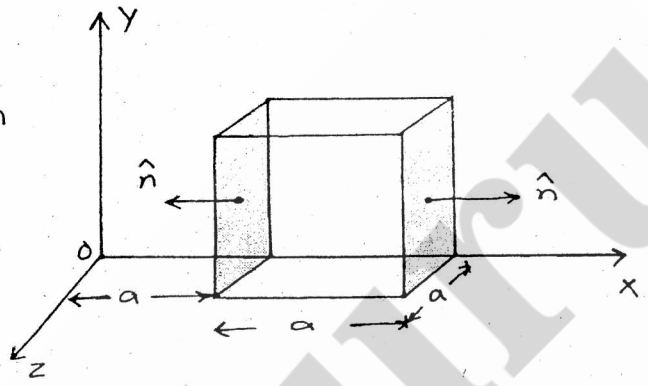
$$\phi = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \times 4\pi r^2 = \frac{q}{\epsilon_0} \quad \left(\oint_s ds = 4\pi r^2 \right)$$

Hence $$\phi = \oint_s \vec{E} \cdot \vec{ds} = \frac{q}{\epsilon_0}$$

It proves the Gauss's theorem.

Q. The electric field components in figure are $E_x = \alpha x^{1/2}$, $E_y = E_z = 0$ in which $\alpha = 800\ N/C\text{-}m^{1/2}$. Consider the cube as shown in fig and calculate –
(a) Total flux through the cube
(b) Charge within the cube. Assume that $a = 0.1\ m$.

Sol. Here $E_x = \alpha x^{1/2}$
$E_y = E_z = 0$,
$\alpha = 800\ N/C\text{-}m^{1/2}$, $a = 0.1\ m$



(a) The electric field has only x component, so
$\phi = \vec{E} \cdot \vec{ds} = 0$ for each of four faces of cube perpendicular to Y-Z plane.

At the left face $x = a$ so $E_L = \alpha a^{1/2}$

so $\phi_L = \vec{E_L} \cdot \vec{\Delta S} = \alpha a^{1/2} \times a^2 \cos 180^\circ = -\alpha a^{5/2}$

At the Right Face $x = a + a = 2a$ so $E_R = \alpha (2a)^{1/2}$

so $\phi_R = \vec{E_R} \cdot \vec{\Delta S} = \alpha (2a)^{1/2} \times a^2 \cos 0^\circ$

$\phi_R = \alpha a^{5/2} \sqrt{2}$

So Net flux through the cube $\phi = \phi_R + \phi_L$

$\phi = \alpha a^{5/2} \sqrt{2} - \alpha a^{5/2} = \alpha a^{5/2} (\sqrt{2} - 1)$

$\phi = 800 (0.1)^{5/2} (\sqrt{2} - 1) = 1.05\ N m^2/C$.

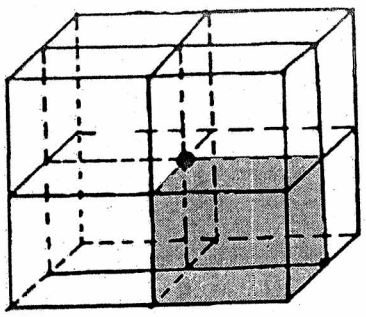
(b) By Gauss's theorem

$q = \epsilon_0 \phi$

so $q = 8.85 \times 10^{-12} \times 1.05 = 9.27 \times 10^{-12}\ C$.

Q. A charge of $2 \times 10^{-9}\ C$ is placed on a corner of a cube of side 1 m. Find the electric flux passing through this cube.

Sol.

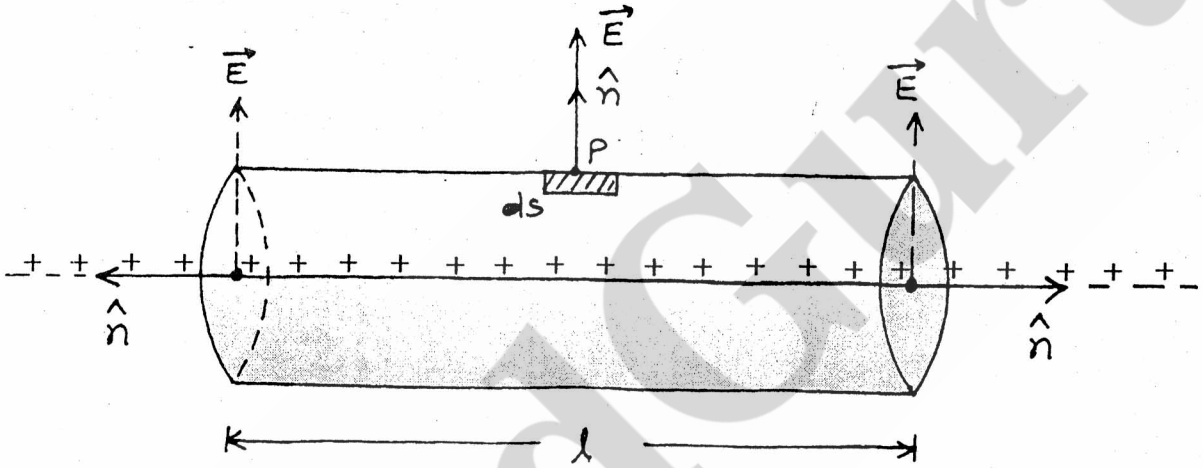


Application of Gauss's Theorem

(1) Field due to an infinitely Long straight Uniformly charged wire -

Consider an infinitely long thin wire with uniform linear charge density λ.

The electric field at every point in the plane cutting the wire normally is radial and its magnitude depends only on the radial distance r.



Now consider a circular closed cylinder of radius r and length l, with the infinitely long line of charge as its axis.

So the total electric flux over the surface -

$$\phi = \oint_S \vec{E}\cdot\vec{ds} = \int_1 E\,ds\cos 0^\circ + \int_2 E\,ds\cos 90^\circ + \int_3 E\,ds\cos 90^\circ$$

$$\phi = E\int_3 ds \quad \text{or} \quad \phi = E \times 2\pi r l$$

Charge inside the cylinder $(q) = \lambda l$

Now according to Gauss's theorem $\phi = \frac{q}{\epsilon_0}$

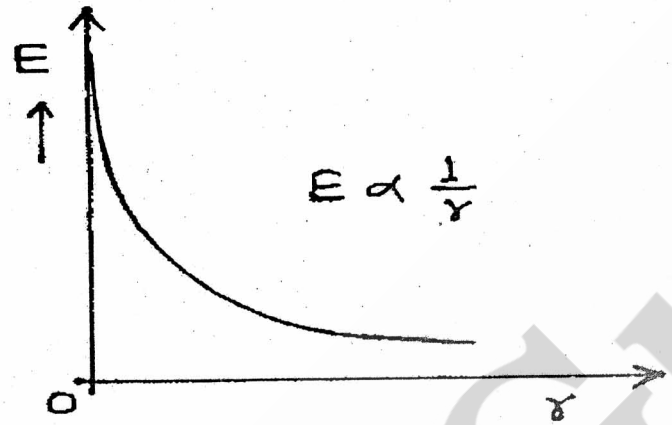
so $$E \times 2\pi r l = \frac{\lambda l}{\epsilon_0}$$

or $$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

or $$\boxed{\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r}\hat{n}}$$

Clearly $E \propto \frac{1}{r}$

- ★ If $\lambda > 0$, the direction of electric field at every point is radially outwards.
- ★ If $\lambda < 0$, the direction of electric field at every point is radially inwards.



Q. A plastic rod of length 2.2 m and radius 3.6 mm carries a negative charge of 3.8×10^{-7} C which is spread uniformly over its surface. What is the electric field near the mid point of the rod, on its surface?

Sol: Given that - $l = 2.2$ m.

$r = 3.6$ mm $= 3.6 \times 10^{-3}$ m., $q = -3.8 \times 10^{-7}$ C.

Linear charge density $\lambda = \frac{q}{l}$

$$\lambda = \frac{-3.8 \times 10^{-7}}{2.2} = -1.73 \times 10^{-7} \text{ C/m}$$

As $$E = \frac{\lambda}{2\pi \epsilon_0 r} = \frac{2\lambda}{4\pi \epsilon_0 r}$$

So $$E = \frac{2 \times 9 \times 10^9 \times (-1.73 \times 10^{-7})}{3.6 \times 10^{-3}}$$

or $$E = -8.6 \times 10^5 \text{ N/C}$$

(2) Electric Field Intensity due to a Uniformly Charged Spherical Shell -

(a) Electric Field Outside the shell -

Consider a thin spherical shell of radius R and centre o with charge +q distributed over the surface of the shell uniformly

To calculate electric field intensity at any point P, imagine a sphere (Gaussian surface) with centre o and radius r.

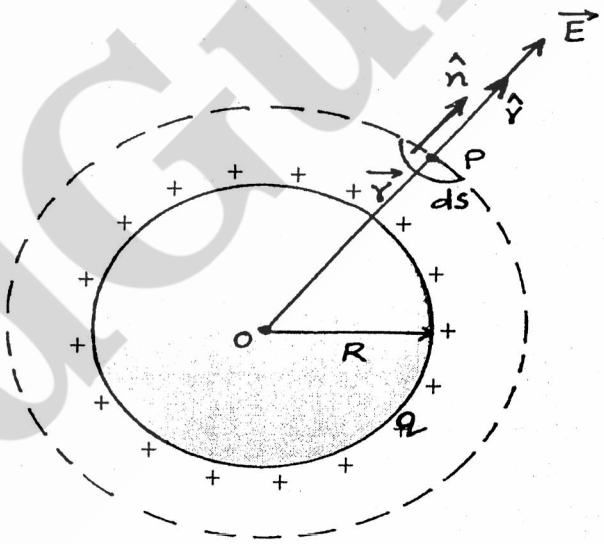
According to Gauss's Law

$$\phi = \oint_s \vec{E}.\vec{ds} = \frac{q}{\epsilon_0}$$

or $$\phi = \oint_s E\,ds\,\cos 0^\circ = \frac{q}{\epsilon_0}$$

$$E \oint_s ds = \frac{q}{\epsilon_0}$$

$$\boxed{E = \frac{q}{4\pi\epsilon_0 r^2}}$$



* If $q > 0$ then $\vec{E}$ is radially outwards and
* If $q < 0$ then $\vec{E}$ is radially inwards.

(b) At a point on the surface of the shell -

At the surface of the spharical shell

$r = R$ so $$\boxed{E = \frac{q}{4\pi\epsilon_0 R^2}}$$

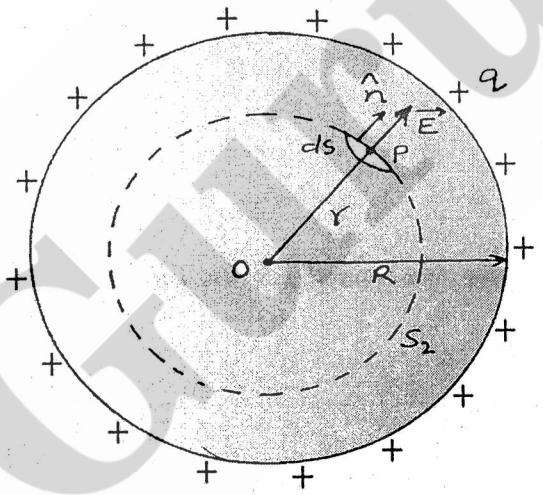
If σ is surface charge density on the shell then -

$$\boxed{E = \frac{\sigma}{\epsilon_0} = \text{Constant}} \qquad \left(\sigma = \frac{q}{4\pi R^2}\right)$$

(C) Electric Field inside the Shell -

To find field at point P inside the shell consider a Gaussian surface of a sphere S_2 passing through P and with centre at O. Here $r < R$.

As the charge inside the spherical shell is zero the Gaussian surface encloses no charge.



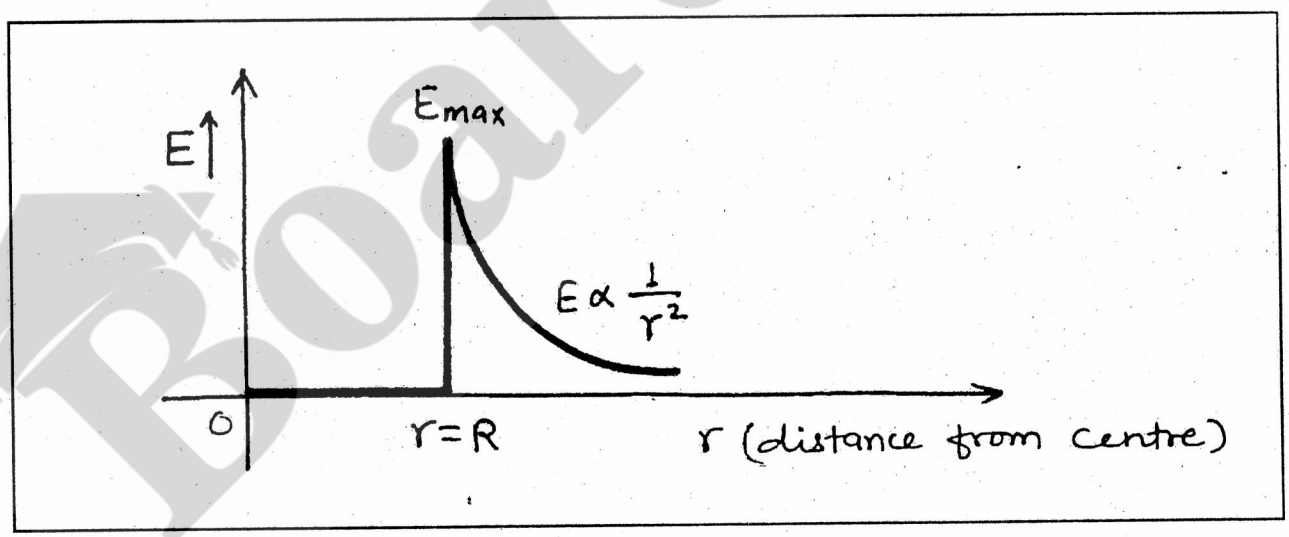
By Gauss's theorem-

$$\phi = \oint_S \vec{E} \cdot \vec{ds} = \frac{q}{\epsilon_0}$$

$$\phi = E \times 4\pi r^2 = 0$$

so $\boxed{E = 0}$ for $r < R$

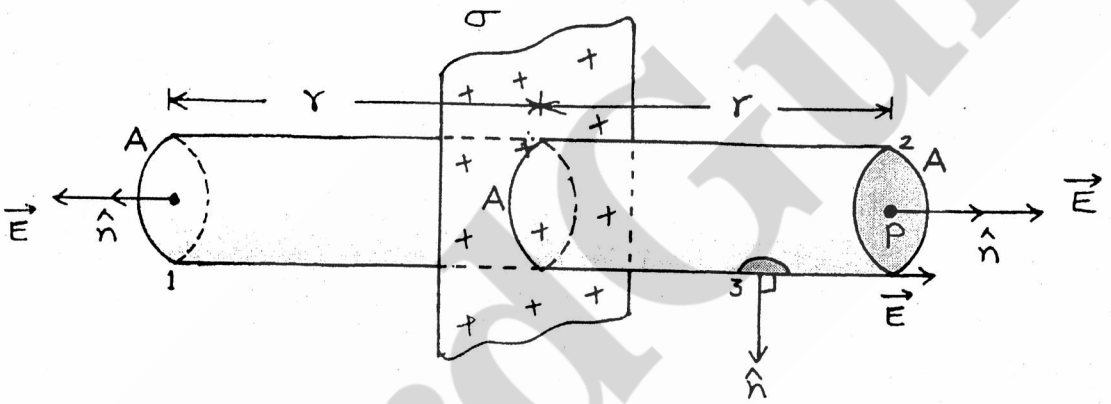
Hence the electric field inside a uniformly charged spherical shell is zero



(3) Electric Field intensity due to a thin infinite plane sheet of charge -

Consider a thin infinite plane sheet of charge with surface charge density σ.
We have to find the electric field at point P at a perpendicular distance r from the sheet.

Now imagine a cylinderical Gaussian surface of cross-sectional area A around P and length $2r$, passing through the sheet.



Total flux over the entire surface of cylinder is -

$$\phi = \int_1 \vec{E}\cdot d\vec{s} + \int_2 \vec{E}\cdot d\vec{s} + \int_3 \vec{E}\cdot d\vec{s}$$

$$\phi = \int_1 E\,ds\cos 0^\circ + \int_2 E\,ds\cos 0^\circ + \int_3 E\,ds\cos 90^\circ$$

$$\phi = 2EA$$

Total Charge enclosed by the cylinder = σA

According to Gauss's Law - $\phi = \frac{q}{\epsilon_0}$

$$2EA = \frac{\sigma A}{\epsilon_0} \qquad E = \frac{\sigma}{2\epsilon_0}$$

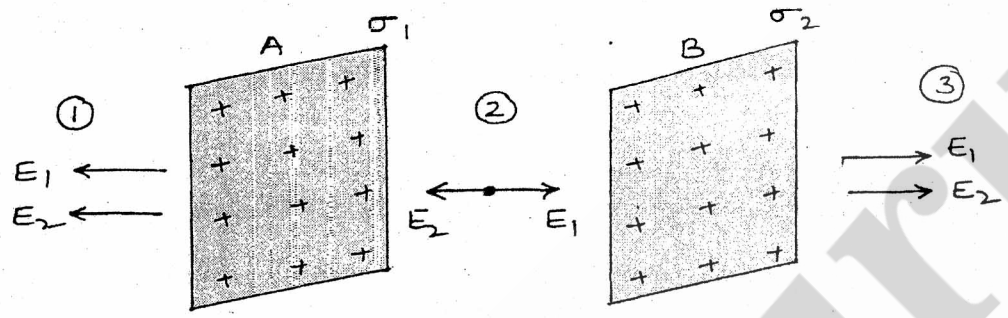
so $$\boxed{\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{n}}$$

★ E is independent of r.

★ If $\sigma > 0$ then electric field is normally outward and
★ If $\sigma < 0$ then electric field is normally inward to the sheet.

(4) Electric Field intensity due to two thin infinite parallel sheets of charge -

Case 1:- Here $\sigma_1 > \sigma_2 > 0$



In region ① & ③

$E = E_1 + E_2$

$E = \frac{\sigma_1}{2\epsilon_0} + \frac{\sigma_2}{2\epsilon_0}$

$$\boxed{E = \frac{1}{2\epsilon_0}(\sigma_1 + \sigma_2)}$$

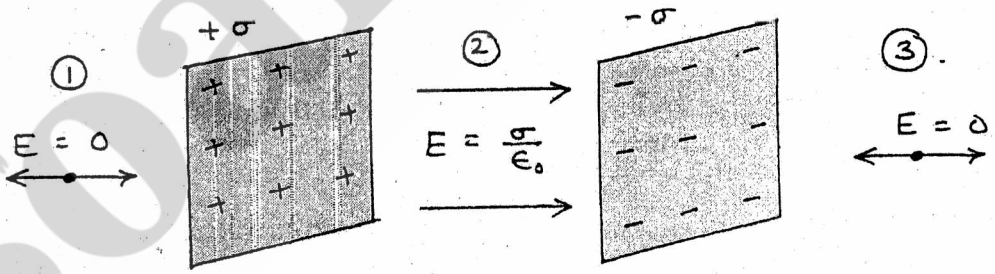
In region ②

$E = E_1 - E_2$

$E = \frac{\sigma_1}{2\epsilon_0} - \frac{\sigma_2}{2\epsilon_0}$

$$\boxed{E = \frac{1}{2\epsilon_0}(\sigma_1 - \sigma_2)}$$

Case 2:- Here $\sigma_1 = \sigma$ and $\sigma_2 = -\sigma$



In region ① & ③

$E = E_1 - E_2$

$E = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0}$

$$\boxed{E = 0}$$

In region ②

$E = E_1 + E_2$

$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0}$

$$\boxed{E = \frac{\sigma}{\epsilon_0}}$$

Q. A Large plane sheet of charge having surface charge density 5×10^{-6} C/m² lie in XY plane. Find the electric flux through a circular area of radius 0.1 m if the normal to the circular area makes an angle of 60° with the z-axis.

Sol. Here $\sigma = 5 \times 10^{-6}$ C/m², $r = 0.1$ m, $\theta = 60°$

So $\phi = \vec{E} \cdot \vec{ds} = E ds \cos\theta$

$$\phi = \left(\frac{\sigma}{2\epsilon_0}\right) \times \pi r^2 \cos\theta$$

$$\phi = \frac{5 \times 10^{-6}}{2 \times 8.85 \times 10^{-12}} \times 3.14 \times (0.1)^2 \cos 60°$$

$$\phi = 4.44 \times 10^3 \text{ Nm}^2/\text{C} \quad \text{— Ans.}$$

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Very Important Questions

1 Mark Questions -

Q.1. Which is bigger, a coulomb or charge on an electron? How many electronic charges form one coulomb of charge?
(Pb. Board 2011)

Q.2. What is the cause of charging ?
(Jharkhand Board 2011)

Q.3. What is ment by Quantization of charge ?
(CBSE 2005) (Bihar Board 2011)

Q.4. What is the value of charge on an electron? Is a charge less than this value possible ?
(Manipur CHSE 2011)

Q.5. Why no two electric lines of force can intersect each other.?
(Hr. Board 2007)

Q.6. Force of attraction between two point charges placed at a distance d is F. What distance apart should they be kept in the same medium so that force between them is F/3 ?
(CBSE 2011.)

Q.7. In Coulomb's Law, on what factors the value of electrostatic force constant K depends ?
(Pb. Board 2011)

Q.8. Define electric dipole moment. write its SI Unit.
(Jharkhand Board 2011, Raj Board 2011, CBSE 2011)

Q.9. In which orientation, a dipole placed in a uniform field is in (a) stable (b) Unstable equilibrium?
(CBSE 2011)

Q.10. Name the physical quantity whose SI Unit is N/C.
(Pb. Board 2011)

2 Marks Questions -

Q.1. Explain what is ment by dielectric constant?
(CBSE 2005)

Q.2. Define electric flux. Write its SI Unit. A charge q is enclosed by a spherical surface of radius R. If the radius is reduced to half, how would the electric flux through the surface change?
(Raj Board 2008, CBSE 2009, 12)

Q.3. A copper sphere of mass 2g. contains nearly 2×10^{22} atoms. The charge on the nucleus of each atom is 29 e. What fraction of the electrons must be removed from the sphere to give it a charge of $+2\,\mu C$?
Ans = 2.16×10^{-11}
(CBSE 2009)

Q.4. An area $5 \times 10^{-3}\hat{j}\ m^2$ is placed in an uniform electric field of $(200\hat{i} + 300\hat{j})$ N/C. Calculate the electric flux passing through this area.
(Raj. Board 2002)
Ans = 1.5 Nm^2/C

Q.5. The magnitude of entered and exerted electric flux from a close surface are 4×10^{12} Nm^2/C and 8×10^{12} Nm^2/C respectively. Calculate the charge enclosed by this surface.
Ans = 35.6 C
(Raj Board 2003)

Q.6. An electric dipole of dipole moment 20×10^{-6} cm is placed inside a closed surface. Calculate the net electric flux passing through this surface.
Ans. = Zero
(CBSE 2005, 06, 11)

Q.7. What do you mean by electric field lines? Give three main properties of electric field lines.
(Raj. Board 2006, CBSE 2005)

Q.8. Plot a graph showing the variation of coulomb force (F) versus $(1/r^2)$, where r is the distance between two charges of each pair of charges (1μC, 2μC) and (2μC, −3μC).
(CBSE 2011)

5 Marks Questions-

Q. 1. Using Gauss's Law, derive an expression for the electric field intensity at any point near a uniformly charged thin wire of charge/length = λ C/m.
(CBSE 2008, 5, 7, 9)

Q. 2. Derive an expression for dipole field intensity at any point on (a) axial line of dipole and (b) on equatorial line of dipole.
(CBSE 2004, 5, 11, 12)

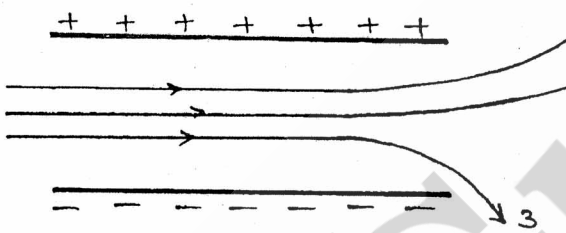
Q. 3. Derive an expression for electric field due to a uniformly charged infinite plane sheet using Gauss's Law.
(CBSE 2009, 12)

Q. 4. State Gauss's Law in electrostatics and derive an expression for electric field intensity at a point due to -
(a) a uniformly charged thin spherical shell (CBSE 3, 5, 7, 9, 11)
(b) two parallel sheets of charge with charge densities $+\sigma$ and $-\sigma$
(CBSE 2009, 6)

Q. 5. Derive an expression for torque acting on an electric dipole in a uniform electric field.
(Raj Board 2008, 11, CBSE 2005)

Q.1. An electric field line is a continuous curve. That is a field line cannot have sudden breaks. Why not?

Q.2. The figure shows tracks of three charges 1, 2 and 3 in a uniform electric field E. Give the signs of three charges. which particle has the highest charge to mass ratio?



Q.3. Two Large conducting spheres carrying charges Q_1 and Q_2 are brought close to each other. Is the magnitude of the electrostatic force between them exactly given by $F = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r^2}$. Where r is the distance b/w their centres?

Q.4. Why must electrostatic field must be normal to the surface at every point of a charged conductor?

Q.5. Define electric dipole moment. Is it a scalar or a vector quantity? what is its SI unit?

Q.6. An electric dipole of dipole moment 20×10^{-6} C.m is enclosed by a closed surface. What is the net electric flux coming out of this surface?

Ans. - Zero

Q.7. Define dielectric constant of a medium. What is the value of dielectric constant for a metal.

Q.8. Two charges $-2Q$ and $+Q$ are located at points $(a, 0)$ and $(4a, 0)$ respectively. What is the electric flux due to these charges through a sphere of radius '3a' with its centre at the origin?

Ans. - $\phi = -2Q/\epsilon_0$

Q.9. If the radius of the Gaussian surface enclosing a charge is halved, how does the electric flux through the Gaussian surface change?

Ans. - No change

Q.10. Two concentric metallic spherical shells of radii R and 2R are given charges Q_1 and Q_2 respectively. The surface charge densities on the outer surfaces of the shells are equal. Determine the ratio $Q_1 : Q_2$.

Ans. - $Q_1/Q_2 = 1/4$

Q.11. Two charges q and $-3q$ are placed fixed on x-axis separated by distance 'd'. Where should a third charge $2q$ be placed such that it will be in equilibrium?

Ans - $x = \frac{d}{2}(1+\sqrt{3})$

Q.12. How many types an object can be charged?

Q.13. In a HCl molecule the distance between H^+ and Cl^- ions is 1.28 A°. Calculate the electric dipole moment of the molecule.

Ans. - $p = 2.048 \times 10^{-29}$ C.M.

Q.14. An electron and proton are released from rest in a uniform electric field. Which of them will have larger acceleration?

Q.15. Guess a possible reason why water has a much greater dielectric constant (= 80) than mica (= 6).

PHYSICS

Unit - 2. Electric Potential and Capacitance

Class - 12th

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2. Electrostatic Potential & Capacitance

Introduction :-

The electrostatic potential of a charged body represents the degree of electrification of the body. It determines the direction of flow of charge between two charged bodies placed in contact with each other.

The charge always flows from a body at higher potential to another body at lower potential. The flow of charge stops as soon as the potentials of the bodies become equal.

Electrostatic Potential Energy

Let us assume an electric field $\vec{E}$ due to a charge $+Q$ placed at the origin.

Let a small test charge $+q$ be brought from a point A to a point B against the repulsive force on it due to charge $+Q$.

We assume that the external force $\vec{F}_{ext}$ applied is just sufficient to counter the repulsive electric force $\vec{F}_E$ on the test charge q so the net force on test charge q is zero and it moves from A to B without any acceleration.

Now the work done by external force is negative of work done by the electric force and gets fully stored in the charge q in the form of its potential energy.

Work done by external force in moving the test charge $+q$ from A to B is -

$$W_{AB} = \int_A^B \vec{F}_{ext} \cdot d\vec{r} = -\int_A^B \vec{F}_E \cdot d\vec{r}$$

This work done against electrostatic force gets stored as potential energy.

$$\Delta U = U_B - U_A = W_{AB}$$

Hence electrostatic potential energy difference between two points B and A is equal to the minimum work done by an external force in moving a test charge q from A to B without acceleration.

Note :-

The work done by an electrostatic field in moving a given charge from one point to another depends only on the positions of initial and final points. It does not depend on the path chosen in going from one point to the other.

* For a charge distribution of finite extent, we choose zero electrostatic potential energy at infinity.

Therefore, when point A is at infinity we get.

$$W_{\infty B} = U_B - U_\infty = U_B - 0 = U_B$$

Hence the electrostatic potential energy of a charge q at a point in an electric field due to any charge configuration is equal to the work done by the external force in bringing the charge q from infinity to that point without any acceleration.

Electrostatic Potential

The electric potential difference between two points B and A in an electric field is equat to the amount of work done in carrying unit positive test charge from A to B without acceleration along any path between the two points.

If V_A and V_B are the electrostatic potentials at A and B respectively then

$$\frac{U_B - U_A}{q} = \frac{W_{AB}}{q} = V_B - V_A = \Delta V$$

We can define electric potential at any point by choosing the potential to be zero at infinity. So if we take point A at infinity then.

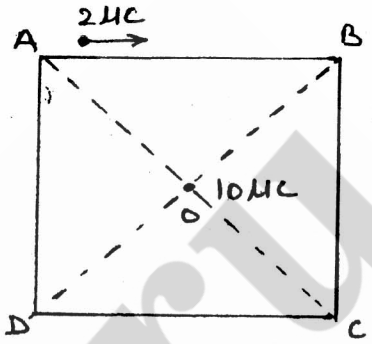
$$V_B = \frac{W_{\infty B}}{q}$$

* The electrostatic potential is a scalar quantity. Its SI unit is Volt.

1 Volt :- The electrostatic potential difference between any two points in an electrostatic field is said to be one volt when 1 Jule of work is done in moving a positive charge of one coulomb from one point to the other against the electrostatic force of the field

$$1\,V = \frac{1\,J}{1\,C} = 1\,JC^{-1} = 1\,NmC^{-1}$$

Q. What is work done in moving a 2 μC point charge from corner A to corner B of a square ABCD when a 10 μC charge exist at the centre of the square.



Sol. As here $\boxed{V_A = V_B}$

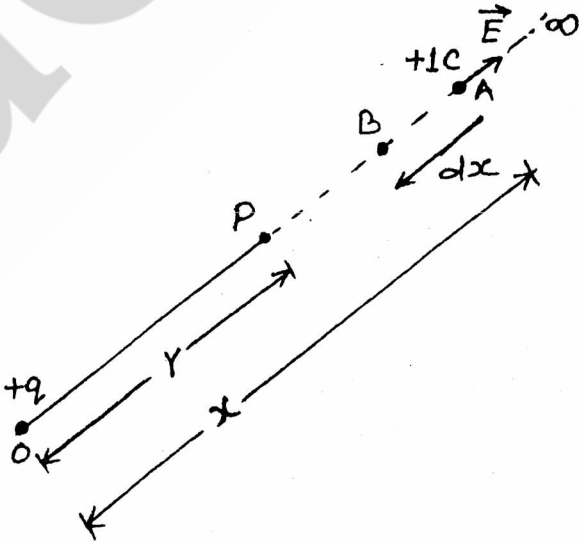
So the work done -

$$W = q(V_B - V_A)$$

$$W = \text{zero}$$

Electrostatic Potential due to a Point Charge

Suppose we have to calculate electrostatic Potential at any point P due to a single point charge +q at O, where OP = r.



By definition electrostatic potential at P is the amount of work done in carrying a unit positive charge from ∞ to P.

At some intermediate point A on this path, where OA = x, the electrostatic force on unit positive charge is -

$$F = qE = E = \frac{q}{4\pi\epsilon_0 x^2} \quad \text{(along OA)}$$

Now small amount of work done in moving a unit positive charge from A to B where $\vec{AB} = \vec{dx}$ is

$$dW = \vec{E}.\vec{dx} = E\,dx\cos 180^\circ = -E\,dx$$

∴ Total work done in moving positive unit charge from ∞ to the point P is -

$$W = \int_{\infty}^{r} -E\,dx = \int_{\infty}^{r} -\frac{1}{4\pi\epsilon_0}\frac{q}{x^2}\,dx$$

$$W = -\frac{q}{4\pi\epsilon_0}\int_{\infty}^{r} x^{-2}\,dx = -\frac{q}{4\pi\epsilon_0}\left[-\frac{1}{x}\right]_{\infty}^{r}$$

So $$W = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r} - \frac{1}{\infty}\right] = \frac{q}{4\pi\epsilon_0 r}$$

So by definition the electrostatic potential at P due to charge +q is -

$$\boxed{V = W = \frac{q}{4\pi\epsilon_0 r}}$$

* When q is positive, the electrostatic potential V is positive and when q is negative, electrostatic potential is negative.
* Electrostatic potential due to a single charge is spherically symmetric.

$$\boxed{V \propto \frac{1}{r}}$$

$$\boxed{E \propto \frac{1}{r^2}}$$

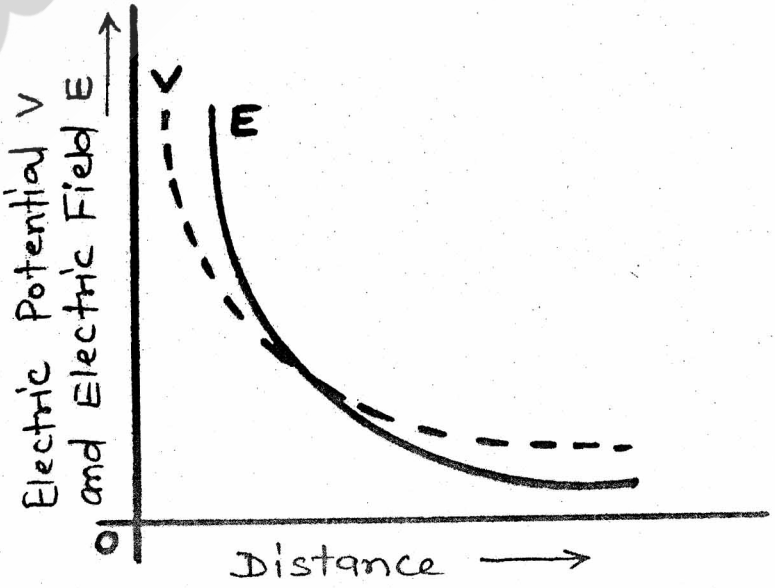


Fig:- Variation of Electrostatic Potential and Electric field with distance.

Q. What is the electrostatic potential at the surface of a gold nucleus of diameter 13.2 fermi. Z for gold is 79.

Sol. Here $r = \frac{13.2}{2} = 6.6$ fermi

or $r = 6.6 \times 10^{-15}$ m. and $q = ne$

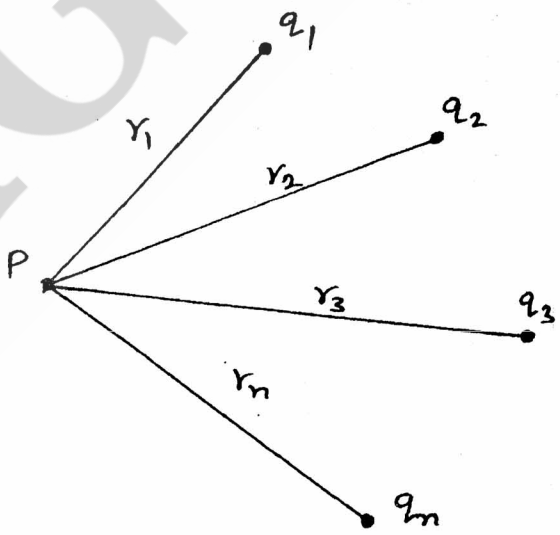
so $q = 79 \times 1.6 \times 10^{-19}$.

so $V = \frac{q}{4\pi\epsilon_0 r} = \frac{9 \times 10^9 \times 79 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-15}}$

$V = 1.7 \times 10^7$ Volt.

Potential at a Point due to group of Electric Charges :-

Suppose there are a number of point charges $q_1, q_2, q_3, \ldots q_n$ at distances $r_1, r_2, r_3, \ldots r_n$ respectively from the point P, where electric potential is to be calculated.



The potential at P due to various charges are -

$V_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1}$, $V_2 = \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_2}$

$V_3 = \frac{1}{4\pi\epsilon_0}\frac{q_3}{r_3}$ and so on $V_n = \frac{1}{4\pi\epsilon_0}\frac{q_n}{r_n}$

The resultant potential at P is

$$V = V_1 + V_2 + V_3 + \ldots V_n$$

$$V = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \ldots \frac{q_n}{r_n}\right)$$

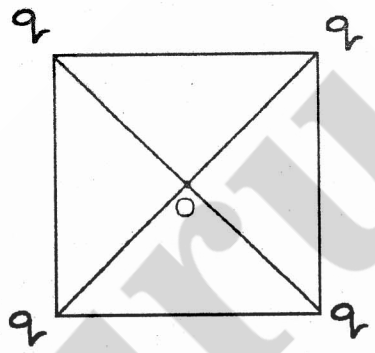
or

$$\boxed{V = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}}$$

Q. A cube of side b has a charge q at each of its vertices. Determine the potential and electric field due to this charge array at the centre of the cube.

Sol. The length of diagonal of the cube of each side b is = $\sqrt{3}\,b$

∴ Distance between centre of cube and each vertex is = $r = \sqrt{3}\,b/2$



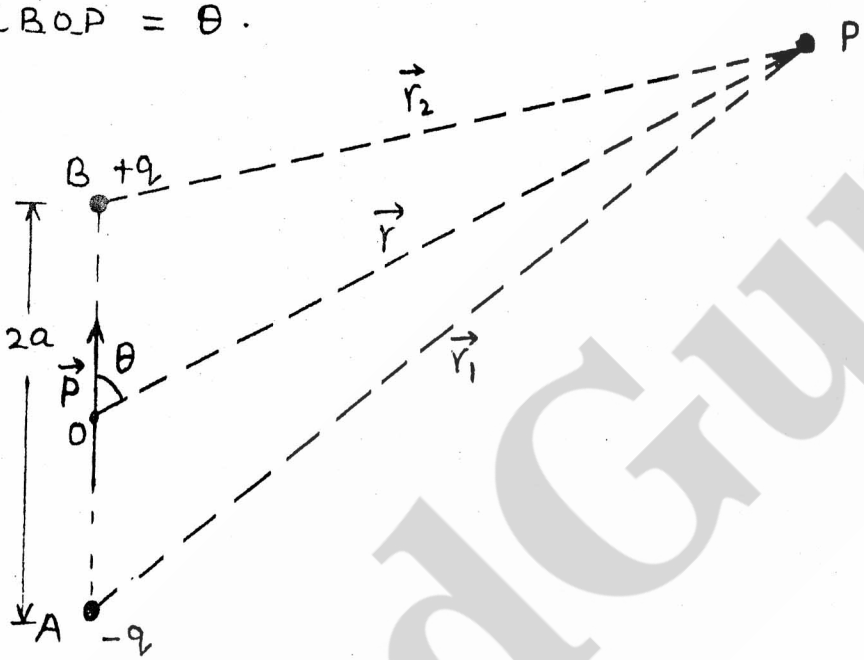
Here 8 charges each of value q are present at the 8 vertices of the cube, therefore total potential is

$$V = V_1 + V_2 + \cdots V_8 = \frac{1}{4\pi\epsilon_0}\frac{8q}{\sqrt{3}b/2} = \frac{4q}{\sqrt{3}\pi\epsilon_0 b}$$

Further electric field at the centre due to all the eight charges is zero, because the field due to individual charges cancel in pairs.

Electrostatic Potential at a point due to an Electric Dipole :-

Let us take the origin at the centre of the dipole. Now we have to calculate electric potential at point P where $\overrightarrow{OP} = \vec{r}$ and $\angle BOP = \theta$.



Let the distance of P from charge $-q$ at A be r_1 and distance of P from charge $+q$ at B is r_2.

Electrostatic potential at P due to $-q$ and $+q$ charge is respectively

$$V_1 = \frac{-q}{4\pi\epsilon_0 r_1} \quad \text{and} \quad V_2 = \frac{q}{4\pi\epsilon_0 r_2}$$

So the potential at P due to the dipole is -

$$V = V_1 + V_2$$

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right] \quad \text{---} \quad (1)$$

Now by geometry -

$$\cos\theta = \frac{a^2 + r^2 - r_2^2}{2ra} \quad \text{(for } \triangle BOP\text{)}$$

or $$r_2^2 = r^2 + a^2 - 2ra\cos\theta$$

Similarly $\cos(180^\circ - \theta) = \frac{a^2 + r^2 - r_1^2}{2ra}$ (for triangle AOP)

or $r_1^2 = r^2 + a^2 + 2ra\cos\theta$

Here we may write -

$$r_1^2 = r^2\left(1 + \frac{a^2}{r^2} + \frac{2a}{r}\cos\theta\right)$$

If $a << r$ then $\frac{a}{r}$ is small and $\frac{a^2}{r^2}$ can be neglected.

$$\therefore \quad r_1^2 = r^2\left(1 + \frac{2a}{r}\cos\theta\right)$$

$$r_1 = r\left(1 + \frac{2a}{r}\cos\theta\right)^{1/2}$$

and $$\frac{1}{r_1} = \frac{1}{r}\left(1 + \frac{2a}{r}\cos\theta\right)^{-1/2}$$

Similarly $$\frac{1}{r_2} = \frac{1}{r}\left(1 - \frac{2a}{r}\cos\theta\right)^{-1/2}$$

so by equation (1) -

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r}\left(1 - \frac{2a}{r}\cos\theta\right)^{-1/2} - \frac{1}{r}\left(1 + \frac{2a}{r}\cos\theta\right)^{-1/2}\right]$$

Using Binomial theorem we get -

$$V = \frac{q}{4\pi\epsilon_0 r}\left[\left(1 + \frac{a}{r}\cos\theta\right) - \left(1 - \frac{a}{r}\cos\theta\right)\right]$$

$$V = \frac{q \times 2a\cos\theta}{4\pi\epsilon_0 r^2}$$

$$V = \frac{P\cos\theta}{4\pi\epsilon_0 r^2} \qquad [\text{As } P = q \times 2a]$$

As $P\cos\theta = \vec{P}\cdot\hat{r}$, where $\hat{r}$ is unit vector along the position vector $\vec{OP} = \vec{r}$

So the electrostatic potential at P due to a dipole is -

$$\boxed{V = \frac{\vec{P}\cdot\hat{r}}{4\pi\epsilon_0 r^2}}$$

(i) On the dipole axis ($\theta = 0^\circ$ or π)

$$V = \pm \frac{P}{4\pi \epsilon_0 r^2}$$

Positive sign for $\theta = 0^\circ$ and negative sign for $\theta = \pi$.

(ii) On the equatorial plane ($\theta = \frac{\pi}{2}$; $\cos\theta = \cos\frac{\pi}{2} = 0$)

$$V = 0$$

So Electrostatic potential at any point in the equatorial plane of dipole is zero.

* The potential due to a dipole is axially symmetric about $\vec{P}$.

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Equipotential Surfaces

That surface at every point of which electric potential is the same is called an equipotential surface.

No work is done on moving the test charge from one point to the other on equipotential surface.

The equipotential surface through a point is normal to the electric field at that point.

* The equipotential surface of a single point charge are concentric spherical surfaces centred at the charge.
* For a uniform electric field say along x-axis, the equipotential surfaces are planes normal to the x axis.

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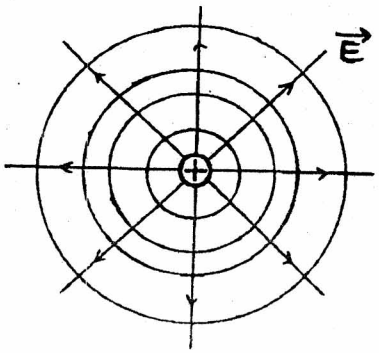
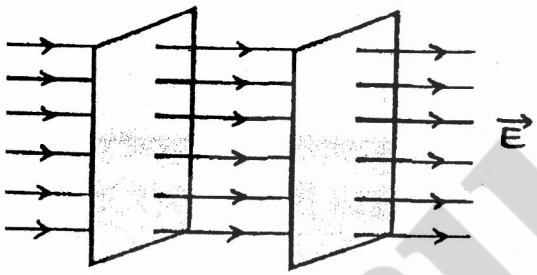
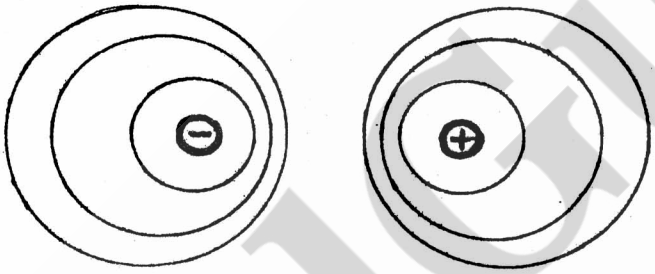
Some equipotential surfaces are shown in figures -



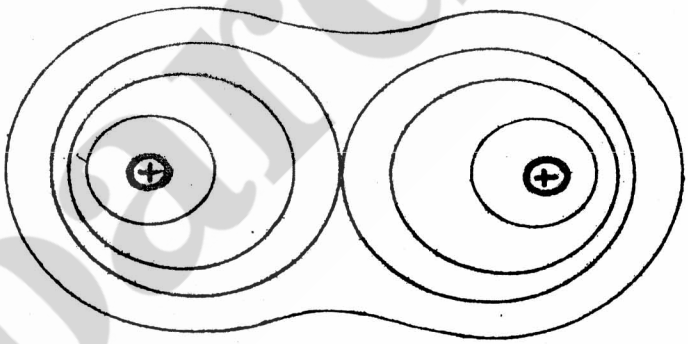
(a) For a point charge



(b) For uniform electric field.



(c) For an electric dipole.



(d) For two identical Positive charges

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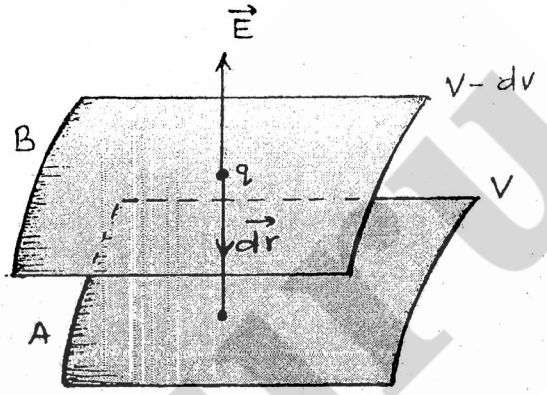
Relation between Electric Field and Electric Potential :-

Let us consider two equipotential surfaces A and spaced closely at a perpendicular distance $d\vec{r}$.

Let the potential of A be $V_A = V$ and potential of be $V_B = V - dV$ where dV is decrease in potential in the direction of electric field $\vec{E}$ normal to A and B.



A unit positive charge ($q = +1$) is taken along this perpendicular distance from the surface B to the surface A against the electric field. So work done

$$W_{BA} = \vec{F} \cdot d\vec{r}$$

$$W_{BA} = -E\,dr \qquad (\text{as } \vec{F} = q\vec{E})$$

$$W_{BA} = V_A - V_B = V - (V - dV) = dV$$

Hence $$-E\,dr = dV$$

or $$\boxed{E = -\frac{dV}{dr}} \quad \ldots\ldots$$

Negative sign shows that the direction of electric field $\vec{E}$ is in the direction of decreasing potential.

* The magnitude of electric field is given by change in magnitude of potential per unit displacement normal to the equipotential surface at the point. This is called potential gradient.

$$|\vec{E}| = -\frac{|dV|}{dr} = -(\text{Potential gradient})$$

Electrostatic Potential Energy of a System of charges :-

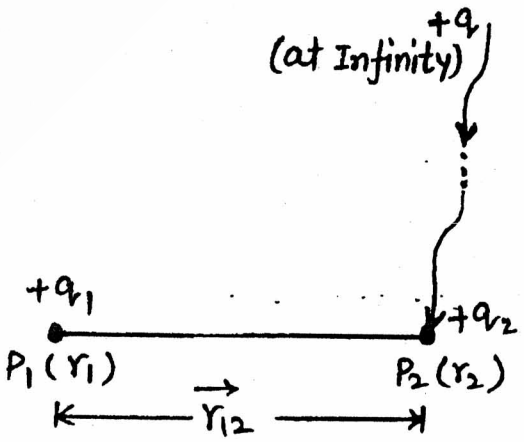
Electrostatic potential energy of a system of point charges is the total amount of work done in bringing the various charges to their respective positions from infinitely large mutual separation.

(i) For a system of two point charges -

Suppose a point charge q_1 is held at a point P_1 with position vector $\vec{r}_1$ in space. Another point charge q_2 is at infinite distance from q_1. This is to be brought to the position P_2 with position vector $\vec{r}_2$.

In bringing the first charge q_1 to position $P_1(\vec{r}_1)$ no work is done, because all other charges are still at infinity, and there is no field.

i.e. $\boxed{W_1 = 0}$



Now, work done in carrying charge q_2 from ∞ to P_2

$W_2 = [\text{Potential due to } q_1] \times q_2$

$$W_2 = \frac{q_1}{4\pi\epsilon_0 r_{12}} \times q_2$$

Total work done $W = W_1 + W_2$

$$\boxed{\text{Electrostatic potential energy } U = W = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}}$$

(ii) For a system of N point charges -

It is equal to the total amount of work done in assembling all the charges at the given positions from infinity.

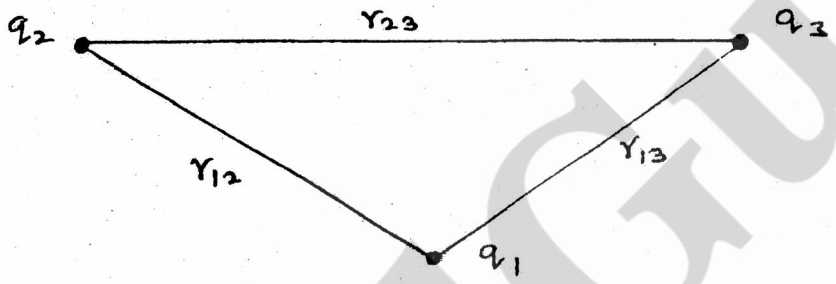
In bringing the first charge q_1 to position $P_1(\vec{r}_1)$ no work is done.

i.e. $W_1 = 0$

When we bring charge q_2 from infinity to $P_2(\vec{r}_2)$ at a distance r_{12} from q_1, work done is

$$W_2 = [\text{Potential due to } q_1] \times q_2$$

$$W_2 = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}$$



In bringing q_3 from ∞ to $P_3(\vec{r}_3)$, work has to be done against electrostatic forces due to both q_1 and q_2

$\therefore \quad W_3 = [\text{Potential due to } q_1 \text{ and } q_2] \times q_3$

$$W_3 = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right) \times q_3$$

$\therefore$ Electrostatic potential energy of a system of three charges

$$U = W_1 + W_2 + W_3 = 0 + \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}} + \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

$$\boxed{U = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)}$$

Proceeding in this way, we can write electrostatic potential energy of a system of N point charges at $\vec{r}_1, \vec{r}_2, \ldots \vec{r}_N$ as -

$$\boxed{U = \frac{1}{4\pi\epsilon_0}\sum_{\text{All Pairs}} \frac{q_j q_k}{r_{jk}}}$$

Potential Energy of a Single charge in an external Field :-

If $V(\vec{r})$ is external potential at any point P of position vector $\vec{r}$, then work done in bringing a charge q from infinity to the point P in the external field is $= q \cdot V(\vec{r})$

This work is stored in the charged particle in the form of its potential energy.

$\therefore$ **Potential energy of a charge q is $(U) = q \cdot V(\vec{r})$**

Potential Energy of a System of two Charges in an external Field :-

Suppose q_1 and q_2 are two point charges at position vectors $\vec{r}_1$ and $\vec{r}_2$ respectively in a uniform external electric field $\vec{E}$.

Now work done in bringing charge q_1 and q_2 from infinity to position $\vec{r}_1$ and $\vec{r}_2$ are

$$W_1 = q_1 \cdot V(\vec{r}_1) \quad \text{and} \quad W_2 = q_2 \cdot V(\vec{r}_2)$$

While bringing q_2 from infinity to position $\vec{r}_2$, work has also to be done against the field due to q_1. This is

$$W_3 = \frac{q_1 q_2}{4\pi \epsilon_0 r_{12}}$$

where r_{12} is the distance between q_1 and q_2.

So Potential energy of the system = total work done

$$U = W_1 + W_2 + W_3$$

$$U = q_1 \cdot V(\vec{r}_1) + q_2 \cdot V(\vec{r}_2) + \frac{q_1 q_2}{4\pi \epsilon_0 r_{12}}$$

Q. (a) Determine the electrostatic potential energy of a system consisting of two charges 7 μC and −2 μC (with no external field) placed at (−9 cm, 0, 0) and (9 cm, 0, 0) respectively.
(b) How much work is required to separate the two charges infinitely away from each other?
(c) Suppose that the same system of charges is now placed in an external field $E = A \times \frac{1}{r^2}$, where $A = 9 \times 10^5$ C/m². What would be the electrostatic energy of the configuration?

Sol.

(a) $U = \frac{q_1 q_2}{4\pi\epsilon_0 r} = \frac{7 \times 10^{-6} \times (-2 \times 10^{-6}) \times 9 \times 10^9}{(9+9) \times 10^{-2}} = -0.7 \text{ J}$

(b) Work required $= U_2 - U_1 = 0 - U = 0.7 \text{ J}$.

(c) In the electric field total P.E. $= \frac{q_1 q_2}{4\pi\epsilon_0 r} + q_1 V(\vec{r}_1) + q_2 V(\vec{r}_2)$

from $E = -\frac{dV}{dr}$; $V = \int -E\,dr = \int -\frac{A}{r^2} dr = \frac{A}{r}$

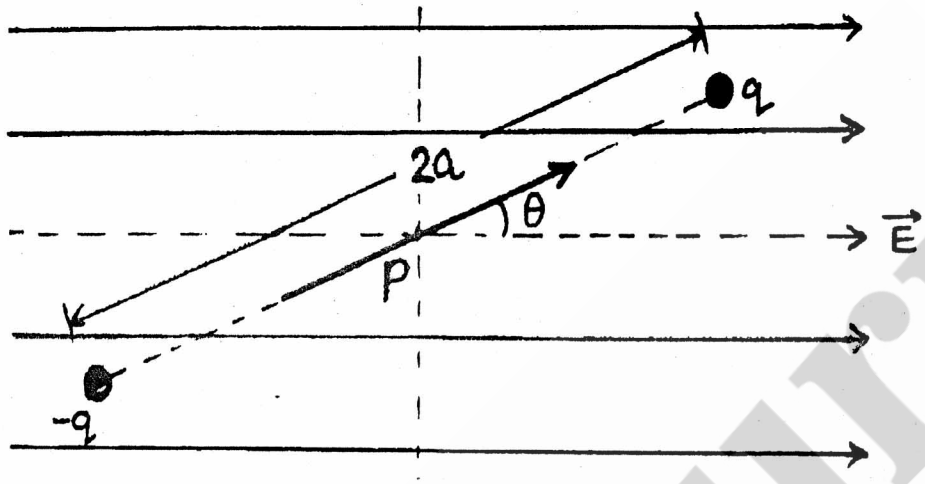
$\therefore$ Total P.E. $= \frac{q_1 q_2}{4\pi\epsilon_0 r} + \frac{A q_1}{r_1} + \frac{A q_2}{r_2}$

$= -0.7 + \frac{9 \times 10^5 (7 \times 10^{-6})}{0.09} + \frac{9 \times 10^5 (-2 \times 10^{-6})}{0.09}$

So Total P.E. $= -0.7 + 70 - 20 = 49.3 \text{ J}$

Potential Energy of a dipole in an external Field :-

Consider a dipole placed in a uniform electric field E.



Here the dipole experiences no net force but experience a torque τ given by -

$$\boxed{\tau = \vec{P} \times \vec{E} = PE \sin\theta}$$

which will tend to rotate it (unless $\vec{P}$ is parallel to $\vec{E}$).

Suppose an external torque τ_{ext} is applied in such a manner that it just neutralises this torque and rotate the dipole from angle θ_1 to angle θ_2 without angular acceleration.

The amount of work done by the external torque will be given by -

$$W = \int_{\theta_1}^{\theta_2} \tau_{ext}(\theta)\, d\theta = \int_{\theta_1}^{\theta_2} PE \sin\theta \, d\theta$$

$$\boxed{W = -PE(\cos\theta_2 - \cos\theta_1)}$$

This work is stored as the potential energy of the system.

If we take $\theta_1 = \pi/2$ where the potential energy U is taken to be zero.

$$\text{So} \quad U(\theta_2) = -PE\left(\cos\theta_2 - \cos\frac{\pi}{2}\right)$$

$$\therefore \quad \boxed{U(\theta) = -PE\cos\theta = -P \cdot E}$$

Electrostatics of Conductors

(1) Electrostatic field is zero inside a conductor.

(2) At the surface of a charged conductor, electrostatic field must be normal to the surface at every point.

If E were not normal to the surface, it would have some non-zero components along the surface. Free charges on the surface of the conductor would then experience force and move. In the static situation therefore E should have no tangential component.

(3) There is no excess charge at any point inside the conductor than the static situation, and any excess charge must reside at the surface.

(4) Electrostatic potential is constant throughout the volume of the conductor and has the same value as inside on its surface.

(5) Electric field at the surface of a charged conductor is

$$\boxed{E = \frac{\sigma}{\epsilon_0}\hat{n}}$$

Here $\hat{n}$ is unit vector normal to the surface and σ is surface charge density.

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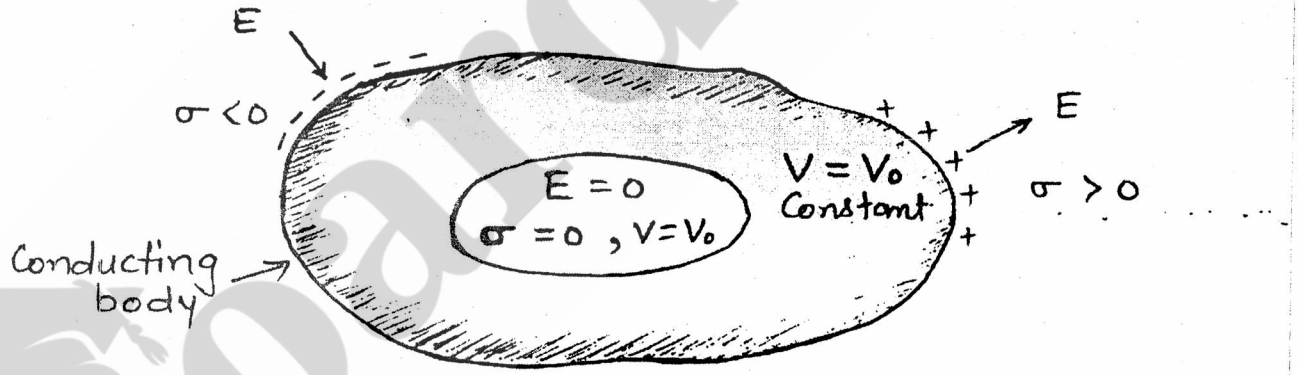
Electrostatic Shielding

Electrostatic shielding / screening is the phenomenon of protecting a certain region of space from external electric field.

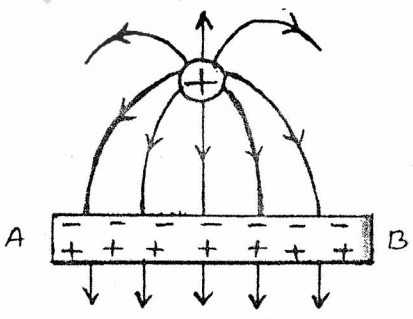
We know that the electric field inside a conductor is zero. Therefore to protect dilicate instruments from external electric fields, we enclose them in hollow conductors. Such hollow conductors are called Faraday cages. They need not be earthed.

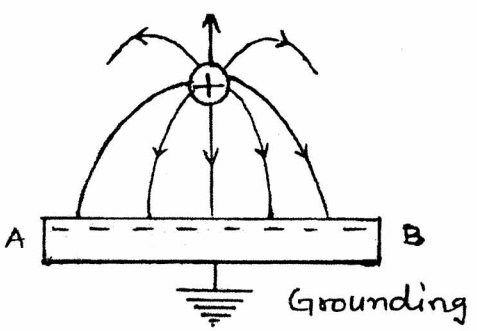
In thunderstorm accompained by lightening, it is safer to be inside a car or bus than to be in the open ground or under a tree etc.. The metallic body of the car or bus provides electrostatic shielding from the lightening.

The fundamental fact is that electric field inside a cavity is zero, whatever be the size and shape of the cavity and whatever be the charge on the conductor and the external field in which it might be placed.



An earthed conductor AB can also act as a screen against the electric field. When AB is earthed, the induced positive charge flow to earth and the field in the region beyond AB disappears.





Non polar and Polar Dielectrics

Dielectrics are insulating materials which transmit electric effects without actually conducting electricity.

Dielectrics are of two types –

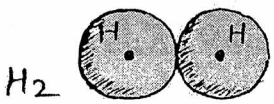
(a) Non polar (b) Polar

(a) Non Polar Dielectrics :-

The molecules in which the centre of positive charge coincide with the centre of negative charge are called non-polar molecules.

Such molecules are symmetric in shape.

Examples – O_2, CO_2, CH_4 etc.



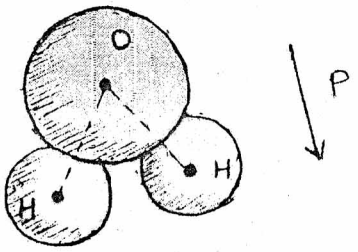
H_2

CO_2

(b) Polar Dielectrics :-

If the centres of positive and negative charges do not coincide because of the asymmetric shape of the molecules, then these are called polar molecules.

Examples – HCl, H_2O, NH_3 etc.



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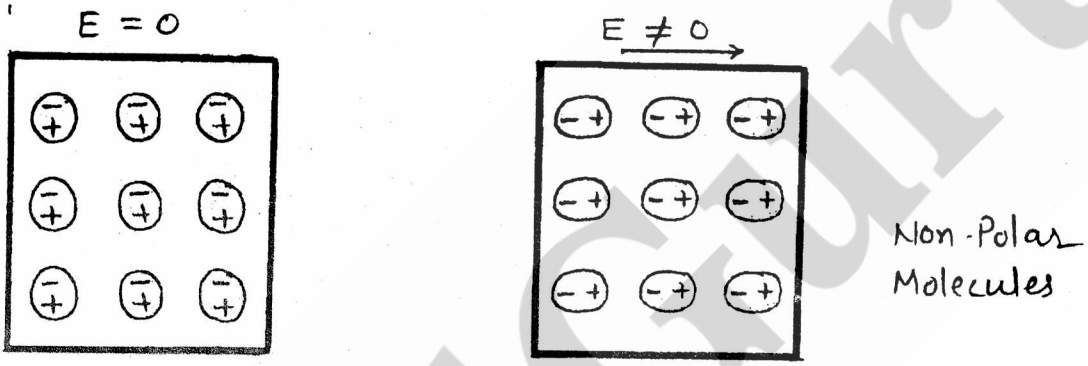
Dielectric Polarization

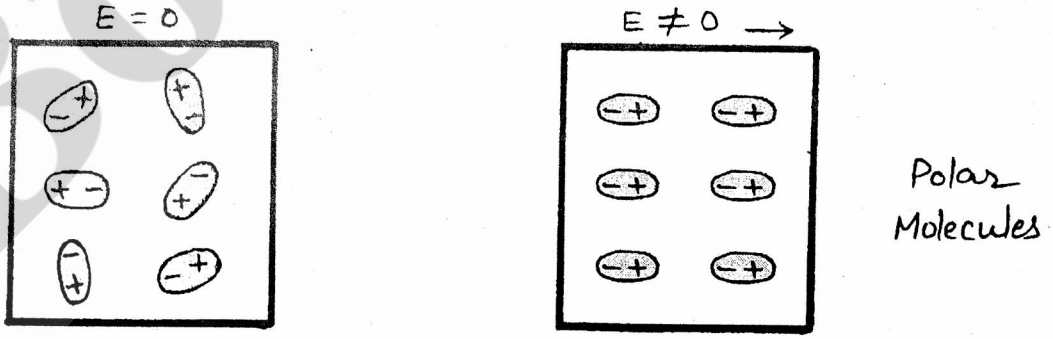
When a non-polar dielectric is held in an external electric field $\vec{E}$, the centre of positive charge (protons) in each molecule is pulled in the direction of $\vec{E}$ and the centre of negative charge (electrons) is pulled in a direction apposite to $\vec{E}$.

Therefore the two centres of positive and negative charges in the molecule are separated. The molecule gets distorted and is said to be polarised.



When no external field is applied to a dielectric with polar molecules then the different permanent dipoles are oriented randomly due to thermal agitation, so the total dipole moment is zero.

When an external electric field is applied, the individual dipole moments tends to align with the field. When summed over all the molecules, there appears some net dipole moment in the direction of the external field, i.e. the dielectric is polarised.



Thus in either case whether polar or non polar a dielectric develops a net dipole moment in the presence of an external field the dipole moment per unit volume is called polarisation and is denoted by P.

Electric susceptibility :-

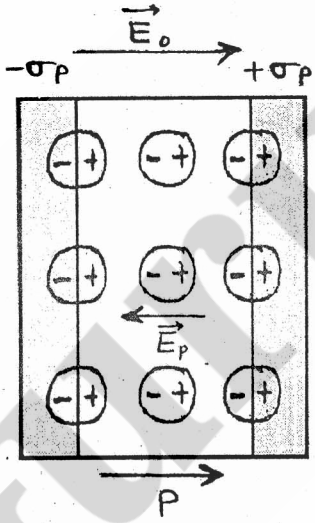
It is found that the electric polarization P is directly proportional to the effective electric field (E) in a polarized dielectric.

$$P \propto E$$

or $$\boxed{P = \chi E}$$

where χ is a constant and is pronounced as 'chi'.

- χ is called electric susceptibility of the dielectric.
- The electric susceptibility describes the electrical behaviour of a dielectric.
- For vacuum $\boxed{\chi = 0}$



Effective Electric field in a polarized dielectric is -

$$\boxed{E = E_0 - E_p}$$

Q. (CBSE 2005) Why does the electric field inside a dielectric decrease when it is placed in an external electric field ?

Sol. When a dielectric is placed in an external field, the field causes polarisation of the dielectric inducing a field in the opposite direction. Thus the net electric field inside the dielectric decreases.

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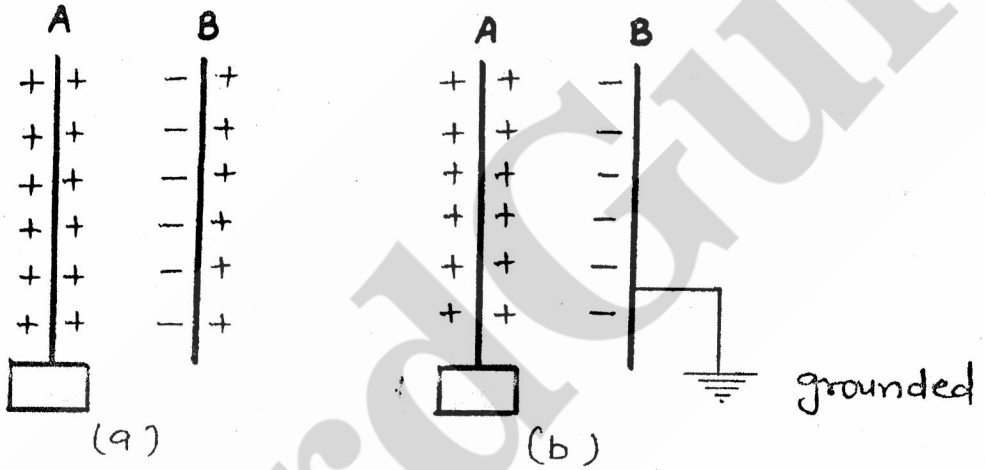
Capacitors and Capacitance

A capacitor is a system of two conductors separated by an insulator. It is an arrangement for storing large amounts of electric charge and hence electrical energy in a small space.

Principle of Capacitor :-

Lets Consider an insulated metal plate A. Let some positive charge be given to this plate, till its potential become maximum.



Now Consider another insulated metal plate B held near the plate A. By induction a negative charge is produced on the nearer face of B and an equal positive charge develops on the farther face of B.

Now Connect the plate B to earth. The induced positive charge on B being free, flow to earth. The induced negative charge on B, however stays on as it is bound to positive charge on A.

Due to the induced negative charge on B, potential of A is greatly reduced. Thus a Large amount of charge can be given to A to raise it to the maximum potential.

Hence we conclude that the capacitance of an insulated conductor is increased considerably by bringing near it an uncharged earthed conductor.

This is the principle of Capacitor.

Electrical Capacitance

The ability of a conductor to store electric charge is called electrical capacitance

If a charge Q given to the conductor raises its potential by V then it is found that

$$Q \propto V \quad \text{or} \quad Q = CV$$

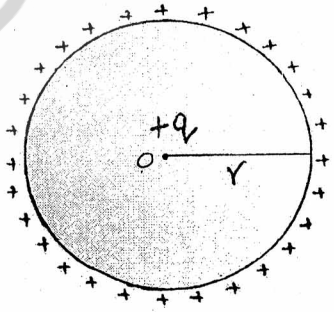
where C is a constant called electrical capacitance.

$$\boxed{C = \frac{Q}{V}}$$ SI unit of capacitance - Farad

A conductor is said to have a capacity of 1 farad, when a charge of 1 coulomb raises its potential by 1 volt.

Capacity of an Isolated Spherical Conductor

Let a charge +q is given to a sphere.



As the sphere behaves as if the entire charge is concentrated at the centre of the sphere, so the potential at any point on the surface of the sphere is -

$$\boxed{V = \frac{q}{4\pi\epsilon_0 r}}$$

As $C = \frac{q}{V}$ so $\boxed{C = 4\pi\epsilon_0 r}$

Capacity of earth :-

We can calculate electrical capacity of earth, taking it to be a conducting sphere of radius $r = 6400$ Km.

$$C = 4\pi\epsilon_0 r = \frac{1}{9\times10^9} \times 6.4\times10^6 = 711\ \mu F$$

Thus the capacity of earth is only 711 μF.

Therefore capacitance will be normally in nanofarad (nF) and picofarad (pF). Farad is too big a unit of capacitance.

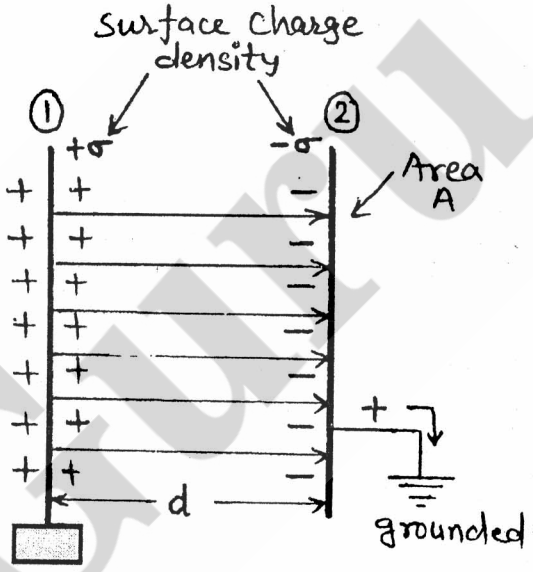
Parallel Plate Capacitor

A parallel plate capacitor consist of two thin conducting plates 1 and 2, each of area A held parallel to each other, suitable distance d apart.

The plates are separated by an insulating medium like air, paper, mica, glass etc.

When a charge +Q is given to the insulated plate 1, then a charge -Q is induced on the nearer face of plate 2,

surface charge density of plate 1 and 2 are σ and $-\sigma$.



In the region between the plates, separated by air / vacuum, electric field intensity is

$$E = \frac{\sigma}{\epsilon_0} = \frac{1}{\epsilon_0} \times \frac{Q}{A}$$

If E is the electric field between the plates, than the potential difference V between the plates is -

$$V = E \times d = \frac{1}{\epsilon_0} \times \frac{Q}{A} d$$

So the capacity of parallel plate capacitor is -

$$C = \frac{Q}{V} \Rightarrow C = \frac{Q}{\frac{1}{\epsilon_0} \times \frac{Q}{A} d}$$

$$\boxed{C = \frac{\epsilon_0 A}{d}}$$

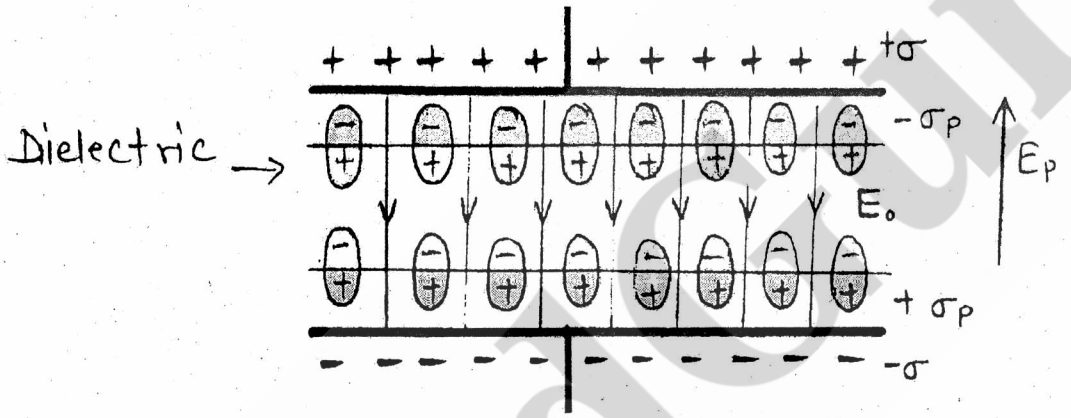
BoardGuru

Effect of Dielectric on Capacitance

When there is a vacuum between the plates of a capacitor having the charge $\pm Q$ on its plates with charge density $\pm\sigma$ then -

$$E_0 = \frac{\sigma}{\epsilon_0} \qquad V_0 = E_0 d \qquad C_0 = \frac{Q}{V_0} = \frac{\epsilon_0 A}{d}$$

Now consider a dielectric is inserted between the plates which fully occupying the intervening region.



The dielectric is polarised by the field with surface charge densities σ_p and $-\sigma_p$ at the surface of the dielectric. So the net surface charge density on the plates is $\pm(\sigma - \sigma_p)$.

So the final electric field inside the dielectric is -

$$E = \frac{\sigma - \sigma_p}{\epsilon_0}$$

and the potential difference between the plates is -

$$V = Ed = \frac{\sigma - \sigma_p}{\epsilon_0} \times d$$

For linear dielectrics, we expect σ_p to be proportional to E_0, i.e. to σ. Thus $(\sigma - \sigma_p)$ is proportional to σ and we can write -

$$\sigma - \sigma_p \propto \sigma$$

$$\sigma - \sigma_p = \frac{\sigma}{K}$$

$K \rightarrow$ dielectric Constant and $K > 1$

So $$\boxed{V = \frac{\sigma d}{\epsilon_0 K} = \frac{Q d}{A \epsilon_0 K}}$$

So the capacitance C with dielectric between the plates is –

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q d}{A \epsilon_0 K}} = \frac{K \epsilon_0 A}{d}$$

Or $$\boxed{C = \frac{K \epsilon_0 A}{d}}$$

Here $K = \frac{\epsilon}{\epsilon_0}$ is called the dielectric constant of the substance.

So that $$\boxed{C = K C_0}$$

- When dielectric is introduced, the field between the plates of the capacitor decreases by K. Due to which V decreases by a factor of K. This implies that the capacitance increase by K.

- Dielectric constant of a medium is defined as the ratio of the capacitance of the capacitor with the dielectric as the medium to its capacitance with vacuum between its plates.

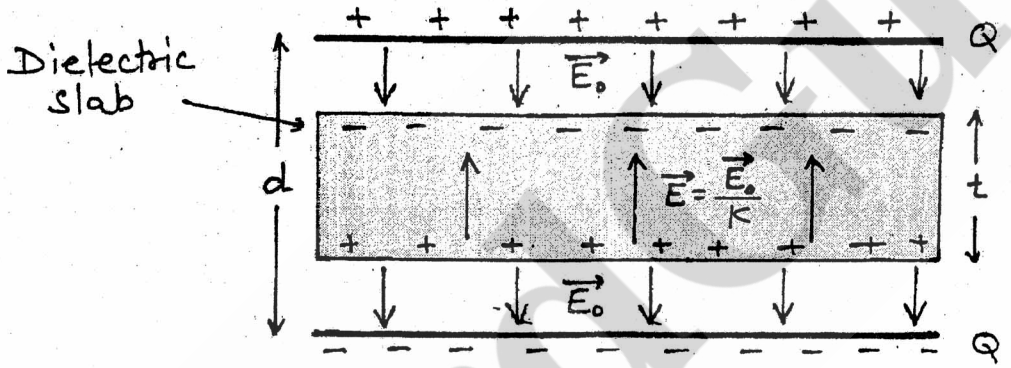
$$\boxed{K = \frac{C}{C_0}}$$

Capacitance of a Parallel plate capacitor with a dielectric slab -

The capacitance of a parallel plate capacitor of plate area A and plate separation d with vacuum or air inbetween is -

$$C_0 = \frac{\epsilon_0 A}{d}$$

Suppose $\pm Q$ are the charges on the capacitor plates which produce a uniform electric field E_0 in the space between the plates.



When a dielectric slab of thickness $t < d$ is introduced between the plates, the molecules in the slab get polarized in the direction of E_0.

Therefore the effective field inside the dielectric is

$$E = \frac{E_0}{K}$$

Outside the dielectric field remains E_0 only.

Therefore, potential difference between the two plates is

$$V = E_0 (d-t) + Et$$

$$V = E_0 (d-t) + \frac{E_0}{K} t$$

$$V = E_0 \left[d - t + \frac{t}{K} \right]$$

As $E_0 = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$

$$\therefore \quad V = \frac{Q}{A\epsilon_0} \left[d - t + \frac{t}{K} \right]$$

∴ Capacitance of the capacitor with dielectric inbetween is

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{A\epsilon_0}\left[d - t + \frac{t}{K}\right]}$$

$$\boxed{C = \frac{\epsilon_0 A}{d - t + \frac{t}{K}}}$$ Here $C > C_0$

★ Hence on introducing a dielectric slab inbetween the plates of a capacitor, its capacitance increases.

Q. CBSE 2008 The two plates of a parallel plate capacitor are 4 mm apart. A slab of dielectric constant 3 and thickness 3mm is introduced between the plates with its faces parallel to them. The distance between the plates is so adjusted that the capacitance of the capacitor become 2/3 of its original value. What is the new distance between the plates?

Sol. Here distance between parallel plates is $d = 4\text{ mm}$
$d = 4 \times 10^{-3}$ m.

$K = 3$, thickness $t = 3\text{ mm} = 3 \times 10^{-3}$ m.

$$\therefore \quad C = \frac{\epsilon_0 A}{d} \quad \text{and} \quad C_1 = \frac{\epsilon_0 A}{d_1 - t\left(1 - \frac{1}{K}\right)}$$

Since $C_1 = \frac{2}{3} C$

$$\therefore \quad \frac{\epsilon_0 A}{d_1 - t\left(1 - \frac{1}{K}\right)} = \frac{2}{3}\frac{\epsilon_0 A}{d}$$

$$\text{or} \quad \frac{1}{d_1 - t\left(1 - \frac{1}{K}\right)} = \frac{2}{3d}$$

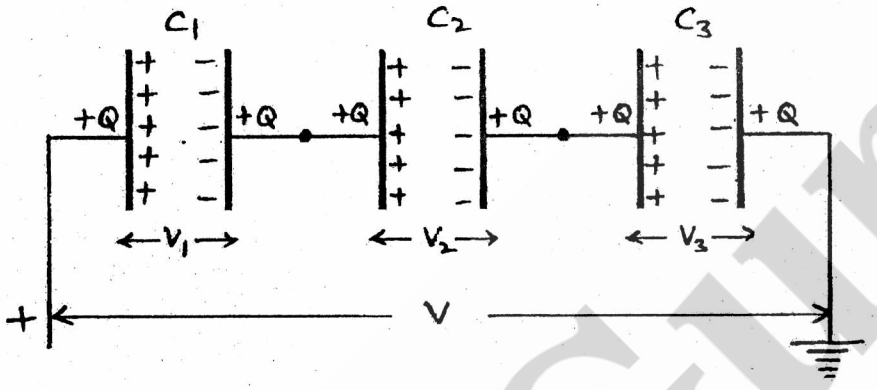
$$\frac{1}{d_1 - 3\times10^{-3}\left(1 - \frac{1}{3}\right)} = \frac{2}{3 \times 4 \times 10^{-3}}$$

So $d_1 = 8 \times 10^{-3}$ m. $= 8$ mm.

Combination of Capacitor

(i) Capacitors in Series

Three capacitors of capacities C_1, C_2 and C_3 are connected in series. V is potential difference applied across the combination.



In series combination each capacitor receives the same charge of Q. As these capacities are different, so the potential differences across the three capacitors are different.

$$V_1 = \frac{Q}{C_1}, \quad V_2 = \frac{Q}{C_2}, \quad V_3 = \frac{Q}{C_3}$$

If C_s is the total capacitance of combination than

$$V = \frac{Q}{C_s}$$

As total potential drop V across the combination is the sum of potential drops V_1, V_2 and V_3 so -

$$V = V_1 + V_2 + V_3$$

$$\therefore \frac{Q}{C_s} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

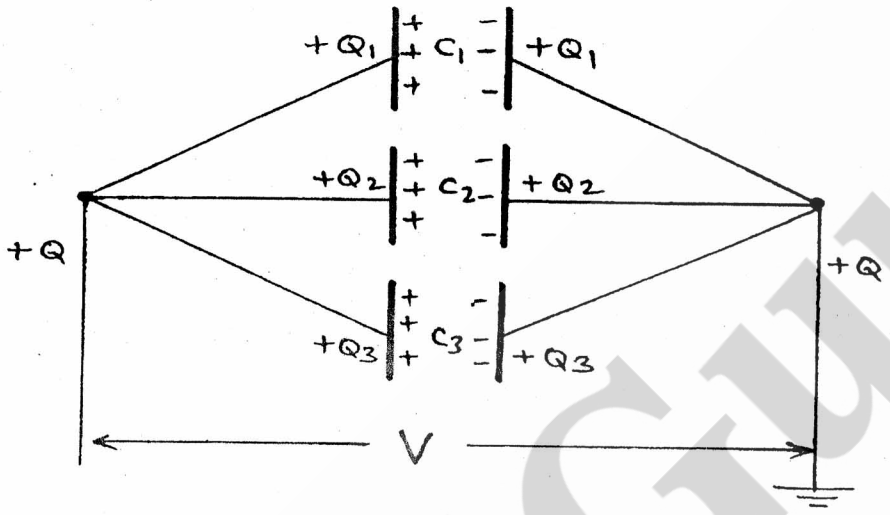
$$\boxed{\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

for n capacitors connected in series, total capacitance would be -

$$\boxed{\frac{1}{C_s} = \sum_{i=1}^{n} \frac{1}{C_i}}$$

(ii) Capacitors in Parallel

Let three capacitors C_1, C_2 and C_3 are connected in parallel and V be the potential difference across the combination.



So the charges on the capacitors will be

$$Q_1 = C_1V \text{ , } Q_2 = C_2V \text{ and } Q_3 = C_3V$$

So the total charge on the capacitors -

$$Q = Q_1 + Q_2 + Q_3$$

Let C_p be the equivalent capacitance in parallel

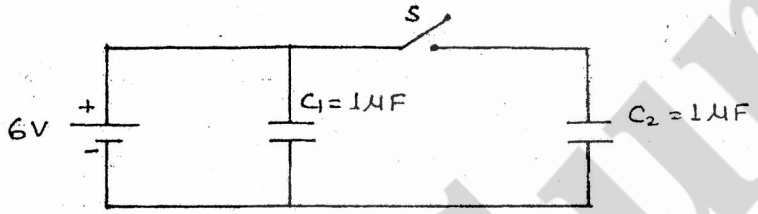
then $C_pV = C_1V + C_2V_2 + C_3V$ ...

$$\boxed{C_p = C_1 + C_2 + C_3}$$

When n capacitors are connected in parallel then -

$$\boxed{C_p = \sum_{i=1}^{n} C_i}$$

Q. CBSE 2010 Fig. Shows two identical capacitors C_1, C_2 each of 1 μF capacitance connected to a battery of 6 V. Initially switch 'S' is closed. After some time 'S' is left open and dielectric slab of dielectric constant K = 3 are inserted to fill the space completely between the plates of the two capacitors. How will the-
(i) charge and (ii) Potential difference between the plates of the capacitors be affected after the slabs are inserted?



Sol: When switch S is closed both C_1 and C_2 are charged to 6 V potential,

i.e. $V_1 = 6V$ and $V_2 = 6V$

When S is left open, C_1 is still connected to battery.

On introducing slab of dielectric constant, K = 3

$$C_1' = KC_1 = 3 \times 1 \times 10^{-6} = 3\,\mu F$$

$$V_1' = V_1 = 6\,V$$

$$Q_1' = C_1' V_1' = 3 \times 10^{-6} \times 6 = 18 \times 10^{-6}\,C$$

However, C_2 is disconnected from battery battery now, when S is left open. Due to introduction of slab,

$$C_2' = KC_2 = 3\,\mu F$$

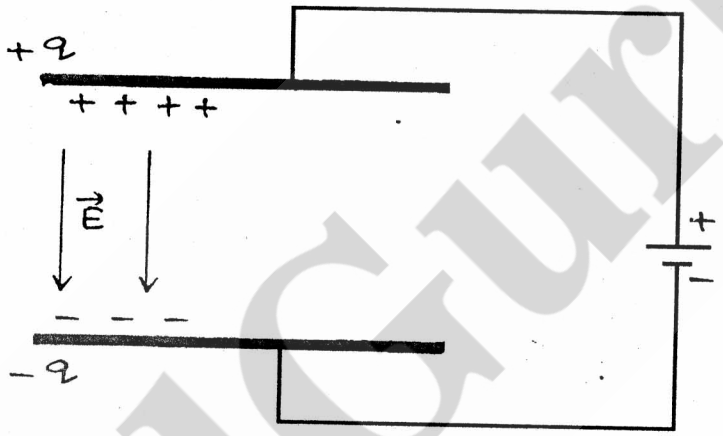
$$Q_2' = Q_2 = C_2 V_2 = 1 \times 10^{-6} \times 6 = 6 \times 10^{-6}\,C$$

$$\therefore \quad V_2' = \frac{Q_2'}{C_2'} = \frac{6 \times 10^{-6}}{3 \times 10^{-6}} = 2\,V$$

Energy Stored in Capacitor

The energy stored in a capacitor is the same as the work done to build up the charge on the plates by external source (battery). of voltage.

Let the two plates of the capacitor are initially uncharged. Now the charge is transferred to one plate in very small instalments of dq and equal opposite charge induces on the other plate.



At any intermediate stage, suppose charge on one plate is q and on other plate is −q.

∴ Potential difference between the plates $= \frac{q}{C}$

Small amount of work done in giving an additional charge dq to the capacitor is

$$dW = \frac{q}{C} \times dq$$

Total work done in giving a charge Q to the capacitor-

$$W = \int_{q=0}^{q=Q} \frac{q}{C}\, dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q$$

$$\boxed{W = \frac{1}{2}\frac{Q^2}{C}}$$

This work is stored in the form of potential energy U.

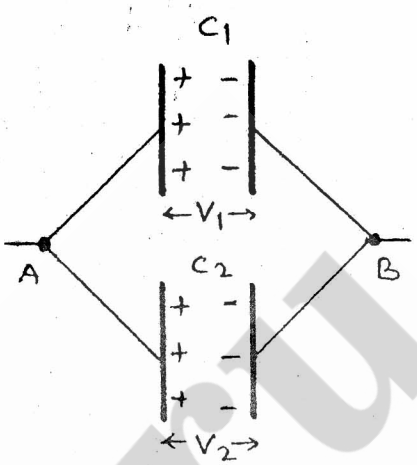
$$U = W = \frac{1}{2}\frac{Q^2}{C}$$

as $Q = CV$ and $CV = Q$ so we can write-

$$\boxed{U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV} \quad \text{Joule}$$

Common Potential

(i) When two capacitors charged to different potentials are connected in parallel with their positive plates at one point and negative plates at another point by a conducting wire, then charge flow from from the one at higher potential to the other at lower potential.



The flow of charge continues till their potentials become equal.

The equal potential of the two capacitors is called common potential.

$$\text{Common Potential} = \frac{\text{Total charge}}{\text{Total Capacity}} \qquad V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

(ii) When the two capacitors are connected with the positive plate of one capacitor to the negative plate of other capacitor, then

$$\text{Common Potential} = \frac{\text{Total charge}}{\text{Total Capacity}} \qquad V = \frac{C_1V_1 - C_2V_2}{C_1 + C_2}$$

Van De Graaff Generator

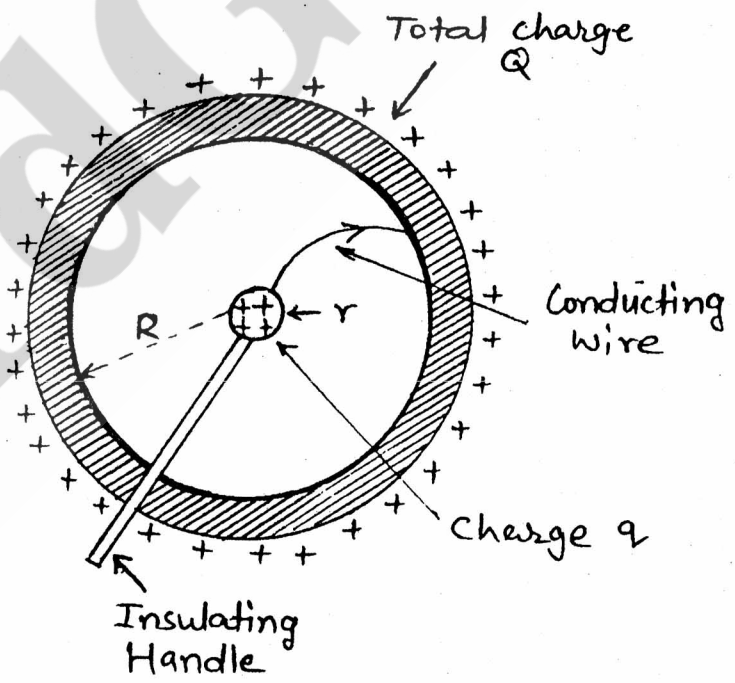
It is a device used to build up high potential difference of the order of few million volts. Such high potential differences are used to accelerate charged particles like electrons, protons, ions etc. needed for various experiments of Nuclear physics.

Principle :-

Suppose we have a large spherical conducting shell of radius R, on which we place a charge Q.

Potential inside conducting spherical shell = Constant

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}$$



Now let we introduce a small sphere of radius r, carrying some charge q, into the large one, and place it at the centre.

Now the potential due to small sphere of radius r carrying charge q can be given as-

at surface of small sphere $= \frac{1}{4\pi\epsilon_0}\frac{q}{r}$

at large shell of radius R $= \frac{1}{4\pi\epsilon_0}\frac{q}{R}$

Taking both charges q and Q into account we have total potential at the two shells and the potential difference as-

$$V(R) = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{R} + \frac{q}{R}\right)$$

$$V(r) = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{R} + \frac{q}{r}\right)$$

$$\boxed{V(r) - V(R) = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r} - \frac{1}{R}\right)}$$

Assume now that q is positive. Then independent of amount of charge Q on the larger sphere and even if it is positive, the inner sphere is always at a higher potential as the difference V(r) - V(R) is positive.

This means that if we now connect the smaller and larger sphere by a wire, the charge q on the small sphere will flow on to the large sphere, even though the charge Q may be quite large, because natural tendency for positive charge is to move from higher to lower potential.

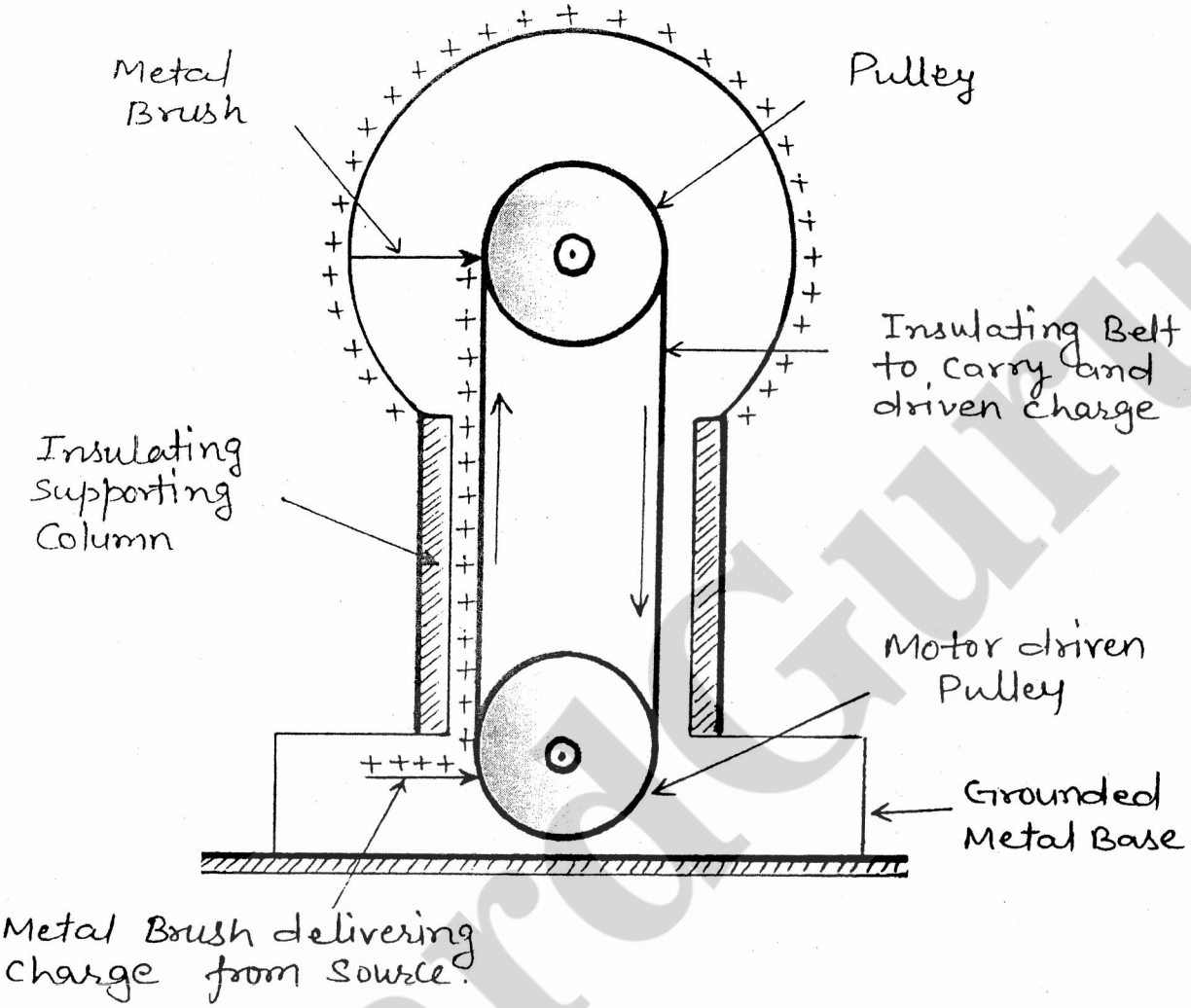
Thus if we are somehow able to introduce the small charged sphere into the larger one. we can keep pilling up larger and larger amount of charge on the large sphere.

This is the principle of Van de Graaff generator.

■ When electric potential on the sphere produces electric field equal to dielectric strength of air or gases then potential leakage takes place with the ionisation of air.

So no more potential difference can be increased. The potential can be increased by enclosing the sphere in highly evacuated chamber.

Construction and Working :-



In Van De Graaff Generator a large spherical conducting shell (of few meter radius) is supported at a height several metre above the ground on an insulating column.

A long narrow endless belt of insulating material like rubber or silk, is wound around two pulleys, one at ground level and one at the centre of the shell.

This belt is kept continuously moving by a motor driven pulley. It continuously carries positive charge sprayed on to it by a brush at ground level, to the top.

There it transfers its positive charge to another conducting brush connected to the large shell. The positive charge is transferred to the shell, where it spread out uniformly on the outer surface.

In this way voltage differences of as much as 6 or 8 million volts (with respect to ground) can be built up.

Q. CBSE 2008 On charging a parallel plate capacitor to a potential V, the spacing between the plates is reduced to half and a dielectric medium of $\epsilon_r = 10$ is introduced between the plates, without disconnecting the DC source. Explain how the-
(i) Capacitance (ii) electric field and
(iii) energy density of the capacitor change?

Sol. Connected battery ensures same potential difference V across capacitor even after reducing the spacing between plates upto half and introducing the dielectric slab.

(i) If initial capacitance $C_1 = \epsilon_0 A/d$

Then final Capacitance $C_2 = \dfrac{10\,\epsilon_0 A}{d/2}$

$$C_2 = \frac{20\,\epsilon_0 A}{d} = 20 C_1$$

Hence $C_2 = 20 C_1$

Capacitance increases to 20 times of original value.

(ii) Initial electric field $E_1 = \dfrac{V}{d}$

Finally $E_2 = \dfrac{V}{d/2} = \dfrac{2V}{d} = 2E_1$

Hence $E_2 = 2E_1$

Electric field intensity gets doubled.

(iii) Energy density of the capacitor $U_1 = \frac{1}{2}\epsilon_0 E_1^2$

and $U_2 = \frac{1}{2}\epsilon_0 E_2^2$

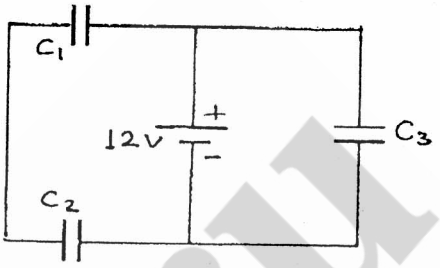
$$U_2 = \frac{1}{2}\epsilon_0 (2E_1)^2 = \frac{1}{2}\epsilon_0 4E_1^2$$

so $U_2 = 4U_1$

Hence energy density becomes 4 times of original value.

Q. CBSE 2009 Three identical capacitors C_1, C_2 and C_3 of capacitance 6 μF each are connected to a 12 V battery as shown. Find

(i) Charge on each capacitor
(ii) Equivalent Capacitance of the circuit
(iii) Energy stored in the network of capacitors.



Sol. (i) Here $V = 12\ V$

and $C_1 = C_2 = C_3 = 6\ \mu F = 6 \times 10^{-6}\ F$

So the Charge on capacitor C_3 is

$$q_3 = C_3 V = 6 \times 10^{-6} \times 12 = 72 \times 10^{-6}$$

$$q_3 = 72\ \mu C$$

Since C_1 and C_3 are in series so the equivalent capacitance $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$

$$\frac{1}{C_s} = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

$$\therefore \quad C_s = 3\ \mu F$$

So the charge on each capacitor C_1 and C_2 is

$$q = C_s V = 3 \times 10^{-6} \times 12 = 36 \times 10^{-6} = 36\ \mu C$$

∴ Charge on each capacitor C_1 and C_2 is 36 μC.

(ii) Since C_1 and C_2 are in series whose equivalent Capacitance is $C_s = 3\ \mu F$

Now C_3 and C_s are in parallel so the equivalent Capacitance of the circuit is $C = C_3 + C_s$

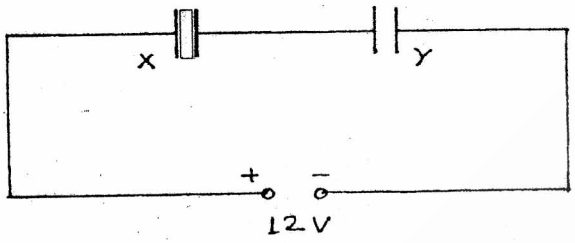
$$C = 6 + 3 = 9\ \mu F$$

(iii) Energy stored $U = \frac{1}{2} C V^2$

$$U = \frac{1}{2} \times 9 \times 10^{-6} \times (12)^2$$

$$U = 648 \times 10^{-6} = 6.48 \times 10^{-4}\ J$$

Q. CBSE 2009 Two parallel plate capacitors X and Y have the same area of plates and same separation between them, Y has air between the plates while X have a dielectric medium of $\epsilon_r = 4$.



(i) Calculate capacitance of each capacitor if equivalent capacitance of the combination is 4 μF.

(ii) Calculate the potential difference between the plates of X and Y.

(iii) What is the ratio of electrostatic energy stored in X and Y ?

Sol. (i) Since capacitance C of the parallel plate capacitor is given by- $C = \frac{\epsilon_0 A}{d}$

$\therefore \quad Y = \frac{\epsilon_0 A}{d}$ and $X = \frac{\epsilon_0 K A}{d} = \frac{4\epsilon_0 A}{d}$

$\therefore \quad X = 4Y$

Since X and Y are in series combination

$\therefore \quad \frac{1}{4} = \frac{1}{X} + \frac{1}{Y} \Rightarrow \frac{1}{4} = \frac{1}{4Y} + \frac{1}{Y}$

or $\frac{1}{4} = \frac{5}{4Y} \Rightarrow Y = 5\ \mu F$

So that $X = 4 \times 5 = 20\ \mu F$

(ii) Since $q = CV$

$q = 4 \times 10^{-6} \times 12 = 48 \times 10^{-6}\ C$

Potential diff. on capacitor X is given by

$V_x = \frac{q}{C_x} = \frac{48 \times 10^{-6}}{20 \times 10^{-6}} = 2.4\ V$

and potential diff. on capacitor Y is given by-

$V_Y = \frac{q}{C_Y} = \frac{48 \times 10^{-6}}{5 \times 10^{-6}} = 9.6\ V$

(iii) Energy stored in capacitor is $= \frac{1}{2} CV^2$

$\therefore$ Ratio of electrostatic energy stored in X and Y is

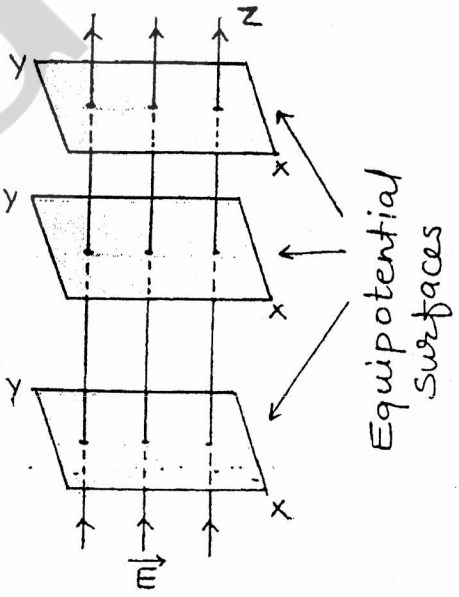
$$\frac{U_x}{U_y} = \frac{\frac{1}{2} C_x V_x^2}{\frac{1}{2} C_y V_y^2} = \frac{20 \times (2.4)^2}{5 \times (9.6)^2}$$

$$\frac{U_x}{U_y} = \frac{1}{4}$$

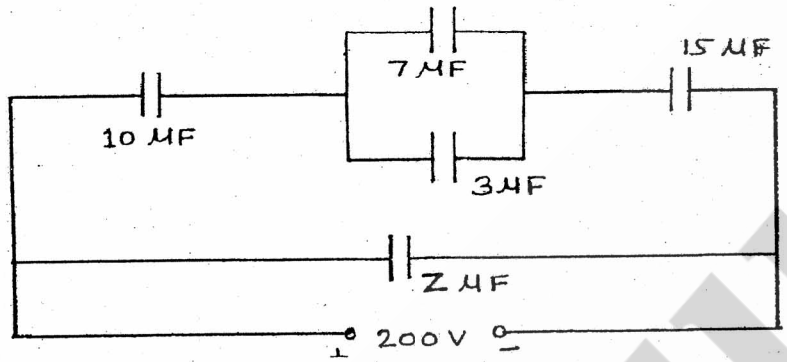
Q. CBSE 2009 Draw 3 equipotential surfaces corresponding to a field that uniformly increases in magnitude but remains constant along z-direction. How are these surfaces different from that of a constant electric field along z-direction?

Sol. The three equipotential surfaces are in xy plane. As magnitude of field increases, distance between successive equipotential surfaces (for given dv) decreases as shown in figure.

If electric field were constant, distance between successive equipotential surfaces would stay constant.



Q. CBSE 2009 A system of capacitors connected as shown, has a total energy of 160 mJ stored in it. Obtain the value of the equivalent capacitance of this system and the value of Z.



Sol. Capacitor of 3 MF and 7 MF are in parallel.

$\therefore \quad C_1 = 3 + 7 = 10 \text{ MF}$

Since 10 MF, 10 MF and 15 MF are in series -

$\therefore \quad \frac{1}{C_2} = \frac{1}{10} + \frac{1}{10} + \frac{1}{15} = \frac{4}{15}$

or $\quad C_2 = \frac{15}{4} \text{ MF}$

Hence total capacitance $\quad C = C_2 + Z$

$$C = \left(\frac{15}{4} + Z\right) \text{MF} \quad \ldots\ldots$$

$\therefore$ Energy stored = 160 mJ

$$\frac{1}{2} C V^2 = 160 \text{ mJ}$$

$$\frac{1}{2} \times \left(\frac{15}{4} + Z\right) \times 10^{-6} \times (200)^2 = 160 \times 10^{-3}$$

$$\Rightarrow \quad \frac{15}{4} + Z = 8$$

$$Z = 8 - \frac{15}{4} = 4.25 \text{ MF}$$

Q. CBSE 2005 A parallel plate capacitor is to be designed with a voltage rating 1 KV using a material of dielectric constant 3 and dielectric strength about 10^7 V/m. For safety we would like the field never to exceed say 10% of the dipole strength. What minimum area of the plates is required to have a capacitance of 50 PF?

Sol. Maximum Permissible Voltage = 1 KV. = 10^3 V

Maximum Permissible electric field = 10% of 10^7 V/m

= 10^6 V/m.

Minimum separation required between the capacitor plates is -

$$d = \frac{V}{E} = \frac{10^3}{10^6} = 10^{-3} \text{ m}$$

So the minimum area required for the capacitor plates

$$A = \frac{Cd}{K\epsilon_0} = \frac{50 \times 10^{-12} \times 10^{-3}}{3 \times 8.85 \times 10^{-12}}$$

$$A = 18.8 \times 10^{-4} \text{ m}^2 \approx 19 \text{ cm}^2.$$

Q. CBSE 2006 The electric field and electric potential at any point due to a point charge kept in air is 20 N/C and and 10 J/C respectively. Compute the magnitude of this charge.

Sol. Electric field at distance r from the point Charge $E = \frac{1}{4\pi\epsilon_0} \times \frac{q}{r^2} = 20$ N/C

Electric Potential at distance r from the point Charge $V = \frac{1}{4\pi\epsilon_0} \times \frac{q}{r} = 10$ J/C

so that $r = \frac{V}{E} = \frac{10}{20} = 0.5$ m

Hence Charge $q = 4\pi\epsilon_0 r \times V = \frac{0.5 \times 10}{9 \times 10^9}$

$q = 0.55 \times 10^{-9}$ C $\quad [\because C = 4\pi\epsilon_0 r]$

Very Important Questions

1 Mark Questions

Q.1. Why does the electric field inside a dielectric decrease when it is placed in an external electric field? (Delhi Board 2005)

Q.2. Define the term 'dielectric constant' of a medium in terms of Capacitance of a capacitor. (Delhi Board 2006, 11)

Q.3. A 500 μC charge is at the centre of a square of side 10 cm. Find the work done in moving charge of 1.0 μC between two diagonally oppasite points on the square. (CBSE 2008)
Answer = zero.

Q.4. What is the electrostatic potential due to an electric dipole at an equatorial point? (CBSE 2009)

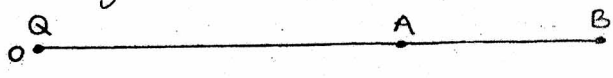
Q.5. What is the work done in moving a test charge q through a distance 1 cm along the equatorial axis of an electric dipole? (CBSE 2009)

Q.6. Define the term potential energy of charge q at a distance 'r' in an external electric field. (CBSE 2009)

Q.7. Name the physical quantity whose SI unit is J/C. It is a scalar or a vector quantity? (CBSE 2010)

Q.8. Draw equipotential surface due to a single point charge. (CBSE 2010)

Q.9. A point charge Q is placed at point O as shown in fig. Is the potential difference $V_A - V_B$ positive, negative or zero if Q is (i) Positive (ii) Negative? (CBSE 2011, 01, 06)



Q.10. A hollow metal sphere of radius 5 cm is charged such that the potential on its surface is 10 V. What is the potential at the centre of the sphere?
(CBSE 2011)

Q.11. Can two equipotential surfaces intersect each other? Justify your answer.
(CBSE 2011, 09)

Q.12. When electrons drift in a metal from lower to higher potential, does it mean that all the free electrons of the metal are moving in the same direction?
(CBSE 2012)

Q.13. Why is electrostatic potential constant throughout the volume of the conductor and has the same value (as inside) on its surface?
(CBSE 2012)

2 Marks Questions / 3 Marks Questions -

Q.1. A parallel plate capacitor with air between the plates has a capacitance of 8 PF. What will be the capacitance if the distance between the plates be reduced by half and the space between them is filled with a substance of dielectric constant K = 6 ?
(CBSE 2005)

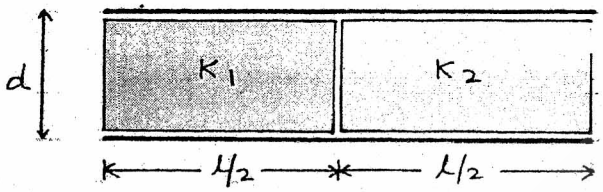
Q.2. A spherical shell of radius b with charge Q is expanded to radius a. Find the work done by the electrical force in the process?
(CBSE 2004)

Q.3. A 12 PF capacitor is connected to a 50 V battery. How much electrostatic energy is stored in the capacitor.
(CBSE 2005)

Q.4. Find out the expression for the potential energy of a system of three charges q_1, q_2 and q_3 located respectively at $\vec{r_1}$, $\vec{r_2}$ and $\vec{r_3}$ with respect to the common origin O.
(CBSE 2010)

Q.5. Two dielectric slabs of dielectric constants K_1 and K_2 are filled in between the two plates, each of area A, of the parallel plate capacitor as shown in figure. Find the net capacitance of the capacitor.

(CBSE 2005)

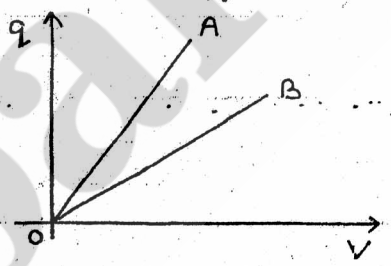


Q.6. A parallel plate capacitor is to be designed with a voltage rating 1 KV using a material of dielectric constant 3 and dielectric strength about 10^7 V/m. For safety we would like the field never to exceed say 10% of the dipole strength. What minimum area of the plates is required to have a capacitance of 50 PF?

(CBSE 2005)

Q.7. The given graph shows the variation of charge q versus potential difference V for two capacitors C_1 and C_2. The two capacitors have some plate separation but the plate area of C_2 is double than that of C_1. Which of the lines in the graph correspond to C_1 and C_2 and why?

(CBSE 2006)



Q.8. Two capacitors of capacitance of 6 μF and 12 μF are connected in series with a battery. The voltage across the 6 μF capacitor is 2 V. Compute the total battery voltage.

(CBSE 2006)

Ans- 3 V.

Q.9. Two point charges 4 μc and -2 μc are separated by a distance of 1 m in air. Calculate at what point on the line joining the two charges is the electric potential zero.

(CBSE 2007)

Ans.- $\frac{2}{3}$ m from 4 μc.

Q.10. Explain the working principle of a parallel plate capacitor. If two similar plates, each of area A having surface charge densities $+\sigma$ and $-\sigma$ are separated by a distance d in air, write expression for-

(i) The electric field at point between the two plates.
(ii) The potential difference between the plates.
(iii) The capacitance of the capacitor so formed

(CBSE 2007)

Q.11. A parallel plate capacitor each with plate area A and separation d is charged to a potential difference V. The battery used to charge it is then disconnected. A dielectric slab of thickness d and dielectric constant K is now placed between the plates. What change will take place in

(i) Charge on the plates.
(ii) Electric field intensity between the plates.
(iii) Capacitance of the capacitor.

(CBSE 2007, 09)

Q.12. Derive the expression for the electric potential at any point along the axial line of an electric dipole?

(CBSE 2008)

Q.13. Derive an expression for the potential energy of an electric dipole of dipole moment $\vec{P}$ in an electric field $\vec{E}$.

(CBSE 2008)

Q.14. Two charges 5 nC and -2 nC are placed at points (5cm, 0, 0) and (23cm, 0, 0) in a region of space where there is no other external field. Calculate the electrostatic potential energy of the system.

Ans = -5×10^{-7} Joule.

(CBSE 2008)

Q.15. Two charges of magnitude 5 nC and -2 nC are placed at points (2cm, 0, 0) and (x cm, 0, 0) in a region of space, where there is no other external field. If the electrostatic potential energy of the system is $-0.5\ \mu J$, what is the value of x?

Ans. = 20 cm. (CBSE 2008)

Q.16. Define an equipotential surface. Draw the equipotential surfaces corresponding to a field that uniformly increases in magnitude but remains constant in direction. (CBSE 2008)

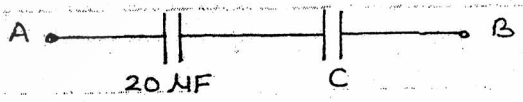
Q.17. The two plates of a parallel plate capacitor are 5 mm apart. A slab of a dielectric, of thickness 4 mm is introduced between the plates with its faces parallel to them. The distance between the plates is adjusted so that the capacitance of the capacitor becomes equal to its original value. If the new distance between the plates equal 8 mm, what is the dielectric constant of the dielectric used?

Ans. = K = 4. (Raj. Board 2008)

Q.18. Two charges -q and +q are located at points A (0, 0, -a) and B (0, 0, +a) respectively. How much work is done in moving a test charge from point P(7, 0, 0) to Q(-3, 0, 0)?

Ans. = Zero. (CBSE 2009)

Q.19. The equivalent capacitance of the combination between A and B in the given figure is 4 MF. (CBSE 2009)



(i) Calculate capacitance of the capacitor C.
(ii) Calculate charge on each capacitor if 12 V battery is connected across terminals A and B.

Q.20. Derive the expression for the potential energy of a system of two point charges q_1 and q_2 brought from infinity to the points $\vec{r}_1$ and $\vec{r}_2$ respectively in the external electric field $\vec{E}$. (CBSE 2009)

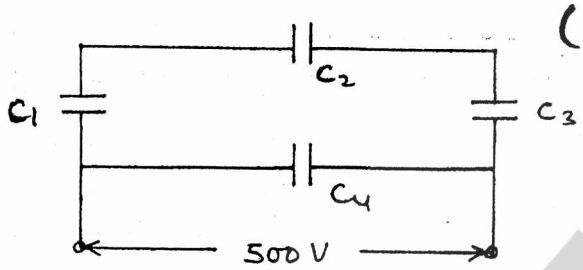
Q. 21. A network of four capacitors each of 12 μF capacitance is connected to a 500 V supply as shown in fig. Determine (a) Equivalent capacitance of the network (b) Charge on each capacitor.

(CBSE 2010)

Ans. (i) 16 μF

(ii) $q_1 = q_2 = q_3 = 2000\ \mu F$

$q_4 = 6000\ \mu F$



Q. 22. Four capacitors of values 6 μF, 6 μF, 6 μF and 2 μF are connected to a 6 V battery as shown in fig. Determine

(i) Equivalent capacitance of the network

(ii) The ch e on each capacitor

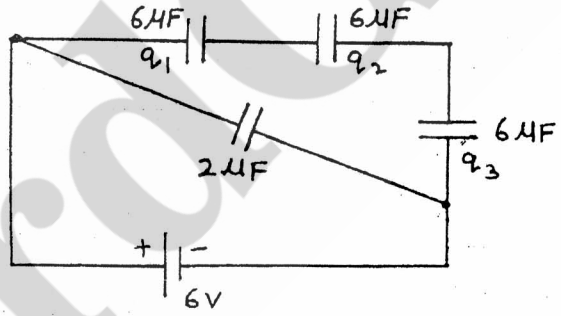
(CBSE 2010)

Ans.

(i) C = 4 μF

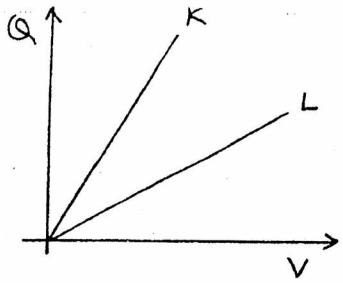
(ii) $q_1 = q_2 = q_3 = 12\ \mu C$

$q_4 = 24\ \mu C$



Q. 23. The following graph shows the variation of charge Q with voltage V, for two capacitors L and K. In which capacitor is more electrostatic energy stored?

(CBSE 2008)



A 4 μF capacitor is charged by a 200 V supply.

Q. 24. The supply is then disconnected and the charged capacitor is connected to another uncharged 2 μF capacitor. How much electrostatic energy of the first capacitor is lost in the process of attaining the steady situation?

(CBSE 2005)

Ans. $\Delta U = 2.67 \times 10^{-2}\ J$

5 Marks Questions -

Q.1. State the principle of the device that can built up high voltage of the order of a few million volts. Draw its Labelled diagram. A stage reached in this device when the potential at the outer sphere cannot be increased further by piling up more charge on it. Explain why?

(Delhi 2012, CBSE 2011)

Q.2. (a) Deduce the expression for the energy stored in a charged capacitor.

(b) show that the effective capacitance C of a series combination of three capacitors C_1, C_2 and C_3 is given by -

$$C = \frac{C_1 C_2 C_3}{C_1C_2 + C_2C_3 + C_3C_1}$$

(CBSE 2010)

Q.3.(a) Show that in a parallel plate capacitor if the medium between the plates of a capacitor is filled with an insulating substance of dielectric constant K, its capacitance increases.

(b) Show that the energy stored in a parallel plate capacitor can be expressed in terms of electric field as $\frac{1}{2}\epsilon_0 E^2 A d$, where A is the area of each plate and d is the separation between the plates.

(CBSE 2007)

Q.1. What is the work done by a field of nucleus in a complete circular orbit of the electron? What if the orbit is elliptical?

Q.2. Guess a possible reason why water has a much greater dielectric constant (=80) than say mica (=6).

Q.3. Why is electrostatic potential constant throught the volume of the conductor and has the same value (as inside) on its surface?

Q.4. Express dielectric constant in terms of capacitance.

Q.5. A hollow metal sphare of radius 5cm is charged such that the potential on its surface is 10 V. What is the potential at the centre of the sphere?

Q.6. A metallic spherical shell has an inner radius R_1 and outer radius R_2. A charge Q is placed at the centre of the spherical cavity. What will be surface charge density on the inner surface and the outer surface? Also show the electric field for this configuration.

Q.7. Consider two conducting spheres of radii R_1 and R_2 with $R_1 > R_2$. If the two are at the same potential. State whether the charge density of the smaller sphere is more or less than that of the Larger one.

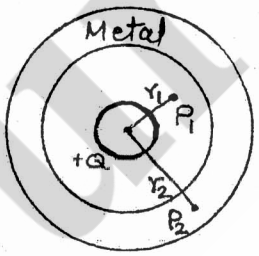
Ans. $\sigma_2/\sigma_1 = R_1/R_2$

Q.8. Two point charges q and $-3q$ are placed fixed on x-axis separated by distance 'd'. where should a third charge $2q$ be placed such that it will be in equilibrium?

Ans. $x = \frac{d}{2}(1+\sqrt{3})$ from q.

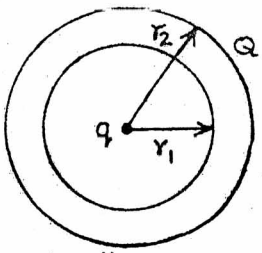
Q.9. Two uniformly charge parallel thin plates having charge densities $+\sigma$ and $-\sigma$ are kept in the x-z plane at a distance 'd' apart. Sketch an equipotential surface due to electric field between the plates. If a particle of mass m and charge '-q' remains stationary between the plates, what is the magnitude and direction of this field?

Q.10. A small metal sphere carrying charge +Q, is located at the centre of a cavity in a large uncharged metal sphere as shown in fig. Use Gauss's theorem to find electric field at points P_1 and P_2.



Ans. - $E_1 = \frac{KQ}{r_1^2}$, $E_2 = 0$

Q.11. A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge 'Q'. A charge 'q' is placed at the centre of the shell. (a) Calculate surface charge density on the (i) Inner surface (ii) Outer surface of the shell.



(b) Write expression for the electric field at a point $x > r_2$ from the centre of the shell.

Ans. - (a) (i) $\sigma = \frac{-q}{4\pi r_1^2}$ (ii) $\sigma = \frac{Q+q}{4\pi r_2^2}$ (b) $E = K\frac{(Q+q)}{x^2}$

Q.12. A thin straight infinitely long conducting wire having charge density λ is enclosed by a cylindrical surface of radius r and length l, its axis coinciding with the length of the wire. Find the expression for the electric flux through the surface of the cylinder.

Ans. - $\phi = \frac{\lambda l}{\epsilon_0}$

PHYSICS

Unit - 3. Current Electricity

Class - 12th

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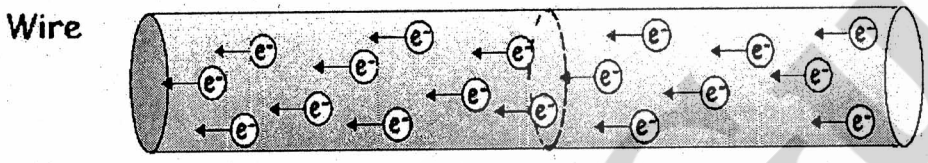
3. Current Electricity

Electric Current -

The time rate of flow of charge through any cross-section of a conductor is called electric current.

$$\text{Electric Current } I = \frac{q}{t}$$



If the rate of flow of charge varies with time, then current at any time (instantaneous current) is given by -

$$I = \frac{dq}{dt}$$

where dq is a small charge passing through any cross-section of the conductor in small time interval dt.

- SI Unit of the current is **Ampere (A)**.

1 Ampere -

The current through a wire is said to be 1 ampere, if one coulomb of charge is flowing per second through any cross section of wire.

$$1 \text{ Ampere (A)} = \frac{1 \text{ Coulomb (C)}}{1 \text{ second (s)}} = 1 \text{ C/s}$$

Current is a scalar quantity.

Though the electric current represents the direction of flow of positive charge, yet it treated as scalar, because current follows, the law of scalar addition and not the Law of vector addition.

Direction of Electric Current -

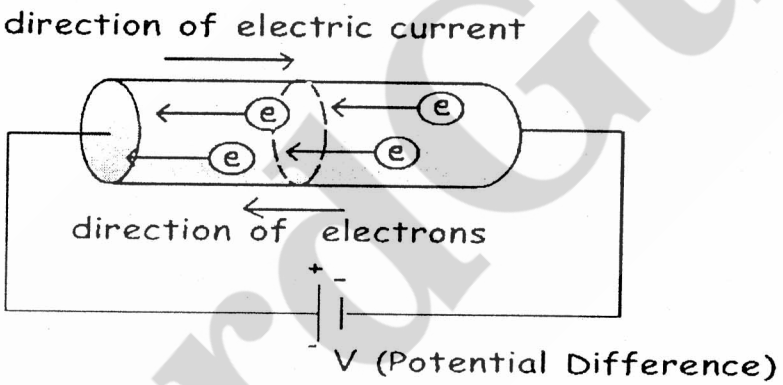
There are two types of current -

(i) Conventional Current -

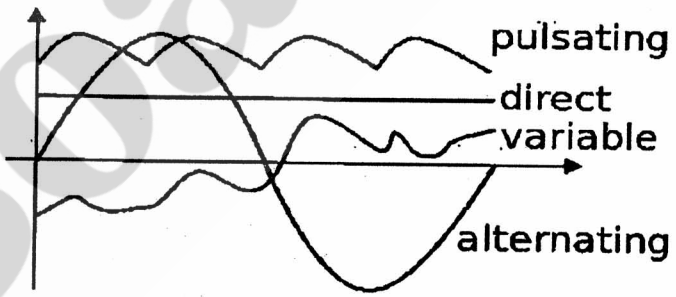
The direction of flow of positive charge gives the direction of current, It is called conventional current.

(ii) Electronic Current -

The direction of flow of electrons gives the direction of electronic current. The direction of electronic current is opposite to the conventional current.



Different types of Current



Current Carriers -

(i) Current Carriers in Solids -

Conductors – Free electrons

Semiconductor – Free electrons & Holes

Insulators – No charge Carriers

(ii) Current Carriers in Liquids -

A conducting liquid is called electrolyte (e.g. $CuSO_4$, NaCl etc.). The electrolyte solution provides positive ions (like Cu^{++}, Na^+) and negative ions (like SO_4^{--}, Cl^-). When electric field (i.e. potential difference) is applied, the positive ions moves in one direction and negative ions in the opposite direction, causing electric current.

* Thus, the current carriers in liquids are positive and negative ions.

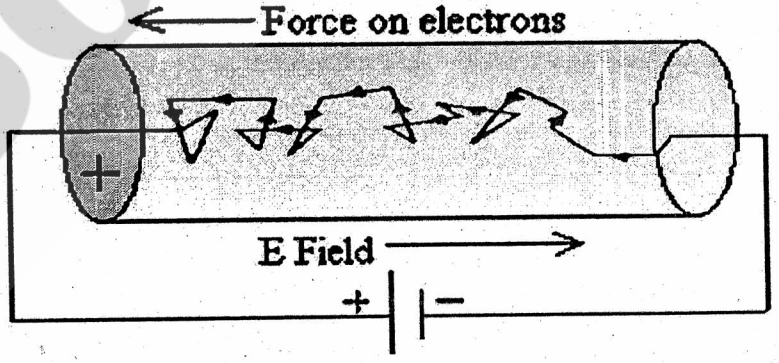
(iii) Current Carriers in gases -

Ordinarily, the gases are insulator of electricity. But they can be ionized by applying a high potential difference at low pressure or by their exposure to X rays etc. The ionized gas contains positive ions and electrons.

* Thus, positive ions and electrons are the current carriers in gases.

Drift Velocity -

It is defined as the average velocity with which the free electrons get drifted towards the positive end of the conductor under the influence of an external electric field applied.



Let E is the strength of applied electric field then the force on electrons due to applied field is -

$$F = eE \qquad (e = \text{Charge on electron})$$

If m is the mass of electron, then acceleration produced is given by $a = \frac{eE}{m}$

If τ is the relaxation time, which is average time that an electron spends between two collisions, then

$$a = \frac{V_d - 0}{\tau} \quad \text{or} \quad V_d = a\tau$$

then $$\boxed{V_d = \left(\frac{eE}{m}\right)\tau}$$

* The average relaxation time (τ) is of the order of 10^{-14} second.
* Average Relaxation Time = $\frac{\text{mean free path of electron}}{\text{drift speed of electron}}$

Mobility (μ)

Mobility of charge carriers, responsible for the current is defined as the magnitude of drift velocity of charge per unit electric field applied.

$$\boxed{\mu = \frac{\text{drift Velocity}}{\text{Electric Field}} = \frac{V_d}{E}}$$

or $$\mu = \frac{qE\tau/m}{E} = \frac{q\tau}{m}$$

Mobility of electrons $\boxed{\mu_e = \frac{e\tau_e}{m_e}}$

Mobility of holes $\boxed{\mu_h = \frac{e\tau_h}{m_h}}$

* SI Unit of mobility is m^2/Vs.

The mobility of electrons is greater than holes. because the electrons are free to move anywhere inside the conductor while holes move from bond to bond.

$\boxed{\mu_e > \mu_h}$

Q. RBSE 2006. Explain how electron mobility changes for a good conductor when (a) the temperature of the conductor is decreased at constant potential difference and (b) applied potential difference is doubled at constant temperature.

Sol.

The electron mobility $\boxed{\mu_e = \frac{e\tau}{m}}$

(a) μ_e increases with the decrease of temperature

(b) μ_e does not change with the increase of potential difference.

Q. A potential difference of 12 V is applied across a conductor of length 0.2 m. If the drift velocity of electron is 3×10^{-4} m/s then calculate the electron mobility.

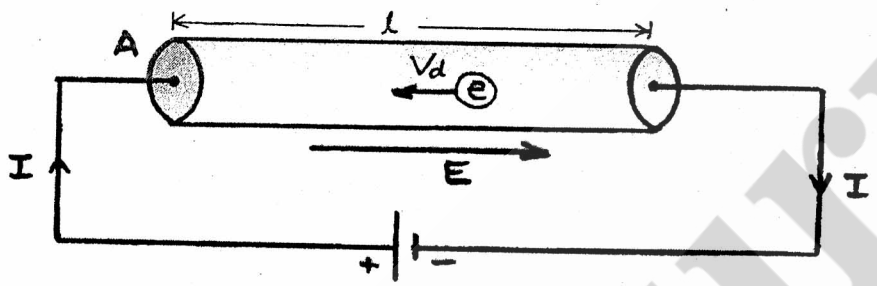
Sol. We Know $E = V/l$

So $E = 12/0.2 = 60$ V/m.

and $\mu = \frac{V_d}{E} = \frac{3 \times 10^{-4}}{60} = 5 \times 10^{-6}$ m^2/Vs

Relation between drift velocity and Current

Consider a conducting wire of length L and having uniform cross section area A in which electric field is present.



Let there are n free electrons per unit volume moving with the drift velocity V_d.

In time interval t each electron advances by a distance $V_d t$ and volume of this portion is $AV_d t$.

Number of free electron in this portion is = $nAV_d t$

all these electrons crosses the area A in time t, Hence charge crossing the area in time t is $Q = neAV_d t$

Current $I = Q/t$

or $$\boxed{I = neAV_d}$$

This is the relation between the electric current and drift velocity.

Q.* The drift velocity of electrons in a conductor is 10^{-4} m/s yet a bulb glows instantly when switched on, why?

Answer – When we close the circuit, the electric field is setup in the entire closed circuit instantly with the speed of electromagnetic waves which causes electron drift at every portion of the circuit. Due to it, the current is setup in the entire circuit instantly.

The current so set up does not wait for the electrons to flow from one end of the conductor to other end.

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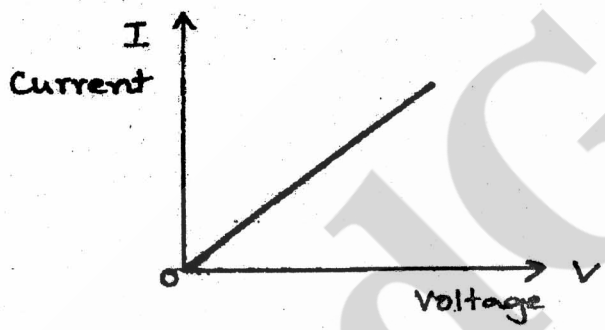
Ohm's Law -

The current flowing through a conductor is directly proportional to the potential difference across the ends of the conductor, if the physical state of the conductor (Temperature, mechanical strain etc.) remains unchanged.

Mathematically $V \propto I$

or $\boxed{V = IR}$

where proportionality constant R is called the resistance of the conductor.



* Value of resistance R depends upon the nature, dimensions of the conductor.
* SI Unit of resistance is ohm (Ω).

Resistance (R)

The resistance of conductor is the opposition offered by a conductor to the flow of electri current through it

$$1 \text{ ohm} = \frac{1 \text{ Volt}}{1 \text{ Ampere}} = \frac{1V}{1A}$$

Thus 1 ohm is the electrical resistance of a conductor through which a current of 1 ampere flows when a potential difference of 1 volt is applied across the ends of the conductor.

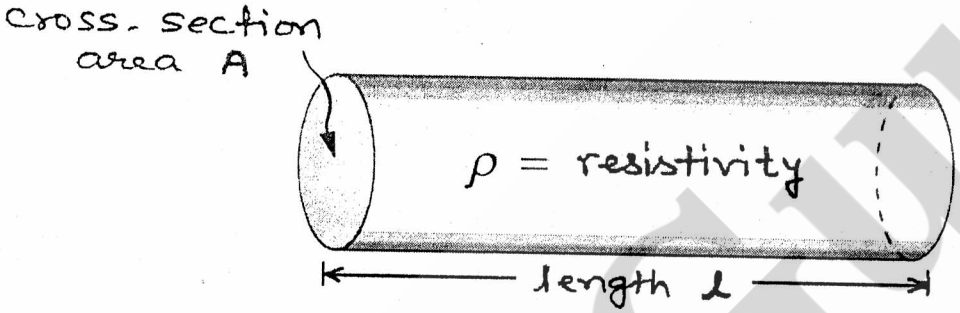
* Dimension of resistance is $[M^1 L^2 T^{-3} A^{-2}]$

Factors on which Resistance depends

(1) Length of Conductor -

Resistance of a conductor is directly proportional to the length of conductor.

$$R \propto l$$



(2) Area of Cross-Section of Conductor -

Resistance of a conductor is inversely proportional to the area of cross-section of conductor.

$$R \propto 1/A$$

(3) Nature of material & Temperature -

From above, $R \propto \frac{l}{A}$

Or $$R = \frac{\rho l}{A}$$

where ρ is known as specific resistance or electrical resistivity of the material of the conductor.

Electrical Resistivity (ρ) -

If $l = 1\ m$, $A = 1\ m^2$ then

$$R = \rho$$

Thus, the resistivity of the material of a conductor is defined as the resistance of a unit length with unit area of cross section of the material of the conductor.

* SI Unit of electrical resistivity is ohm-meter ($\Omega \cdot m$)

factors affecting Electrical Resistivity -

By ohms Law, $V = IR$

so $V = neAV_d R = neA\left(\frac{eE\tau}{m}\right)R$

$V = Rne^2A\left(\frac{V}{l}\right)\frac{\tau}{m}$ $\left(\because E = \frac{V}{l}\right)$

or $R = \frac{m}{ne^2\tau} \times \frac{l}{A} = \frac{\rho l}{A}$

so $\boxed{\rho = \frac{m}{ne^2\tau}}$

Thus resistivity of Conductor depends upon -

(i) $\rho \propto 1/n$. Since the value of n depends upon the nature of material, hence the resistivity of the conductor depends upon the nature of material.

(ii) $\rho \propto 1/\tau$. With the increase in temperature of metal conductor, τ decreases hence its ρ increases.

* $\rho \propto 1/n$, so that If $n \uparrow$ then $\rho \downarrow$
* $\rho \propto 1/\tau$, so that If $T \uparrow$ then $\tau \downarrow$ so $\rho \uparrow$

Q. Two wires A and B with circular cross-section have identical lengths and are made of the same material. Yet, wire A has four times the resistance of wire B. How many times greater is the diameter of wire B than wire A ?

Sol.

Variation of Resistivity / Resistance with Temperature

Resistivity of the metallic conductor increases with increase on temperature.

For small temperature variation, resistivity of the most of the metal varies according to the given relation -

$$\rho = \rho_0 [1 + \alpha (T - T_0)]$$

Where ρ and ρ_0 are the resistivity of the material at temperature T and T_0 respectively and α is the constant for given material and is known as coefficient of resistivity.

We know that $R = \rho L/A$

Hence temperature variation of the resistance can be given as -

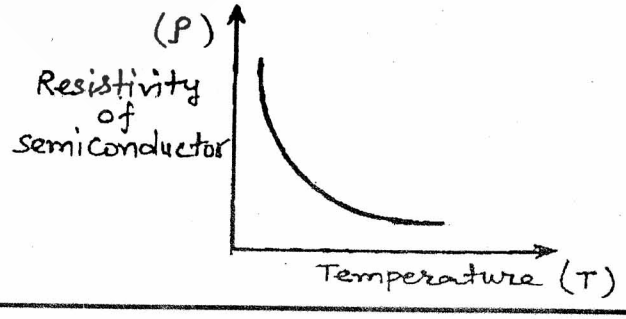
$$R = R_0 [1 + \alpha (T - T_0)]$$

or

$$\alpha = \frac{R - R_0}{R_0 \times (T - T_0)}$$

Thus temperature coefficient of resistance is defined as the increase in resistance per unit original resistance per degree celsius or Kelvin rise of temperature.

* For metals α is positive, therefore resistance of a metal increases with rise in temperature.
* For insulator and semiconductors α is negative, so the resistance decreases with rise in temperature.



Current Density - (J)

current density (J) at a point in a conductor is defined as the amount of current flowing per unit area of the conductor around that point provided the area is held in a direction normal to the current.

$$J = \frac{I}{A}$$

$$J = \frac{I}{A} = neV_d \quad \text{as} \quad I = neAV_d$$

The unit of current density is ampere/metre^2 [A/m^2]

Conductivity (σ) -

The inverse of resistivity (ρ) of a conductor is called its electrical conductivity.

Conductivity $\sigma = \frac{1}{\rho}$

The unit of electrical conductivity is mho/meter.

Conductance (G) -

The inverse of resistance (R) is called Conductance of a conductor.

Conductance $G = \frac{1}{R}$

The unit of Conductance is mho ($\mho$)

Relation between J, σ and E -

We know that $I = neAV_d$

$$I = neA\left(\frac{eE\tau}{m}\right) = \frac{ne^2A\tau E}{m}$$

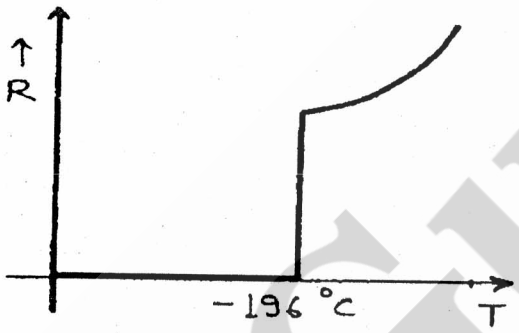
or $$\frac{I}{A} = \frac{ne^2\tau E}{m}$$

or $$J = \frac{1}{\rho}E$$

$$\therefore \quad J = \sigma E$$

Superconductivity -

In some metals and alloys when the material is cooled to a critical temperature (−196 °c) the resistance of the material falls to zero. This state of zero resistance is when the material become superconducting.



Use- Superconducting materials are used when very strong electromagnets are required, in MRI scanners or to reduce losses in power cables.

Super conductivity is used to produce very high speed computers.

Super conductors are used for the transmission of electric power.

Limitation of Ohm's Law / Non-Ohmic Devices-

Those devices which do not obey ohm's Law are called non-ohmic devices.

Examples - Vacuum tubes, semiconductor diode, transistor liquid electrolyte etc.

★ The semiconductor junction diode is also used to convert alternating current to direct current and are used to perform various logic functions.

The voltage - current relation for a diode is shown in the given figure.

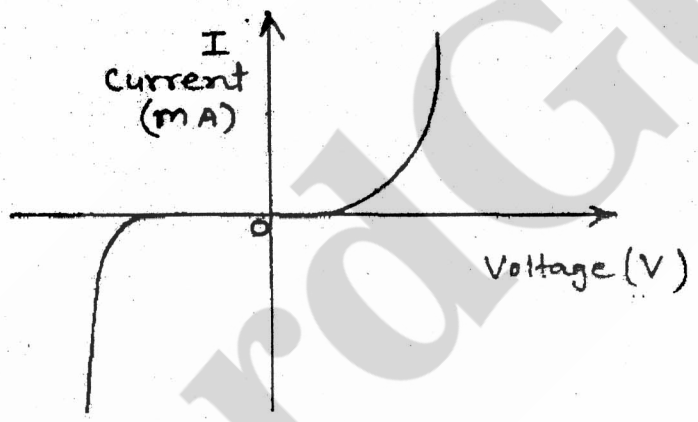
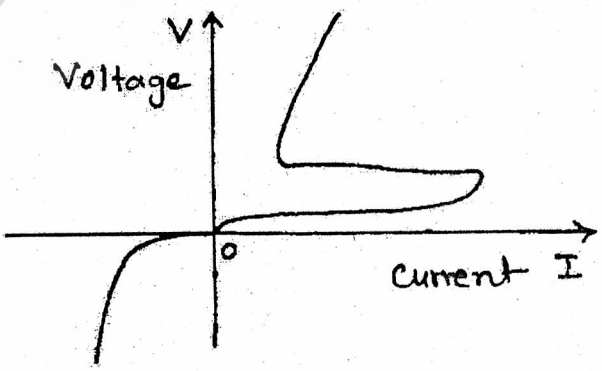


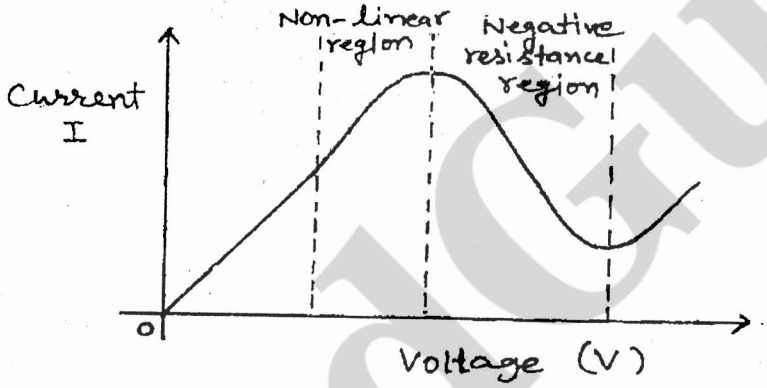
fig- Diode V-I characteristics

★ The following graph shows the characteristics of a device known as a thyristor, which consist of four alternating layers of P-type and N-type semiconductors. Here the current carried by the device increases as the voltage decreases.



★ A more fundamental breakdown of Ohm's Law occurs in certain alloys for example GaAs at very low temperatures (at 4K).

It is observed that the resistance of a uniform wire made of the alloy of length $2l$ is more than twice the resistance of the wire of length l. This can be understood in terms of the wave character of electrons, which leads to interference effects. This increases resistance beyond what would be if electrons flowed like water



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Color Code for Carbon Resistors -

Commercially resistors of different type and values are available in the market but in electronic circuits carbon resistors are more frequently used.

In carbon resistors value of resistance is indicated by four color bands marked on its surface as shown in figure.

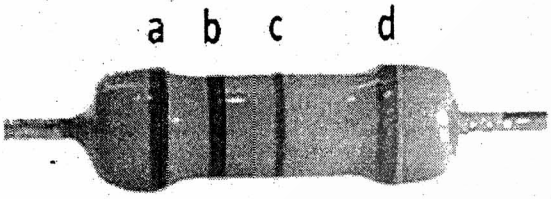


Fig: Carbon Resistor with color strips

The first three bands a, b and c determine the value of the resistance and fourth band d gives the tolerance of the resistance.

Color	Figure(first and second band)	Multiplier(for third band)	tolerance
Black	0	1	-
Brown	1	10	-
Red	2	10^2	-
Orange	3	10^3	-
Yellow	4	10^4	-
Green	5	10^5	-
Blue	6	10^6	-
Violet	7	10^7	-
Gray	8	10^8	-
white	9	10^9	-
Gold	-	10^{-1}	5%
Silver	-	10^{-2}	10%
No Color	-	-	20%

The color of first and second band respectively gives the first and second significant figure of resistance and third band c gives the power of the ten by which two significant digits are multiplied, to obtain the value of the resistance.

Q. In a given resistor the first strip be red, second strip be brown and third strip be orange and fourth be gold then calculate the resistance of the resistor.

Sol.

$$21 \times 10^3 \pm 5\%$$

Q. The resistance of the given carbon resistor is $2.4 \times 10^6\ \Omega \pm 5\%$. What is the sequence of colors on the strips provided on resistor?

Sol.

$$R = 2.4 \times 10^6\ \Omega \pm 5\%$$

or

$$R = 24 \times 10^5\ \Omega \pm 5\%.$$

The colors attached to the numbers 2, 4 and 5 are red, yellow and green. The color for 5% is gold.

Therefore, the colors of strips in sequence for the given carbon resistor are red, yellow, green and gold.

Q. Delhi 2010 The sequence of colored bands in two carbon resistor R_1 and R_2 is

(a) Brown, green, blue, Silver

(b) Orange, black, green

Find the ratio of their resistances.

Sol. According to color codes, resistance of two resistors are -

(a) $R_1 = 15 \times 10^6\ \Omega \pm 10\%$

(b) $R_2 = 30 \times 10^5\ \Omega \pm 20\%$

∴ Ratio of their resistances $\frac{R_1}{R_2} = \frac{15 \times 10^6}{30 \times 10^5}$

so $\frac{R_1}{R_2} = 5$

Q. CBSE 2008 Two metallic wires of the same material have the same length but cross sectional area is in the ratio 1 : 2. They are connected (a) in series and (b) in parallel. Compare the drift velocities of electrons in the two wires in both the cases.

Sol. Let each of two conductors is of length L.
Since, drift velocity of electron is given by -

$$V_d = \frac{eE\tau}{m} = \frac{e\tau V}{m l} \qquad \left(\because E = \frac{V}{l}\right)$$

$$V_d = \left(\frac{e\tau}{m}\right) \times \frac{V}{L}$$

At given temperature, $\boxed{V_d \propto \frac{1}{l}}$ and potential difference.

(a) In series combination effective length $L_1 = 2l$

(b) In parallel Combination effective length $L_2 = l$.

$$\therefore \quad \frac{V_{d_1}}{V_{d_2}} = \frac{l_2}{l_1} = \frac{l}{2l}$$

$$\text{So} \qquad V_{d_1} : V_{d_2} = 1 : 2$$

Q. CBSE 2007 A voltage of 30 V is applied across a carbon resistor with first, second and third rings of blue, black and yellow colors respectively. What is the value of current in mA, through the resistor.

Sol. Resistance of Carbon resistor

$$R = 60 \times 10^4\ \Omega \pm 20\ \%$$

$\therefore$ Current through the resistor $I = \frac{V}{R}$

$$I = \frac{30}{60 \times 10^4} = 0.5 \times 10^{-4}\ A$$

$$\text{or} \qquad I = 0.05 \times 10^{-3}\ A = 0.05\ mA.$$

Q. CBSE 2009 A wire of 15 Ω resistance is gradually stretched to double its original length. It is then cut into two equal parts. These parts are then connected in parallel across a 3 V battery. Find the current drawn from the battery.

Sol: Let original cross-sectional are and length of 15 Ω resistance are A and L and after stretching they become A' and L' respectively.

Initial resistance $R = \rho L/A = 15$

In case of stretching, volume of the wire remains same, so

$$AL = A'L'$$

so that $A' = \frac{A}{2}$ $\quad [L' = 2L]$

∴ Resistance after stretching

$$R' = \frac{\rho L'}{A'} = \rho\left(\frac{2L}{A/2}\right) = 4\left(\frac{\rho L}{A}\right)$$

so $R' = 4R = 4 \times 15$

Now resistance $R' = 60\ \Omega$

After dividing into parts, resistance of each part is 30 Ω.

∴ Effective resistance after connecting them into parallel combination is

$$R_{effective} = \frac{30}{2} = 15\ \Omega$$

Applied Potential diff., $V = 3V$

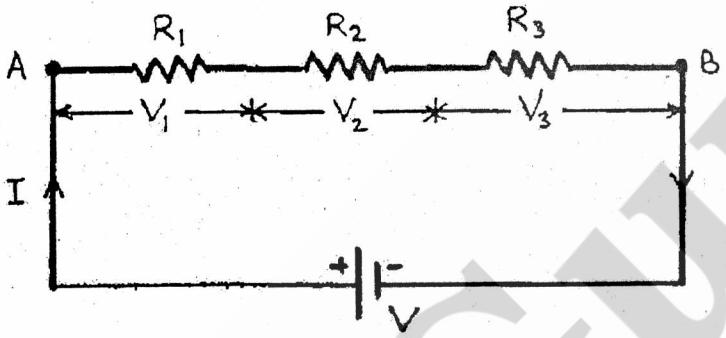
∴ Current drawn from the battery $I = \frac{V}{R_{effective}}$

$$I = \frac{3}{15} = \frac{1}{5}\ A$$

Combination of Resistors

(A) Resistors in Series -

Resistors are said to be connected in series combination, if same current flows through each resistor when same potential difference is applied across the combination.



Let three resistors of resistance R_1, R_2 and R_3 are connected in series combination.

If V_1, V_2 and V_3 is the potential differene across each resistor R_1, R_2 and R_3 respectively due to the battery of potential difference V then

According to Ohm's Law -

$$V_1 = IR_1 \text{ , } V_2 = IR_2 \text{ , and } V_3 = IR_3$$

Since in series combination current remains same but potential is divided so.

$$\boxed{V = V_1 + V_2 + V_3}$$

or $$IR_s = IR_1 + IR_2 + IR_3$$

If R_s is the equivalent resistance of the series combination of R_1, R_2 and R_3 then -

$$V = IR_s$$

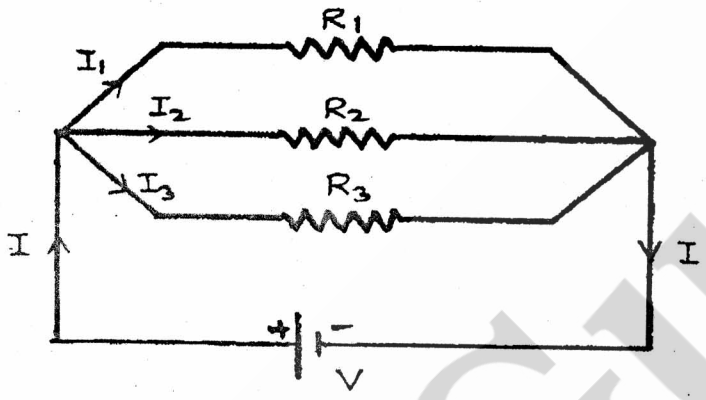
where

$$\boxed{R_s = R_1 + R_2 + R_3}$$

Thus equivalent resistance of a number of resistors Connected in series is equal to the sum of individual resistances.

(B) Resistors in parallel -

Resistors are said to be connected in parallel combination if potential difference across each resistance is same.



Let a battery of potential difference V is connected across parallel combination of resistors R_1, R_2 and R_3.

Total current in the circuit is -

$$I = I_1 + I_2 + I_3$$

by applying Ohm's Law, we get -

$$V = I_1 R_1 = I_2 R_2 = I_3 R_3$$

Or $$\frac{V}{R_p} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

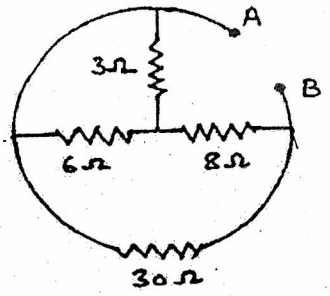
$$\boxed{\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Where R_p is the equivalent resistance of parallel combination.

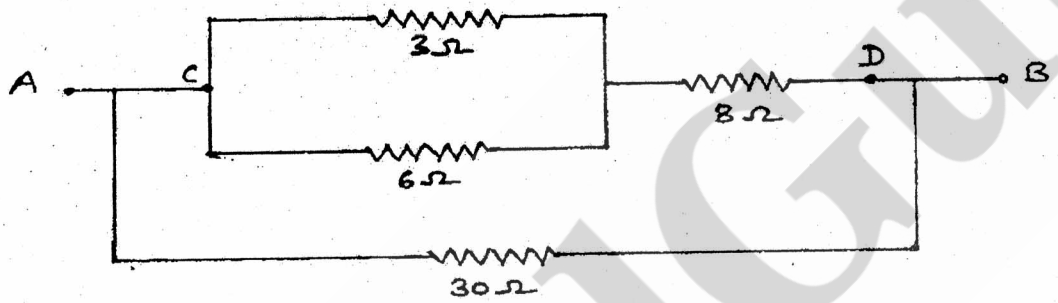
Hence for resistors connected in parallel combination, reciprocal of equivalent resistance is equal to the sum of reciprocal of individual resistances.

* The value of equivalent resistance for resistors connected in parallel combination is always less than the value of the smallest resistance in circuit.

Q. Find the equivalent resistance between A and B of the given circuit.



Sol. Equivalent circuit of the given circuit is -



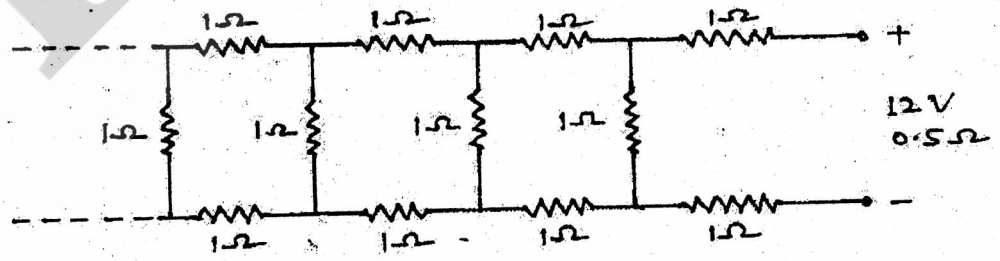
Equivalent resistance between C and D is

$$R_{CD} = \frac{3 \times 6}{3+6} + 8 = 10\ \Omega$$

Now in the given figure 10 Ω and 30 Ω are in parallel so equivalent resistance of the given circuit is -

$$R_{eq} = \frac{10 \times 30}{10 + 30} = 7.5\ \Omega$$

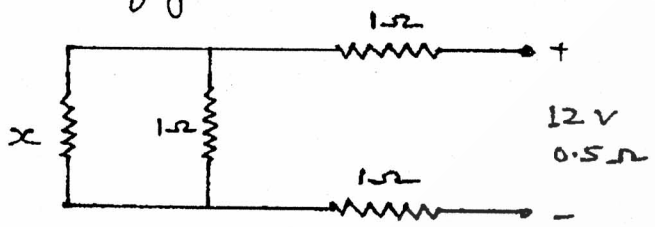
Q. Determine the current drawn from a 12 V supply with internal resistance 0.5 Ω by the following infinite network; each resistor has 1 Ω resistance.



Sol: Let x be the equivalent resistance of infinite network.

Since the network is infinite, therefore the addition of one more unit of three resistances each of value 1 Ω across the terminals will not change the total resistance of network i.e. it should remain x.

Therefore the network would appear as shown in the given figure and its total resistance remain x.



Here the parallel combination of x and 1 Ω is in series with the two resistors of 1 Ω each.

The resistance of parallel combination is –

$$\frac{1}{R_P} = \frac{1}{x} + \frac{1}{1} = \frac{1+x}{x}$$

or $$R_P = \frac{x}{1+x}$$

Total resistance of the network will be given by.

$$x = 1+1+\frac{x}{x+1} = 2+\frac{x}{x+1}$$

$$x(x+1) = 2(x+1)+x$$

$$x^2+x = 2x+2+x$$

or $$x^2 - 2x - 2 = 0$$

or $$x = \frac{2 \pm \sqrt{4+8}}{2} = \frac{2 \pm \sqrt{12}}{2}$$

$$x = 1 \pm \sqrt{3}$$

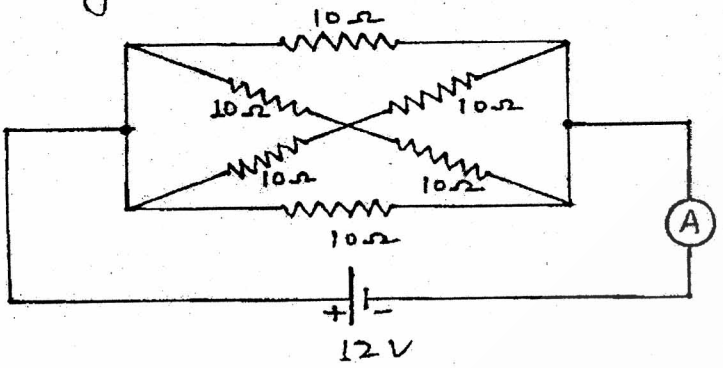
The value of resistance can not be negative, therefore the resistance of the network is.

$$x = 1+\sqrt{3} = 2.73\ \Omega$$

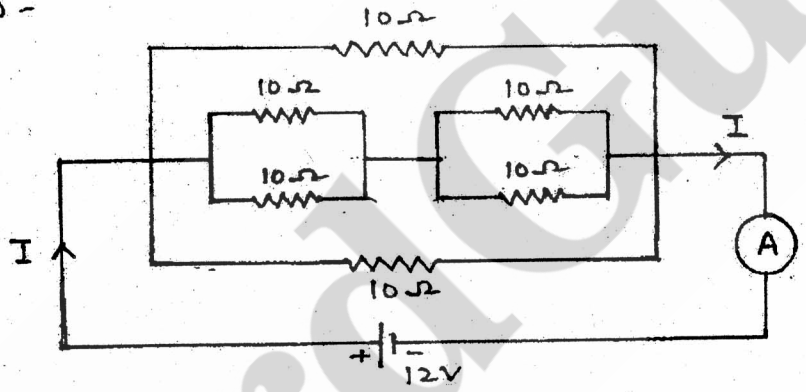
Total resistance of the network = 2.73 + 0.5 = 3.23 Ω

∴ Current drawn $I = \frac{12}{3.23} = 3.72$ A

Q. Calculate the current shown by the ammeter A in the given circuit.



Sol. The equivalent circuit of given network is as given below-



The resistance of the middle arm is -

$$= \frac{10 \times 10}{10+10} + \frac{10 \times 10}{10+10}$$

$$= 5 + 5 = 10\ \Omega$$

So the Effective resistance of the circuit is -

$$\frac{1}{R} = \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{3}{10}$$

$$\text{So} \qquad R = \frac{10}{3}\ \Omega$$

Current Shown by ammeter $I = \frac{V}{R}$

$$I = \frac{12}{10/3} = 3.6\ A$$

Electrical Power -

The rate at which electric work is done by the source of emf in maintaining the current in electric circuit is called electric power of the circuit.

$$\text{Electrical Power} = \frac{\Delta U}{\Delta t} = \frac{V \times \Delta q}{\Delta t}$$

$$\boxed{P = V \times I = I^2R = \frac{V^2}{R}}$$

Unit of electrical Power is 'Watt'.

1 Watt -

The power of an electrical circuit is said to be one watt if one ampere current flows in it against a potential difference of 1 volt.

Electrical Energy -

The total work done in maintaining the current in electric circuit by the source of emf is called electrical energy given to the circuit.

$$\text{Electrical Energy} = \text{Electrical Power} \times \text{time}$$

$$W = (V \times I) \times t$$

$$\text{or} \quad W = I^2Rt = \frac{V^2}{R}t$$

If this work done appears as heat then amount of heat produced (H) is given by -

$$\boxed{H = W = I^2Rt = \frac{V^2}{R}t}$$

Above equation is known as Joule's law of heating.

EMF, Internal Resistance and Terminal Potential Difference of a Cell -

EMF- EMF of a cell is the maximum potential difference between two electrodes of the cell when no current is drawn from the cell. It is denoted by E.

Internal Resistance -

Internal resistance of a cell is defined as the resistance offered by the electrolyte and electrodes of a cell when the electric current flows through it.

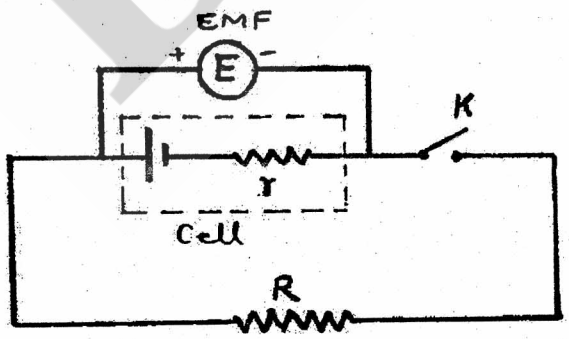
Internal Resistance of a cell depends on the following factors -

(i) Distance between the electrodes
(ii) Nature, Concentration and temperature of electrolyte
(iii) The nature of Electrodes.
(iv) Area of Electrodes, immersed in the electrolyte

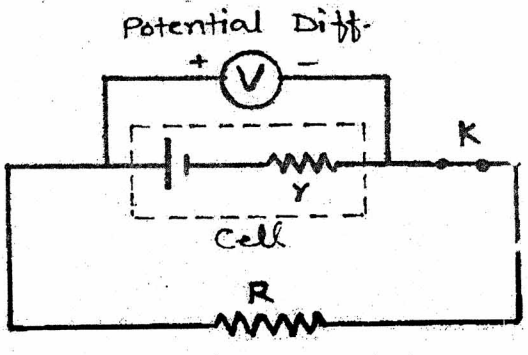
Terminal Potential Difference -

Terminal Potential difference of a cell is defined as the potential difference between the two electrodes of a cell in a closed circuit, when current is drawn from the cell. It is denoted by V.

* The terminal potential difference of a cell is always less than the emf of the cell because of the fall of potential across the internal resistance of the cell.



Open Circuit (EMF is measured)



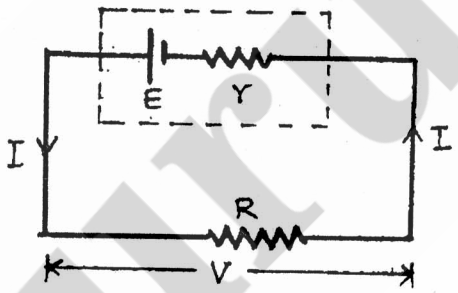
Closed Circuit (P.D. is measured)

According to the definition of terminal potential difference -

$$V = E - Ir$$

Internal Resistance of a Cell in terms of E, V & R

The terminal potential difference of a cell is equal to the potential difference across the external resistance R of the circuit, so -



$$V = IR$$

or $$I = \frac{V}{R}$$

We know that. $$Ir = E - V$$

So $$r = \frac{E-V}{I} = \frac{E-V}{V/R}$$

$$r = \left(\frac{E}{V} - 1\right) R$$

Difference between E.M.F. and Potential difference

S.N.	EMF	S.N.	Potential Difference
1.	The emf of a cell is the maximum potential difference between the two points when the cell is in the open circuit	1.	The potential difference between the two points is the difference of potential between those two points in a closed circuit.
2.	It is independent of the resistance of the circuit and depends upon the nature of electrodes and electrolyte of cell.	2.	It depends upon the resistance between the two points of the circuit and current flowing through the circuit.
3.	The term emf is used only for the source of emf.	3.	It is measured between any two points of the circuit.
4.	It is a cause	4.	It is an effect.

Q. CBSE 2008 A (a) series (b) Parallel Combination of two given resistors is connected, one-by-one across a cell. In which case will the terminal potential difference, across the cell have a higher value?

Sol. We know that equivalent resistance of the combination of resistance is - (a) greater than the greatest resistance in series combination and (b) smaller than the least value of resistance in parallel combination.

The terminal potential difference across the cell is higher in series combination as

$$V = E - Ir$$

and due to higher resistance current I is less in series combination.

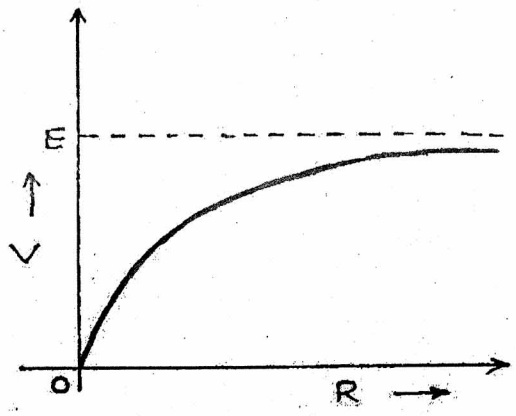
Q. Delhi 2009 A cell of emf E and internal resistance r is connected across a variable resistor R. Plot a graph showing the variation of terminal potential V with resistance R.

Sol. Terminal Voltage V is given by -

$$V = IR = \frac{E}{(R+r)} \times R$$

$$V = \frac{E}{(1 + r/R)}$$

$\Rightarrow$ so V increases with the increase of R.



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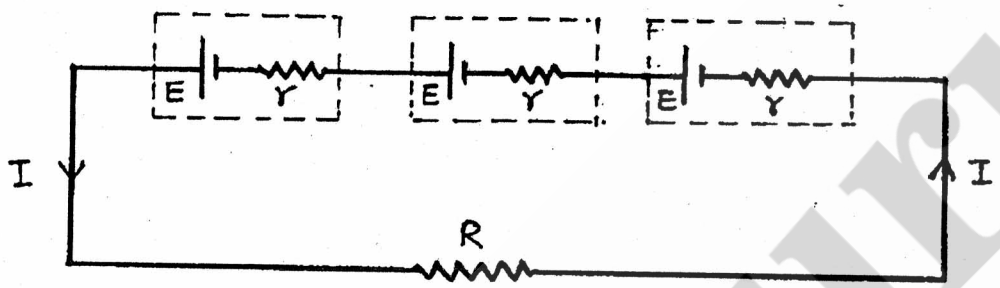
Cells in Series Combination -

Cells are connected in series when they are joined end to end so that the same quantity of electric current must flow through each cell.



Note-

(1) The emf of the battery is the sum of the individual emfs.

(2) The total internal resistance of the battery is the sum of the individual internal resistances.

Total emf of the battery = nE (For n cells)

Total internal resistance of the battery = nr

Total resistance of the circuit = $nr+R$

So $$\text{Current } I = \frac{nE}{nr+R}$$

(a) If $R << nr$, then $I = E/r$

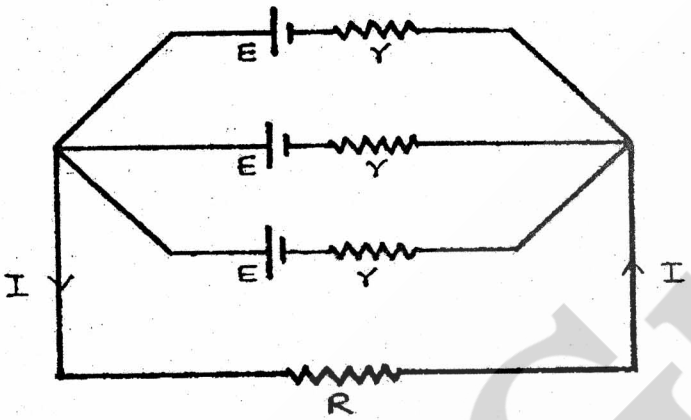
(b) If $R >> nr$, then $I = nE/R$

Conclusion -

When internal resistance is negligible in comparison to the external resistance, then the cells are connected in series to get maximum current.

Cells in parallel Combination -

Cells are said to be connected in parallel, if positive terminal of each cell is connected to one point and negative terminal of each cell is connected to the other point.



Note -

(1) The equivalent emf of the cells is the same as that of a single cell.

(2) The reciprocal of the total internal resistance is the sum of the reciprocal of the individual internal resistances.

Total EMF of the battery = E

Total internal resistance of the battery = r/n

Total resistance of the circuit = $\frac{r}{n} + R$

So

$$\text{Current} \quad I = \frac{nE}{nR + r}$$

(a) If $R << r/n$, then $I = nE/r$

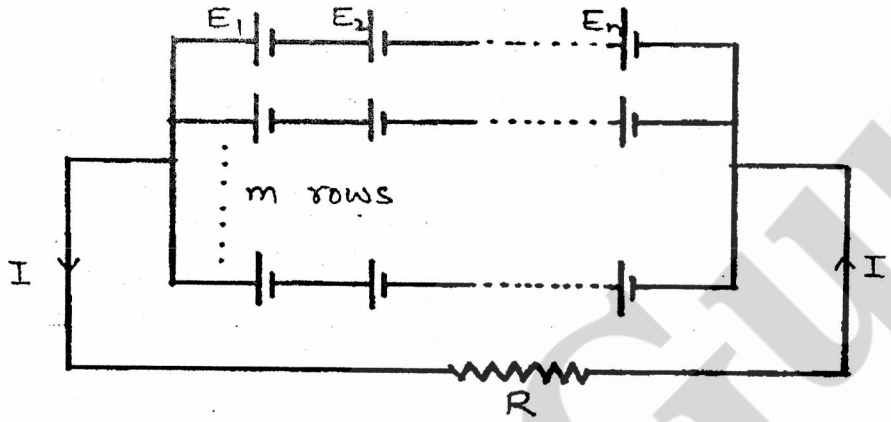
(b) If $R >> r/n$, then $I = E/R$

Conclusion -

When external resistance is negligible in comparison to the internal resistance, then the cells are connected in parallel to get maximum current.

Mixed Grouping of Cells -

If some cells are connected in series to form a row and number of such rows of cells are connected in parallel, then they are said to be connected in mixed grouping.



The emf of cells in a row = nE

Total emf of the combination = nE

Total internal resistance = $\frac{nr}{m}$

Total internal resistance of the circuit = $R + \frac{nr}{m}$

so $$\boxed{\text{Current } I = \frac{nE}{R + nr/m}}$$

or $$I = \frac{mnE}{(mR + nr)}$$

The current I will be maximum if $(mR + nr)$ is minimum.

$$\left[\left(\sqrt{mR}\right)^2 + \left(\sqrt{nr}\right)^2 - 2\sqrt{mnRr}\right] + 2\sqrt{mnRr} = \text{minimum}$$

or $$\left(\sqrt{mR} - \sqrt{nr}\right)^2 + 2\sqrt{mnRr} = \text{minimum}$$

It will be so if $\sqrt{mR} - \sqrt{nr} = 0$

or $$\sqrt{mR} = \sqrt{nr}$$

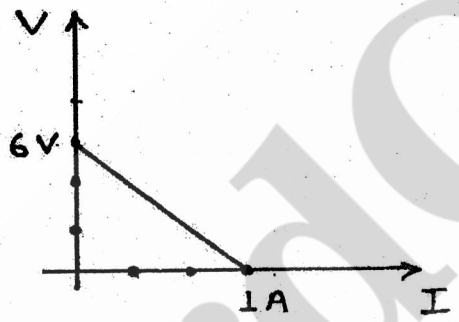
or $$mR = nr$$

$$R = \frac{nr}{m} \quad \text{or} \quad mR = nr$$

Conclusion –

For the maximum current to flow through the external circuit, the external resistance should be equal to the total internal resistance.

Q. Delhi 2008 The plot of the variation of potential difference across a combination of three identical cells in series versus current is as shown in figure. What is the emf of each cell?



Sol. Terminal Potential difference across the cell combination can be given as –

$$V = E - Ir$$

when $I = 0$ then $V = E$

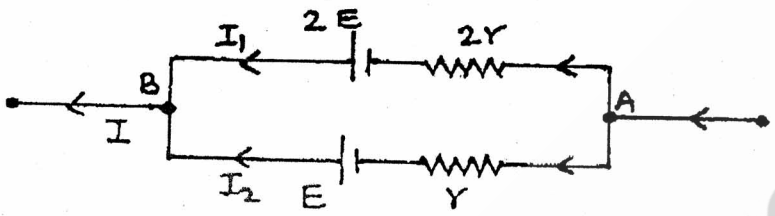
So that from the graph when $I = 0$, $V = 6$ Volt

Hence emf $E = 6$ Volt.

BoardGuru

Q. CBSE 2020 Two cells of emf 2E and E, and interal resistance 2r and r respectively, are connected in parallel. Obtain the expressions for the equivalent emf and the internal resistance of the combination.

Sol.



Net Current $I = I_1 + I_2$ —— (1)

For cell – (1)

$$V = 2E - I_1(2r)$$

$$\Rightarrow I_1 = \frac{2E - V}{2r} \quad \text{—— (2)}$$

For Cell – (2)

$$V = E - I_2 r$$

$$\Rightarrow I_2 = \frac{E - V}{r} \quad \text{—— (3)}$$

by substituting the value of I_1, I_2 in eq. (1) –

$$I = \frac{2E - V}{2r} + \frac{E - V}{r}$$

on rearrangin the term, we get –

$$V = \frac{4E}{3} - I\left(\frac{2r}{3}\right)$$

But the equivalent of Combination

$$V = E_{eq} - I(r_{eq.})$$

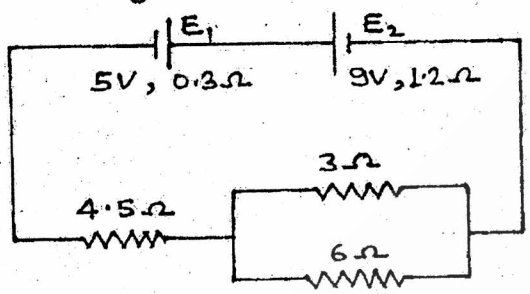
On Comparing,

$$E_{eq} = \frac{4E}{3} \quad \text{and} \quad r_{eq} = \frac{2r}{3}$$

Note - For parallel Combination of Two cells

$$E_{eq} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} \quad \text{and} \quad r_{eq} = \frac{r_1 r_2}{r_1 + r_2}$$

Q. Delhi 2006 Two cells E_1 and E_2 in the given circuit diagram have an emf of 5V and 9V and internal resistance of 0.3 Ω and 1.2 Ω respectively. Calculate the value of current flowing through the resistance of 3 Ω.



Sol.

Net emf = 9 − 5 = 4 V

Equivalent resistance of 6 Ω and 3 Ω is

$$= \frac{6 \times 3}{6+3} = 2\ \Omega$$

Net resistance across the circuit

$$R = 0.3 + 1.2 + 2 + 4.5$$

$$R = 8\ \Omega$$

So current $I = \frac{V}{R} = \frac{4}{8} = 0.5$ A

Potential difference across 4.5 Ω is

$$V = IR$$

$$V = 0.5 \times 4.5 = 2.25 \text{ Volt}$$

∴ Potential difference across 6 Ω and 3 Ω

$$= 4 - 2.25 = 1.75 \text{ Volt}$$

∴ Current through resistance 3 Ω

$$I' = \frac{V}{R} = \frac{1.75}{3} = 0.586 \text{ A}$$

Q. CBSE 2007 A cylinderical metallic wire is stretched to increase its length by 10 %. Calculate the percentage change in its resistance.

Sol. Volume of the wire $V = Al$

$$\Rightarrow \frac{\Delta V}{V} \times 100 = \frac{\Delta A}{A} \times 100 + \frac{\Delta L}{L} \times 100$$

But $\Delta V = 0$ (Volume remains Constant)

$$\Rightarrow \frac{\Delta A}{A} \times 100 = -\frac{\Delta l}{l} \times 100$$

Negative sign imply that increase in length of conductor would create decrease in area.

$\because R = \rho L/A$

For given resistor ρ = Constant

$$\text{So } \frac{\Delta R}{R} \times 100 = \left(\frac{\Delta l}{l} \times 100\right) - \left(\frac{\Delta A}{A} \times 100\right)$$

Here $\frac{\Delta l}{l} \times 100 = 10\ \%$

Therefore $\frac{\Delta A}{A} \times 100 = -10\ \%$

$$\therefore \frac{\Delta R}{R} \times 100 = 10\ \% - (-10\ \%)$$

$$\frac{\Delta R}{R} \times 100 = 20\ \%$$

$\therefore$ Percentage Rise of resistance = 20 %.

Kirchhoff's Law

(1) Kirchhoff's Current Law (KCL)-

The algebric sum of the currents meeting at a junction in a closed electric circuit is zero.

$$I_{exiting} + I_{entering} = 0$$
$$\sum I = 0$$

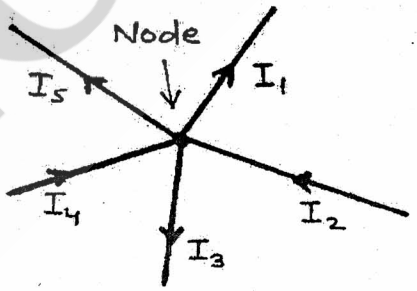
Sign Convention-

(1) The incoming currents towards the junction are taken positive.

(2) The outgoing currents away from the junction are taken negative.

By Kirchhoff's Current Law-

$$I_2 + I_4 = I_1 - I_3 - I_5 = 0$$



* Kirchoff's current Law supports law of conservation of charge.

The charges cannot accumulate at a junction. The number of charges that arrive at a junction in a given time must leave in the same time in accordance with conservation of charges.

(2) Kirchhoff's Voltage Law (KVL)-

The algebric sum of all the potential drops and emf's along any closed path in an electrical network is always zero.

$$\sum \Delta V = 0$$

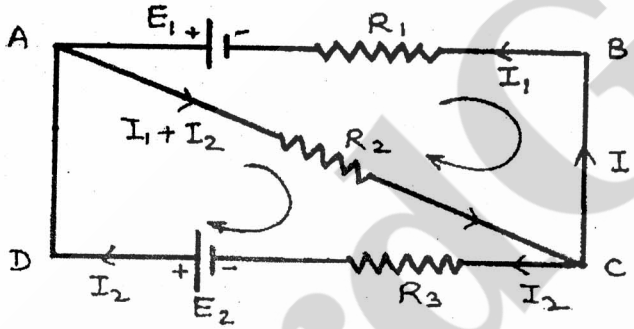
Sign Convention-

(1) The emf is taken positive when we traverse from positive to negative terminal of the cell.

(2) The emf is taken negative when we traverse from negative to positive terminal of the cell.

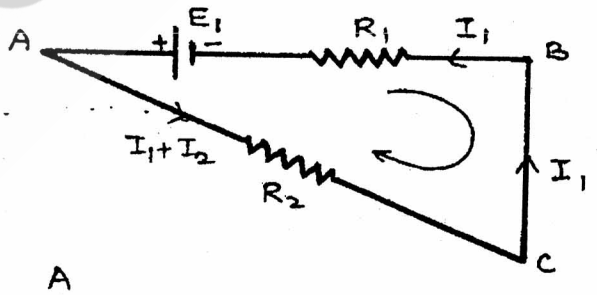
(3) If the resistor is being traversed in the direction of the current then potential difference across it, is taken positive i.e. IR.

(4) If the resistor is being traversed in the direction opposite to the current then potential difference across it, is taken negative i.e. -IR.



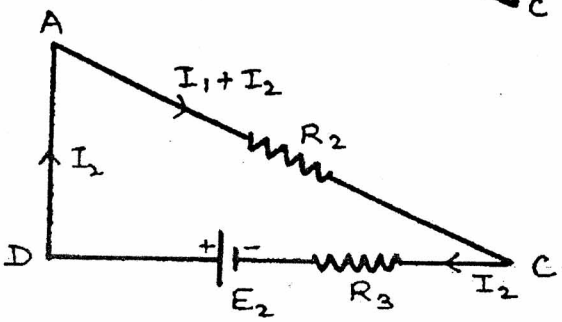
For Loop ABCA

$$E_1 - I_1R_1 - (I_1+I_2)R_2 = 0$$



For Loop ACDA

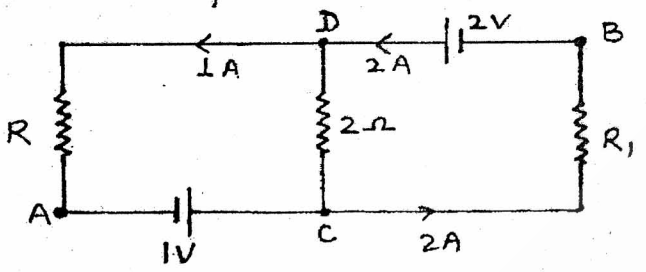
$$(I_1+I_2)R_2 + I_2R_3 - E_2 = 0$$



* Kirchoff's Voltage Law (KVL) supports the Law of conservation of Energy.

Q. All India 2011

In the given circuit, assuming point A to be at zero potential, use Kirchhoff's Laws to determine the potential at point B.



Sol.

By KCL at point D

$$I_{DC} + 1 = 2$$
$$I_{DC} = 1\ A$$

Along the ACDBA

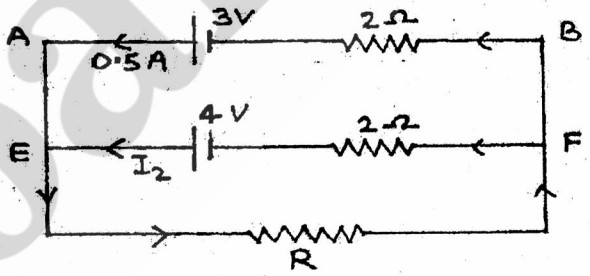
$$V_A - V_B = -1 - 2\times1 + 2 = -1$$

But $V_A = 0$, so that

$$V_B = 1\ V$$

Q. All India 2011

Using Kirchhoff's rules in the given circuit, find
(a) the voltage drop across the unknown resistor R.
(b) the current I_2 in the arm EF.



Sol. Applying KVL in the closed loop ABFEA

$$3 - 1 + 2I_2 - 4 = 0$$
$$I_2 = 1\ A$$

So the current I_2 in the arm EF is 1 A.

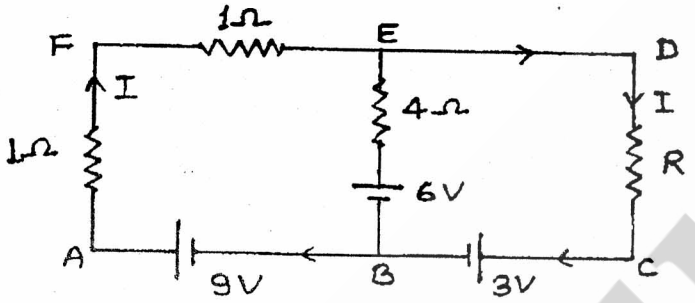
Potential drop across R, EF and AB is same because these are parallel.

So that $V = V_E - V_F = 4 - 2\times1$

$V = 2$ Volt.

BoardGuru

Q. Delhi 2012 Using Kirchoff's rules, determine the value of unknown resistance R in the circuit, so that no current flows through 4Ω resistance. Also find the potential difference between points A and D.



Sol. Given that no current should flow through 4Ω resistance, so Let Current I flows through the loop AFEDCBA.

By applying KVL in loop AFEBA -

$$1 \times I + 1 \times I + 6 - 9 = 0$$

So $$I = \frac{3}{2} A$$

Now applying KVL in loop AFDCA -

$$1 \times \frac{3}{2} + 1 \times \frac{3}{2} + R \times \frac{3}{2} + 3 - 9 = 0$$

$$R \times \frac{3}{2} = 3$$

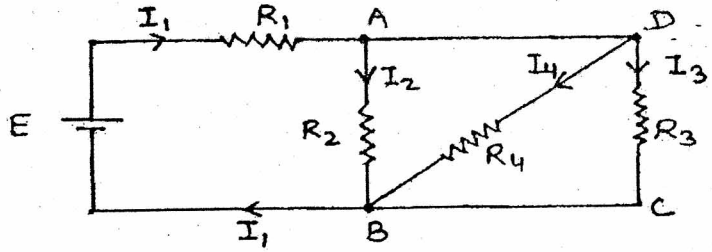
So $$R = 2\ \Omega$$

For potential difference across points A and D, along AFED -

$$V_A - V_D = 1 \times \frac{3}{2} + 1 \times \frac{3}{2}$$

$$V_A - V_D = 3\ V$$

Q. Delhi 2011 In the given circuit $R_1 = 4\Omega$, $R_2 = R_3 = 15\Omega$, $R_4 = 30\Omega$ and $E = 10\ V$. Calculate the equivalent resistance of the circuit and the current in each resistor.



Sol. According to the figure R_2, R_3 and R_4 are in parallel, so their equivalent resistance R' is -

$$\frac{1}{R'} = \frac{1}{15} + \frac{1}{30} + \frac{1}{15} = \frac{5}{30} = \frac{1}{6}$$

so that $R' = 6\Omega$

Now R' ($= 6\Omega$) is in series with 4Ω, so their equivalent resistance R_{eq} is -

$$R_{eq} = R' + 4 = 6 + 4$$

$$R_{eq} = 10\ \Omega$$

By KCL at node A we get -

$$I_1 = I_2 + I_3 + I_4 \quad \text{---} \quad (1)$$

By Applying KVL in loop ADBA -

$$I_4 \times 30 - 15 \times I_2 = 0$$

$$I_4 = I_2/2$$

By KVL in loop BDCB -

$$-30 \times I_4 + 15 \times I_3 = 0$$

$$I_4 = I_3/2$$

By applying KVL in loop ABEA -

$$4I_1 + 15I_2 - 10 = 0$$

$$4I_1 + 15I_2 = 10 \quad \text{---} \quad (2)$$

By applying KVL in loop ABCDA -

$$15I_2 - 15I_3 = 0$$

$$I_2 = I_3$$

By equation (1) we get -

$$I_1 = I_2 + I_3 + \frac{I_2}{2} \Rightarrow I_1 = \frac{5}{2} I_2$$

From equation (2) we get -

$$4 \times \frac{5}{2} I_2 + 15 I_2 = 10$$

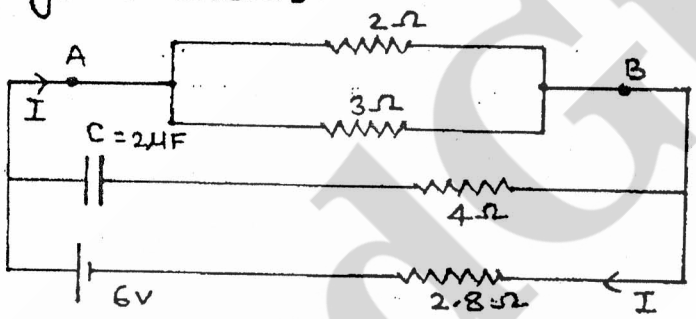
$$I_2 = \frac{10}{25} = \frac{2}{5} \text{ A}$$

So $I_2 = I_3 = \frac{2}{5}$ A

and $I_4 = \frac{I_2}{2} = \frac{1}{5}$ A

and $I_1 = \frac{5}{2} I_2 = \frac{5}{2} \times \frac{2}{5} \Rightarrow I_1 = 1$ A

Q. CBSE 2010 Calculate the steady current through the 2Ω resistor in the given circuit



Sol. No current flows through 4Ω resistor in steady state as Capacitor offers infinite resistance in DC circuit.

Here 2Ω and 3Ω are in parallel combination

So $R_{AB} = \frac{2 \times 3}{2+3} = 1.2$ A

So Equivalent Resistance of the circuit is

$$R_{eq} = 1.2 + 2.8 = 4\ \Omega$$

so total Current $I = \frac{6}{4} = 1.5$ A

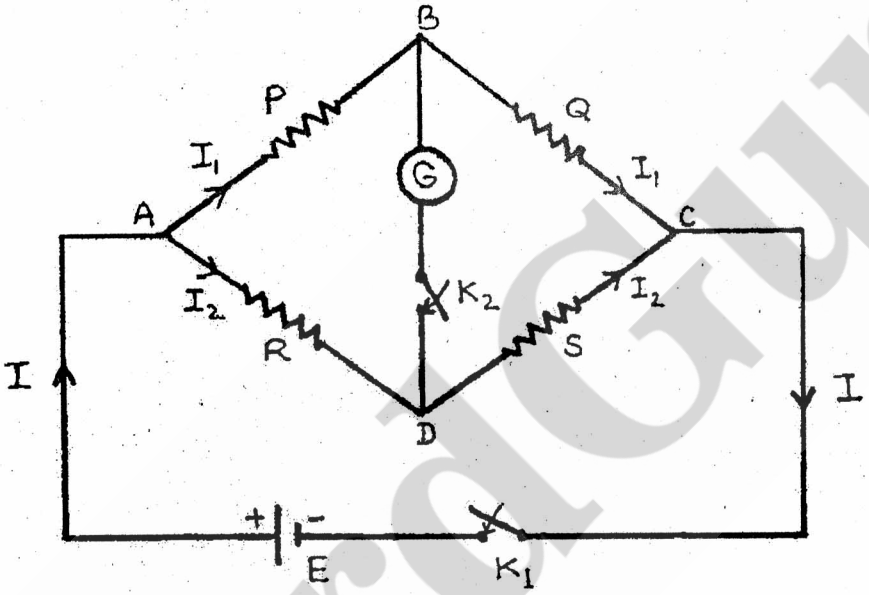
and Current through 2Ω resistor is given by -

$$I' = 1.5 \times \frac{3}{(2+3)} \Rightarrow I' = 0.9 \text{ A}$$

Wheatstone Bridge Principle -

It is an arrangement of four resistors used to determine resistance of one resistors in terms of other three resistors.

Consider the figure given below, which is an arrangement of resistors and is known as wheatstone bridge.



Wheatstone bridge consist of four resistance P, Q, R and S, with a battery of emf E.

On pressing Key K_2 if galvanometer does not show any deflection then wheatstone bridge is said to be balanced.

Galvanometer is not showing any duflection this means that no current is flowing through the galvanometer and derminal B and D are at the same potential.

Thus for a balanced bridge $\boxed{V_B = V_D}$

Now we have to find the condition for the balanced wheatstone bridge. For this by applying KVL to the loop ADBA -

$$I_2R - I_1P = 0$$

or $$I_1 P = I_2 R \quad \text{— (1)}$$

Again by applying KVL to the loop BCDB -

$$I_1 Q - I_2 S = 0$$

or $$I_1 Q = I_2 S \quad \text{— (2)}$$

from equation (1) and (2) we get -

$$\frac{I_1}{I_2} = \frac{R}{P} = \frac{S}{Q}$$

or $$\boxed{\frac{P}{Q} = \frac{R}{S}}$$

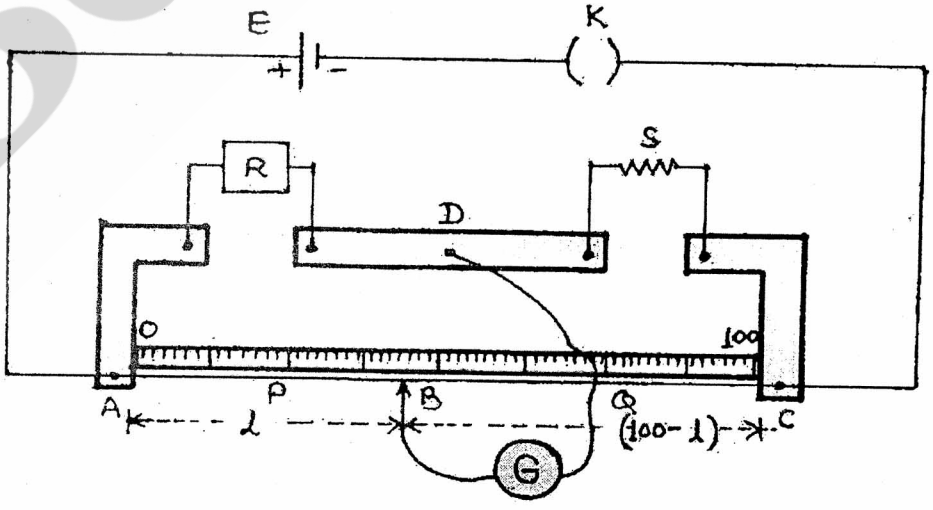
This equation gives the condition for the balanced Wheatstone bridge.

Thus if the ratio of the P and Q and the resistance R is known then unknown resistance S can easily be calculated.

Meter Bridge -

A slide wire bridge is a practical form of Wheatstone bridge.

It consist of a wire AC of constantan or manganin of 1 meter length and of uniform area of cross-section.



The wire is stretched between the copper strips on a horizontal wooden board. A meter scale is also fitted on the wooden board parallel to the length of the wire.

There is another copper strip fitted on the wooden board in order to provide two gaps in strips. Across one gap, a resistance box R and in another gap the unknown resistance S are connected.

A jokey B can be slided on the bridge wire. One terminal of the galvanometer G is connected to jokey B and the other to the terminal D.

The positive pole of the battery E is connected to terminal A and the negative pole of the battery to terminal C through Key K.

Working -

Close key K and take out suitable resistance R from resistance box. Adjust the position of jockey on the wire (say at B) where on pressing, the galvanometer shows no deflection. Note the length AB ($= l$) of the wire. Find the length BC ($= 100 - l$) of the wire.

Now as the bridge is balanced, therefore, according to Wheatstone bridge principle -

$$\frac{P}{Q} = \frac{R}{S}$$

If r is the resistance per centimeter length of wire then -

P = resistance of the length l of wire AB $= lr$

Q = resistance of the length $(100 - l)$ of the wire

BC $= (100 - l)r$

$$\therefore \quad \frac{lr}{(100-l)r} = \frac{R}{S}$$

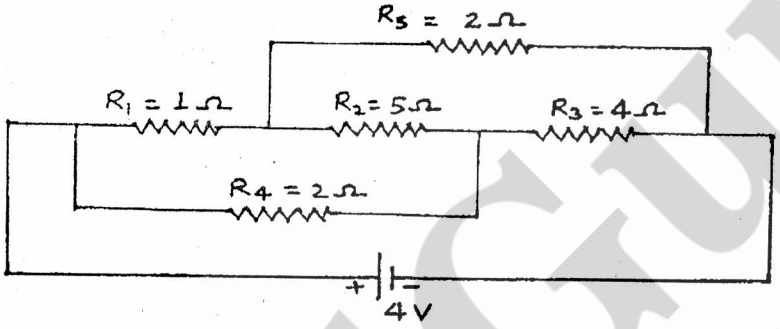
or

$$\boxed{S = \left(\frac{100-l}{l}\right) \times R}$$

So by knowing l and R we can calculate S.

* Characteristics of mangnin wire which make it suitable for making a standard resistance -
  (i) Low temperature coefficient of resistance
  (ii) High Value of resistivity of material of Manganin.

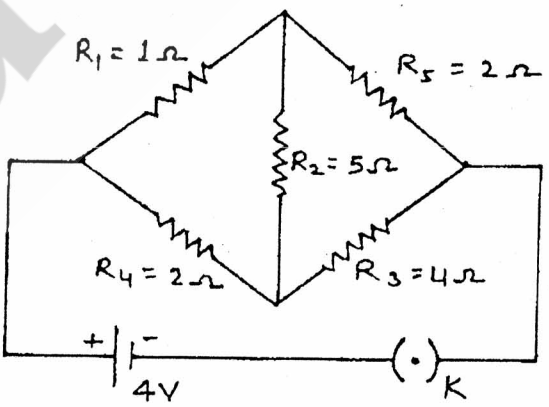
Q. All India 2009 Calculate the current drawn from the battery in the given network.



Sol. The given circuit can be redrawn as given below.

Here $\frac{R_1}{R_5} = \frac{R_4}{R_3}$

$\Rightarrow \frac{1}{2} = \frac{2}{4}$



Wheatstone bridge is balanced so there will be no current in the resistance R_2 and it can be withdrawn.

The equivalent resistance would be equivalent to a parallel combination of two rows which consist of series combination of R_1 and R_5, and R_4 and R_3.

$$\frac{1}{R} = \frac{1}{1+2} + \frac{1}{2+4} = \frac{1}{2}$$

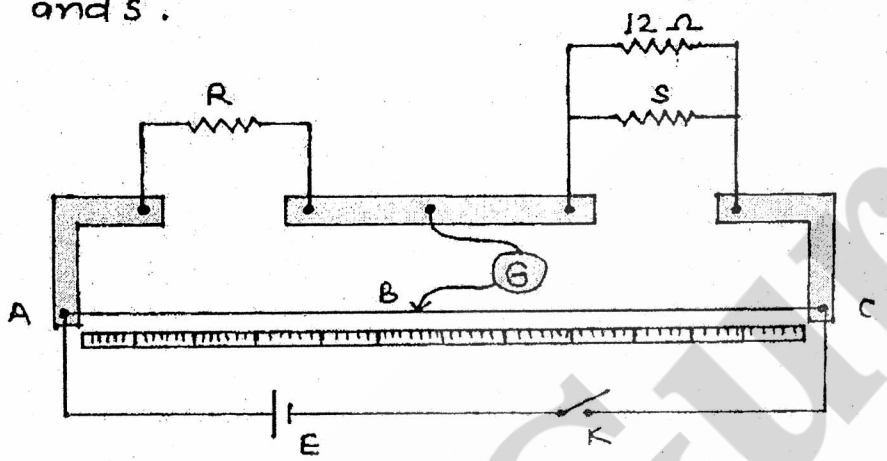
$$R = 2\Omega$$

$$\therefore I = \frac{V}{R} = \frac{4}{2} = 2\ A$$

$$I = 2\ A$$

Q. Delhi 2010 In a meter bridge the null point is found at a distance of 40 cm from A. If a resistance of 12 Ω is connected in parallel with S, the null point occurs at 50 cm from A. Determine the value of R and S.

Sol.



In the null point condition the bridge is balanced. so by applying the condition of balanced wheatstone bridge,

$$\frac{R}{S} = \frac{l}{100-l} = \frac{40}{100-40} = \frac{2}{3} \Rightarrow \frac{R}{S} = \frac{2}{3}$$

The equivalent resistance of 12 Ω and S in parallel is

$$= \frac{12S}{12+S} \ \Omega$$

Again applying the condition

$$\frac{R}{\left(\frac{12S}{12+S}\right)} = \frac{50}{50} = 1$$

$$R = \frac{12S}{12+S}$$

or $$\frac{2}{3}S = \frac{12S}{12+S} \qquad \left[\text{as } R = \frac{2}{3}S\right]$$

$$12 + S = 18$$

$$S = 6\ \Omega$$

$$R = \frac{2}{3}S = \frac{2}{3} \times 6 = 4$$

So $$R = 4\ \Omega$$

Potentiometer -

Potentiometer is an accurate instrument used to compare emf of cells, to measure the emf of a cell or potential difference between two points in an electrical circuit accuratety.

Principle -

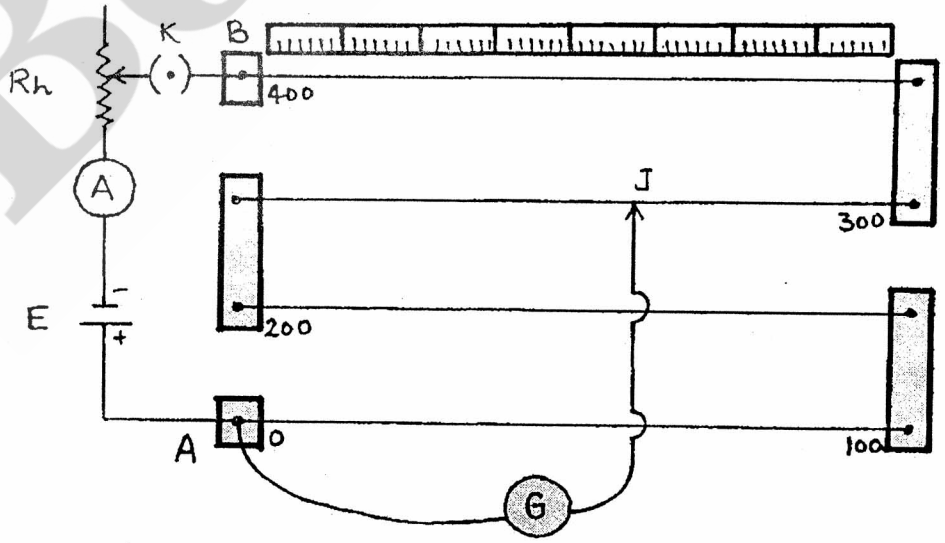
potentiometer is based on the principle that potential drop across any portion of the wire of uniform cross-section is proportional to the length of that portion of the wire when a constant current flows through the wire.

Construction & Working -

A potentiometer consist of a long uniform wire of manganin or constantan, stretched on a wooden board. A meter scale is fixed on the board parallel to the length of the wire.

The potentiometer is provided with a jockey J with the help of which, the contact can be made at any point of the wire.

A battery E is connected across A and B, which sends the current through the wire which is kept constant by using a rheostat R_h.
This circuit is called an auxillary circuit.



Suppose A and ρ are the area of cross-section and resistivity of the material of the wire, then

$$\text{Resistance} \quad R = \frac{\rho l}{A}$$

where l = length of the wire.

If Current I flowing through the wire then by ohm's Law

$$V = IR$$

where V is the potential difference across the wire of length l, thus -

$$V = IR = \frac{I\rho l}{A} = Kl$$

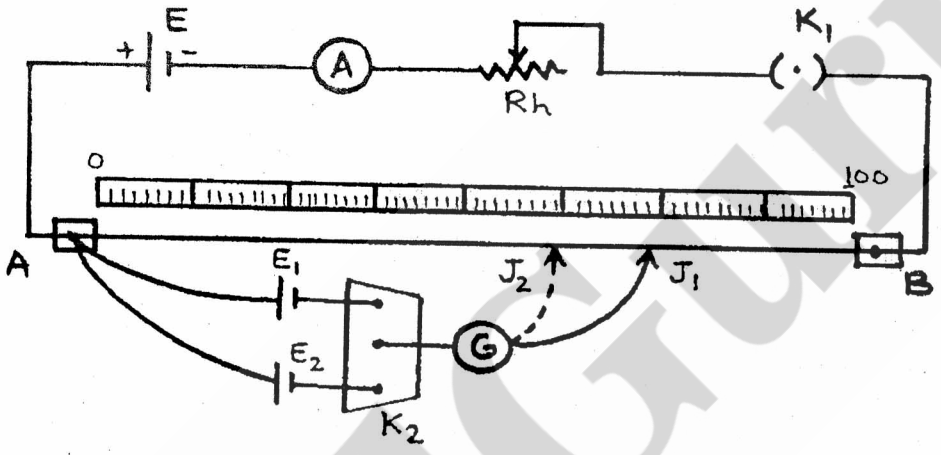
$$\therefore \quad \boxed{V \propto l} \qquad \left(\text{where } K = \frac{I\rho}{A}\right)$$

Hence potential difference across any portion of potentiometer wire is directly proportional to length of the wire of that portion.

* Here $K = \frac{V}{l}$ is called potential gradient i.e. the fall of potential per unit length of the wire.
* sensitivity of a potentiometer depends on its potential gradient. If the potential gradient of a potentiometer is small than the potentiometer is more sensitive and more accurate.

(A) Comparison of EMF's of two cells using Potentiometer –

Positive terminals of two cells of emf's E_1 and E_2 (whose emf are to be compared) are connected to the terminal A and negative terminals are connected to jockey through a two way Key K_2 and a galvanometer.



➪ Now key K_1 is closed to establish a potential difference between the terminals A and B then by closing Key K_2 introduce cell of emf E_1 in the circuit.

Now determine the null point J_1 with the help of jockey. If the null point on the wire is at length l_1 from A then –

$$E_1 = Kl_1$$

where K is the potential gradient of wire.

➪ Similarly cell of emf E_2 is introduced in the circuit and again null point J_2 is determined. If length of this null point from A is l_2 then

$$E_2 = Kl_2$$

Therefore, we get $\boxed{\frac{E_1}{E_2} = \frac{l_1}{l_2}}$

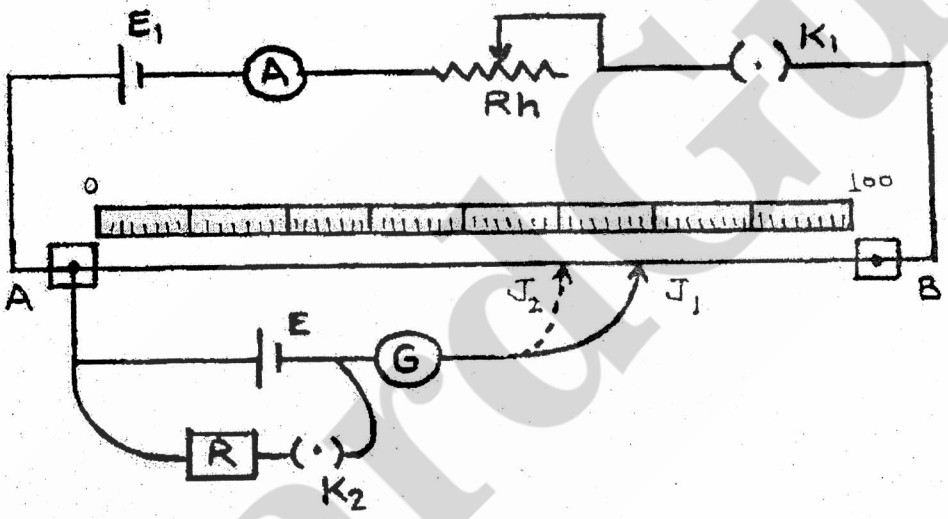
Hence we can find the ratio E_1 / E_2.

* If the emf of one cell is known then the emf of other cell can be known easily.

* A balance point can not be obtained on the wire if the fall of potential along the potentiometer wire due to driving cell is less than the emf of the emf of the cell to be balanced.

(B) Determination of internal resistance of the cell

The cell whose internal resistance is to be determined is connected to the terminal A of the potentiometer across a resistance box through a key K_2.



⇨ First close the Key K_1 and obtain the null point. Let l_1 be the length of this null point from terminal A then -

$$E = K l_1 \qquad (\text{where } l_1 = AJ_1)$$

⇨ when Key K_2 is closed, the cell sends current through resistance box (R).

If V is the terminal potential difference and null point is obtained at length l_2, then -

$$V = K l_2 \qquad (\text{where } l_2 = AJ_2)$$

Thus $$\boxed{\frac{E}{V} = \frac{l_1}{l_2}}$$

We know that the internal resistance r of a cell of emf E, when a resistance R is connected in its circuit is given by -

$$r = \left(\frac{E}{V} - 1\right) R$$

$$\boxed{r = \left(\frac{l_1}{l_2} - 1\right) R}$$

Thus by knowing the value of l_1, l_2 and R, the internal resistance r of the cell can be determined.

Difference between voltmeter and potentiometer

S.N.	Voltmeter	Potentiometer
1.	Its resistance is high but finite.	Its resistance is infinite
2.	It draws some current from source of emf.	It does not draw any current from the source of emf.
3.	Its sensitivity is low	Its sensitivity is high.
4.	It is based on deflection method.	It is based on null deflection method.
5.	The potential difference measured by it is lesser than the actual value.	The potential difference measured by it is equal to actual value.

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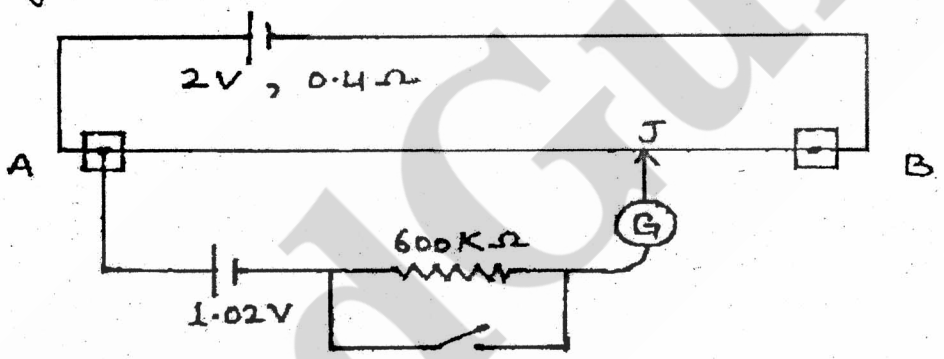
Q. Figure shows a potentiometer with a cell of 2 V and internal resistance 0.4 Ω maintaining a potential drop across the resistor wire AB. A standard cell which maintains a constant emf of 1.02 V gives a balance point at 67.3 cm. length of the wire. To ensure very low currents drawn from the standard cell, a very high resistance of 600 KΩ is put in series with it, which is shorted close to the balance point.
The standard cell is then replaced by a cell of unknown emf E and the balance point found at 82.3 cm length of the wire.



(a) What is the value of E ?
(b) What purpose does the high resistance of 600 KΩ have?
(c) Is the balance point affected by the high resistance?
(d) Is the balance point affected by the internal resistance of the driver cell ?
(e) Would the method work in the absolute situation, if the driver cell of potentiometer had an emf of 1 V instead of 2 V ?
(f) Would the circuit work well for determining very small emf (of the order of a few mV) ? If not, how will you modify the circuit ?

Sol. (a) Here $E_1 = 1.02$ V, $l_1 = 67.3$ cm.
$E_2 = E = ?$, $l_2 = 82.3$ cm.

Since $\frac{E_2}{E_1} = \frac{l_2}{l_1}$

$\therefore \quad E = \frac{l_2}{l_1} \times E_1 = \frac{82.3}{67.3} \times 1.02 = 1.247$ V

$E = 1.247$ V

(b) The purpose of using high resistance of 600kΩ is to allow very small current through the galvanometer when the movable contact is far from the balance point.

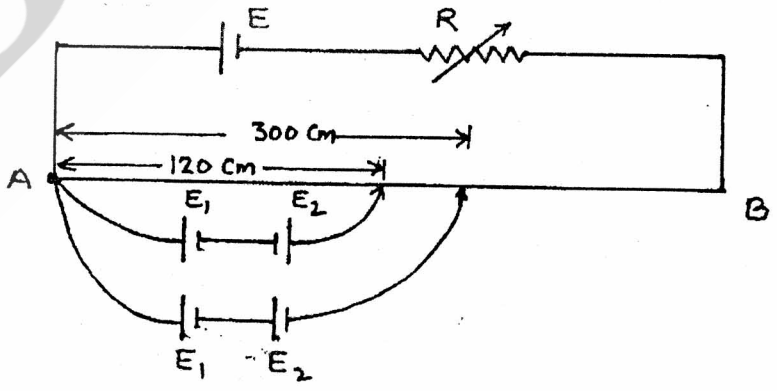
(c) No, the balance point is not affected by the presence of this resistance.

(d) No, the balance point is not affected by the internal resistance of the driver cell.

(e) No, the method will not work as the balance point will not be obtained on the potentiometer wire if the emf of the driver cell is less than the emf of the other cell.

(f) The circuit will not work for measuring extremely small emf because in that case, the balance point will be just close to the end A. To modify the circuit we have to use a suitable high resistance in series with the cell of 2V. This would decrease the current in the potentiometer wire. So the potential difference/cm of the wire will decrease.

Q. Delhi 2012 In a potentiometer the null point for the two primary cells of emf E_1 and E_2 connected in the given manner, are obtained at a distance of 120 Cm. and 300 Cm. from the end A. Find (a) E_1/E_2 and (b) position of null point for the cell E_1.
How is the sensitivity of a potentiometer increased?



Sol. (a) Let the potential gradient be K.

$\therefore \quad E_1 - E_2 = K \times 120$ (Cells are connected in opposite manner)

and $\quad E_1 + E_2 = K \times 300$ (Cells are connected in supporting manner)

so that $\dfrac{E_1 + E_2}{E_1 - E_2} = \dfrac{K \times 300}{K \times 120} = \dfrac{5}{2}$

Now apply Componendo and dividendo -

$$\frac{(E_1 + E_2) + (E_1 - E_2)}{(E_1 + E_2) - (E_1 - E_2)} = \frac{5 + 2}{5 - 2}$$

$$\text{So} \quad \frac{E_1}{E_2} = \frac{7}{3}$$

(b) $\quad E_2 = \dfrac{3}{7} E_1$

$$\text{or} \quad E_1 - \frac{3}{7}E_1 = K \times 120$$

$$\frac{4}{7} E_1 = K \times 120$$

$$E_1 = K \times 210$$

Hence null point for E_1 is obtained at 210 cm.

* The sensitivity of potentiometer can be increased by increasing the length of wire.

Very Important Questions

1 Mark Questions -

Q.1 Show on a graph, the variation of resistivity with temperature for a typical semiconductor.

(Delhi 2012)

Q.2. Two wires of equal length, one of Copper and the other of manganin have the same resistance. Which wire is thicker ?

(All India 2012)

Q.3. Define resistivity of a conductor. Write its SI unit.

(All India 2012)

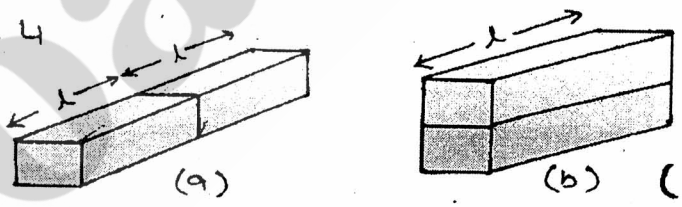
Q.4. A wire of resistance 8R is bent in the form of a circle. what is the effective resistance between the ends of a diameter AB ?

Ans. = 2R

(Delhi 2010)

Q.5. Two identical slabs, of a given metal are joined togather, in two different ways, as shown in fig (a) and (b). What is the ratio of the resistances of these two combinations ?

Ans. = $\frac{R_1}{R_2} = 4$



(Delhi 2010)

Q.6. The three Coloured bands, on a carbon resistor are red green and yellow respectively. Write the value of its resistance.

Ans. = $25 \times 10^4 \Omega \pm 20\%$

(All India 2009)

Q.7. Write an expression for the resistivity of a metallic conductor showing its variation over a limited range of temperatures.

(Delhi 2008)

Q.8. A physical quantity, associated with electrical conductivity, has the SI unit Ω-m. Identify this physical quantity. (All India 2008)

Q.9. Explain how electron mobility changes for a good conductor when (a) the temperature of the conductor is decreased at constant potential diff. (b) applied potential difference is doubled at constant temperature. (All India 2006)

Q.10. A resistance R is connected across a cell of emf E and internal resistance r. Now a potentiometer measures the potential difference between the terminals of the cell as V. write the expression for r in terms of E, V and R. (Delhi 2010, 2011)

Q.11. In an experiment on meter bridge, if the balancing length AC is x, what would be its value, when the radius of the meter bridge wire AB is doubled? (All India 2011)

Q.12. In a meter bridge, two unknown resistances R and S when connected in the two gap, give a null point at 40 cm from one end. What is the ratio of R and S ? (Delhi 2010)

Ans. = 2 : 3

2 Marks Questions -

Q.1 A Conductor of length l is connected to a DC source of potential V. If the length of the Conductor is tripled by gradually stretching it, Keeping V Constant how will the (a) Drift speed of electrons and (b) resistance of the conductor be affected ?

(RBSE 2012)

Q.2. You are required to select a carbon resistor of resistance 47 KΩ ± 10% from a large collection. what should be the sequence of color bands used to code it ?

Write two characteristic of manganin which make it suitable for making standard resistances.

(Delhi 2011)

Q.3. The sequence of colored bands in two carbon resistor R_1 and R_2 is
(a) Brown, Green, blue
(b) Orange, black, green
Find the ratio of their resistances.

(Delhi 2010)

Q.4. Derive an expression for drift velocity of free electrons in a conductor in terms of relaxation time.

(Delhi 2009)

Q.5. Derive an expression for the current density of a conductor in terms of the drift speed of electrons.

(All India 2008)

Q.6. You are given n resistors, each of resistance r. These are first connected to get minimum possible resistance. In the second case, these are again connected differently to get maximum possible resistance. Compute the ratio between the minimum values and maximum values of resistance so obtained.

Ans. = $1 : n^2$

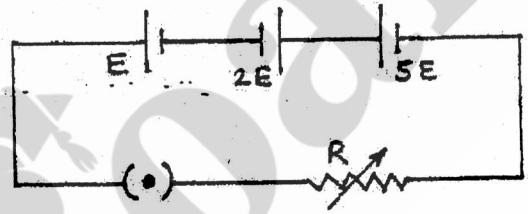
(All India 2009)

Q.7. Derive an expression for the resistivity of a good conductor, in terms of the relaxation time of electrons. **(All India 2008)**

Q.8. Write the mathematical relation between mobility and drift velocity of charge carriers in a conductor. Name the mobile charge carriers responsible for conduction of electric current in (a) an electrolyte (b) an ionized gas. **(Delhi 2006)**

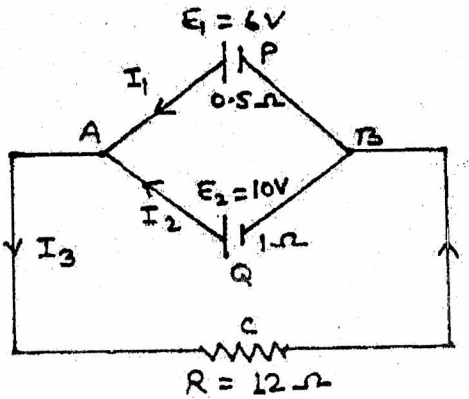
Q.9. Define the term current density of a metallic conductor. Deduce the relation connecting current density (J) and the conductivity (σ) of the conductor, when an electric field (E) is applied to it. **(Delhi 2006)**

Q.10 Three cells of emf E, 2E and 5E having internal resistances r, 2r, and 3r, variable resistance R is shown in the figure. Find the expression for the current. Plot a graph for variation of current with R. **(All India 2010)**



Ans. = $\frac{4E}{6r+R}$

Q.11 Apply Kirchhoff's Laws to the loop ACBPA and ACBQA to determine the current I_1, I_2 and I_3 in the given network. **(All India 2010)**



Ans. = $I_1 = -2.27\ A$, $I_2 = 2.86\ A$
$I_3 = 0.6\ A$

3 Marks Questions -

Q.1 Define relaxation time of the free electrons drifting in a conductor. How is it related to the drift velocity of free electrons ? Use this relation to deduce the expression for the electrical resistivity of the material.

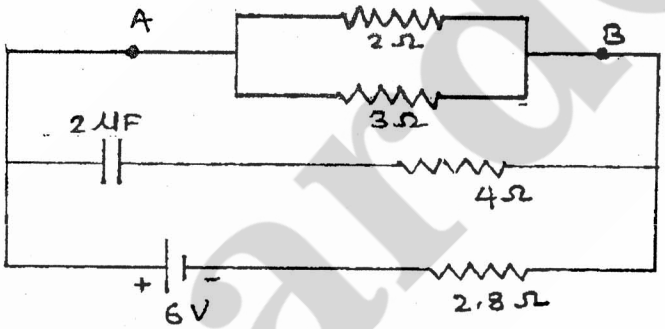
(All India 2012)

Q.2. Write mathematical relation for the resistivity of a material in terms of relaxation time, number density and mass and charge of charge carriers in it. Explain why the resistivity of metal increases and semiconductor decreases with rise in temperature.

(Delhi 2007)

Q.3. Calculate the steady current through the 2 Ω resistor in the given circuit.

(CBSE 2010)

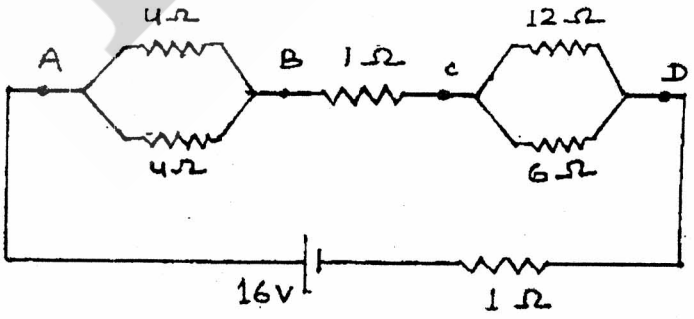


Ans = 0.9 A

Q.4. A network of resistors is connected to a 16V battery of internal resistance of 1 Ω as shown in the figure.

(a) Compute the equivalent resistance of the network.
(b) Obtain the voltage drops V_{AB} and V_{CD}.

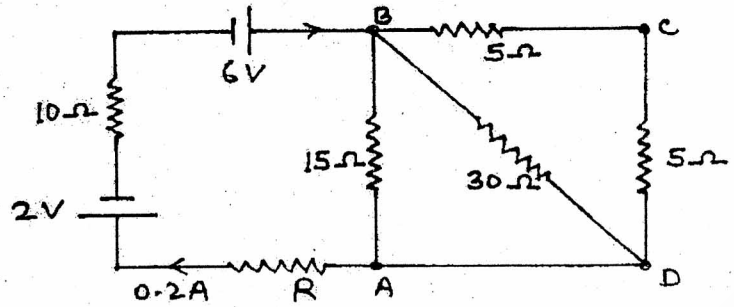
(CBSE 2010)



Ans.: (a) $R_{eq} = 8\ \Omega$
(b) $V_{AB} = 4V$
$V_{CD} = 8V$

Q.5. Calculate the value of the resistance R in the given circuit, so that the current in the circuit is 0·2 A. What would be the potential difference between points A and B ?

(All India 2012)



Ans. $R = 5\,\Omega$

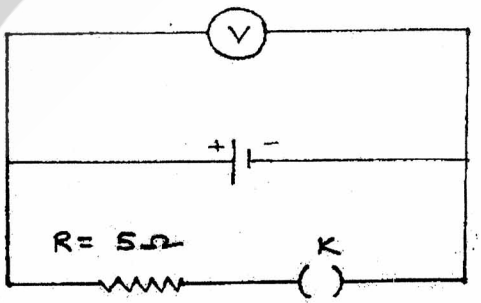
$V_{AB} = -1\,V$

Q.6. Draw the circuit diagram of a potentiometer which can be used to determine the internal resistance r of a given cell of emf E. Explain briefly how the internal resistance of the cell is determined?

(Delhi 2010, CBSE 2008)

Q.7. Write any two factors on which internal resistance of a cell depends.

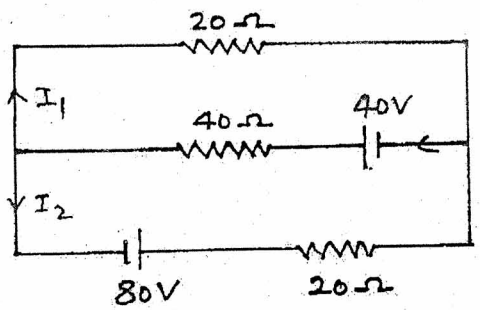
The reading on a voltmeter, when a cell is connected across it, is 2·2 V. When the terminals of the cell are also connected to a resistance of 5Ω as shown in fig, the voltmeter reading drops to 1·8 V. Find the internal resistance of the cell.



(All India 2010)

Ans. = $r = \frac{10}{9}\,\Omega$

Q.8. State Kirchoff's rules of current and voltage in an electrical network. Using these rules determine the value of the current I_1 in the given network.



Ans = $I_1 = -1.2\,A$

(CBSE 2007)

5 Marks Questions -

Q.1 (a) State the working principle of a potentiometer. Draw a circuit diagram to compare emf of two primary cells. Derive the formula used.

(b) Which material is used for potentiometer wire and why?

(c) How can the sensitivity of a potentiometer be increased?

(CBSE 2007, 2011)

Q.2. State the working priciple of a potentiometer. In a potentiometer arrangement, a cell of emf 1.2 V gives a balance point at 30 cm. length of the wire. Now this cell is replaced by another cell of unknown emf. If the ratio of the emf of the two cell is 1:5, calculate the difference in the balancing length of the potentiometer wire in the two cases.

Ans. = 10 cm.

(Delhi 2006)

Q.3. State the principle of working of a meter bridge.

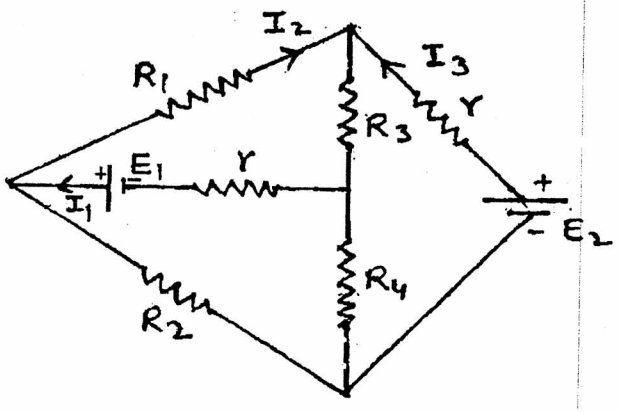
In a meter bridge the null point is found at a distance of 60 cm from A. If a resistance of 5Ω is connected in series with S, the null point occurs at 50 cm from A. Determine the value of R and S.

(Delhi 2010)

Ans: = $R = 15\ \Omega$, $S = 10\ \Omega$

Q.4. State the Kirchoff's Laws. Use these rules and write the equations of three unknown currents in the branches of the given circuit.

(All India 2008)



Q.1. Out of two bulbs marked 25W and 100W, which has higher resistance? give reason.

Ans. - 25 W bulb

Q.2. Two materials Si and Cu, are cooled from 300K to 60K. what will be the effect on their resistivity?

Q.3. A wire of resistivity ρ is stretched to double its length. what will be its new resistivity?

Q.4. Two conducting wires X and Y of same diameter but different materials are joined in series across a battery. If the number density of electrons in X is twice that in Y, find the ratio of drift velocity of electrons in the two wires.

Ans. $\frac{V_x}{V_y} = \frac{1}{2}$

Q.5. A carbon resistor is marked in colour bands of red, black, orange and silver. What is the resistance and tolerance value of the resistor?

Ans - $R = 20\,K\Omega \pm 10\%$

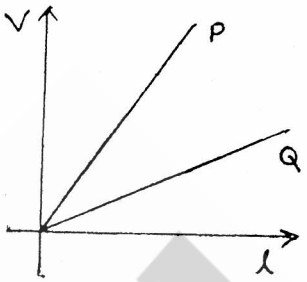
Q.6. A conductor of length 'L' is connected to a dc source of potential 'V'. If the length of the conductor is tripled by gradually stretching it, keeping V constant then how will (i) drift speed of electrons (ii) resistance of the conductor be affected?

Ans. - (i) $V_d' = \frac{V_d}{3}$ (ii) $R' = 9R$

Q.7. Explain the variation of conductivity with temperature for (i) a metallic conductor (ii) ionic conductors

Q.8. How does drift velocity of electrons in a metallic wire vary with the rise of temperature?

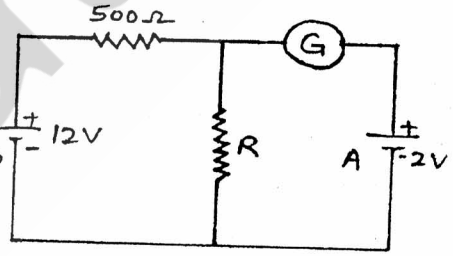
Q.9. The variation of potential difference V with length L in the case of two potentiometer P and Q is as shown in fig. Which of these two will you prefer for comparing the emfs of two primary cells ? Give reason.



Q.10. Calculate the temperature at which the resistance of a conductor becomes 20% more than its resistance at 27°C. The value of the temperature coefficient of resistance of the conductor is $2 \times 10^{-4} K^{-1}$.

Ans. - $T_2 = 1300K$

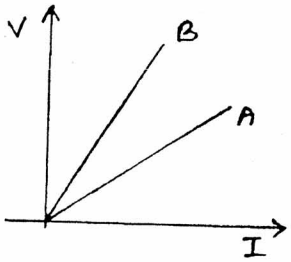
Q.11. In the circuit shown in fig., the galvanometer G gives zero deflection. If the batteries A and B have negligible internal resistance find the value of the resistor R.



Ans. $R = 100\Omega$

Q.12. V-I graphs for parallel and series combinations of two metallic resistors are shown in fig. which graph represents parallel combination?

Ans. - graph A



Q.13. You are given n resistors each of resistance r. They are first connected to get the minimum possible resistance. In the second case, these are again connected differently to get the maximum possible resistance. Calculate the ratio between minimum and maximum values of resistance.

Ans. $\frac{r_{min}}{r_{max}} = \frac{1}{n^2}$

Q.14. Electrons are continuously in motion within a conductor but there is no current in it unless some source of emf is applied across its ends. Give reason.

PHYSICS

Unit - 4. Moving Charges and Magnetism

Class - 12th

BoardGuru

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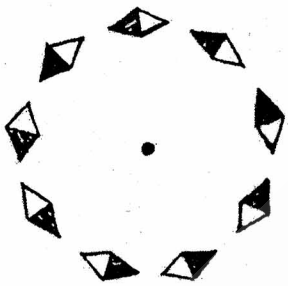
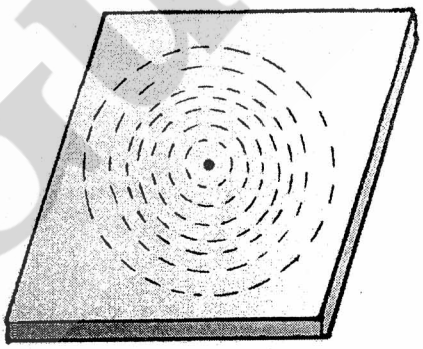
4. Moving Charges And Magnetism

Introduction -

In the year 1820 Oersted realised that electricity and magnetism were intimately related. He shows that the electric current through a straight wire causes noticeable deflection of the magnetic compass needle held near the wire.

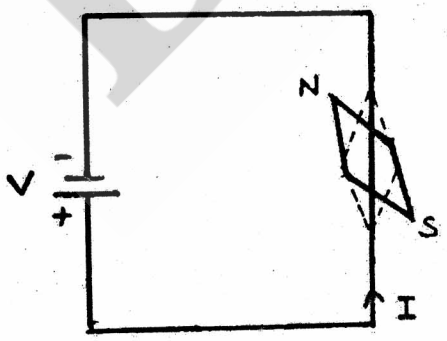
He also found that the magnetic needle is aligned tangentially to an imaginary circle which has the straight current carrying wire as its centre and has its plane perpendicular to the wire.

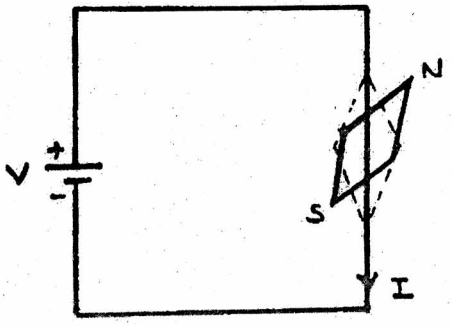




If the direction of the current in wire is reversed, the direction of deflection of the magnetic needle is also reversed. The deflection increases on increasing the current in the wire or bringing the magnetic compass needle closer to the wire.

He also found that the iron filing sprinkled around the wire, arrange themselves in concentric circles with the wire as centre in the plane perpendicular to the wire.





Magnetic Field -

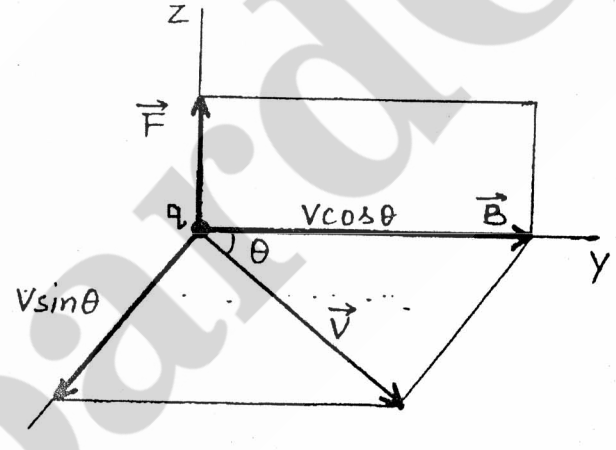
The magnetic field is a space around a conductor carrying current or the space around a magnet in which its magnetic effect can be felt.

Magnetic field is a vector quantity and is denoted by $\vec{B}$

Force on a moving charge in a magnetic field -

Consider a positive charge q moving in a uniform magnetic field $\vec{B}$, with a velocity $\vec{V}$. Let the angle between $\vec{V}$ and $\vec{B}$ be θ

Due to interaction between the magnetic field produced due to moving charge (i.e current) and magnetic field applied, the charge q then experiences a force, which depends on the following factors -



(i) The magnitude of force $\vec{F}$ experienced by the moving charge is directly proportional to the magnitude of the charge -

$$F \propto q$$

(ii) The magnitude of the force $\vec{F}$ is directly proportional to the component of velocity acting perpendicular to the direction of the magnetic field

$$F \propto V \sin\theta$$

(iii) The magnitude of force $\vec{F}$ is directly proportional to the magnitudes of the magnetic field applied.

$$F \propto B$$

so by combining the above factor, we get

$$F \propto KqVB\sin\theta$$

Here K is a constant. Its value is found to be 1.

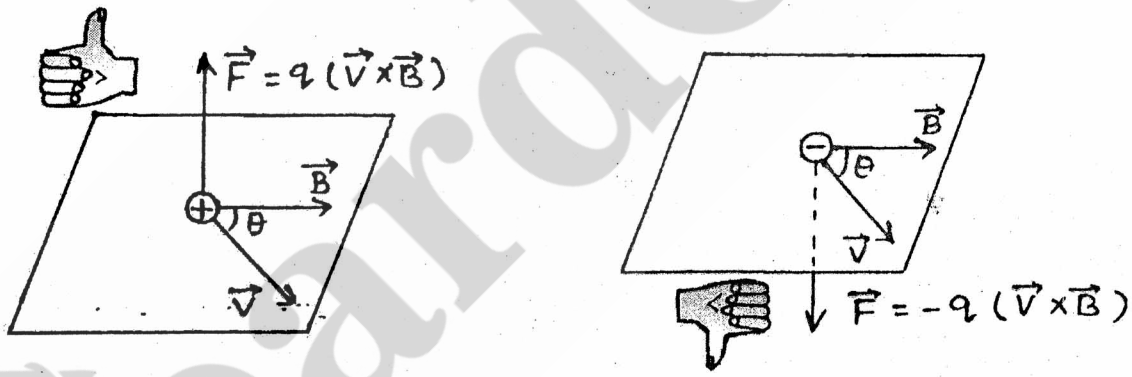
$$F = qVB\sin\theta$$

$$\vec{F} = q(\vec{V} \times \vec{B})$$

$$B = \frac{F}{qV\sin\theta}$$

The direction of $\vec{F}$ is the direction of cross-product of velocity $\vec{V}$ and magnetic field $\vec{B}$, which is perpendicular to the plane containing $\vec{V}$ and $\vec{B}$.

Its direction is given by the 'Right hand Rule'.



If $\vec{V}$ and $\vec{B}$ are in the plane of paper then according to the 'right hand Rule', the direction of $\vec{F}$ on positively charged particle will be perpendicular to the plane of paper upwards.

On negatively charged particle the direction of force $\vec{F}$ will be perpendicular to the plane of paper downwards.

Unit - SI unit of $\vec{B}$ is Tesla (T) or Weber/metre^2 or $N A^{-1} m^{-1}$.

Dimensions of $\vec{B}$ = $[M^1 A^{-1} T^{-2}]$

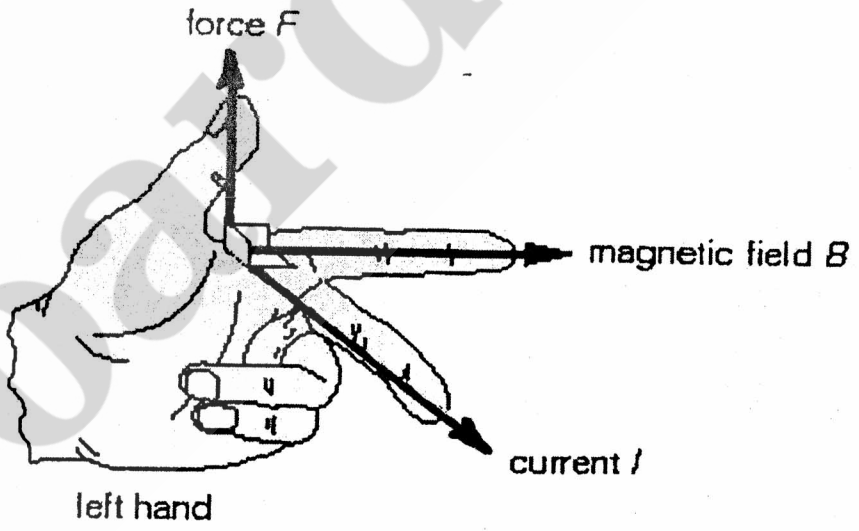
1 Tesla -

The magnetic field $\vec{B}$ at a point is said to be 1 tesla if a charge of one coulomb while moving at right angle to a magnetic field with a velocity of 1 m/s experience a force of 1 Newton at that point.

$$B = \frac{F}{qv\sin\theta} \qquad \text{so} \quad 1\,T = \frac{1N}{1C \times 1\,m/s} = 1\,NA^{-1}m^{-1}$$

Fleming Left hand Rule -

If we stretch the first finger, the central finger and the thumb of left hand mutually perpendicular to each other such that the first finger points towards the direction of magnetic field, the central finger points towards the direction of electric current (motion of the positive charge) then the thumb represents the direction of the force experienced by the charged particle.



Q. An α particle of mass 6.65×10^{-27} kg is travelling at right angles to a magnetic field with a speed of 6×10^5 m/s. The strength of the magnetic field is 0.2 T. Calculate the force on the α particle and its acceleration.

Sol. Given that- $m = 6.65 \times 10^{-27}$ kg

$q = 2e = 2 \times 1.6 \times 10^{-19}$ C , $V = 6 \times 10^5$ m/s

$B = 0.2$ T and $\theta = 90^\circ$

So the force on the α particle in magnetic field is

$$F = qVB\sin\theta$$
$$F = 2 \times 1.6 \times 10^{-19} \times 6 \times 10^5 \times 0.2 \times \sin 90^\circ$$
$$F = 3.84 \times 10^{-14} \text{ N}$$

and acceleration of the α particle is -

$$a = \frac{F}{m} = \frac{3.84 \times 10^{-14}}{6.65 \times 10^{-27}} = 5.77 \times 10^{12} \text{ m/s}^2$$

Q. All India 2011 Write the expression for Lorentz magnetic force on a particle of charge q moving with velocity V in a magnetic field B. Show that no work is done by this force on the charged particle.

Sol. Lorentz force always acts along the direction perpendicular to the direction of velocity of the particle.

Magnetic Lorentz force $F_m = q(V \times B)$

$\because F \perp V$

Force is perpendicular to displacement made by charge particle.

$\therefore W = F d \cos 90^\circ = 0$ [Force & displacement are perpendicular]

$W = 0$

So No work is done by magnetic Lorentz force on charge particle.

Magnetic Force on a Current Carrying Conductor

Consider a wire of uniform cross-sectional area A and length l. We shall assume one kind of charge carriers (electrons) as in a conductor.

Let there are n electrons per unit volume so the total number of electrons in it is nAl, and the drift velocity of the electrons is V_d.

So the force on these electrons is -

$$\vec{F} = -(nAL)e\,\vec{V_d} \times \vec{B} \qquad [q = -nAle\ ]$$

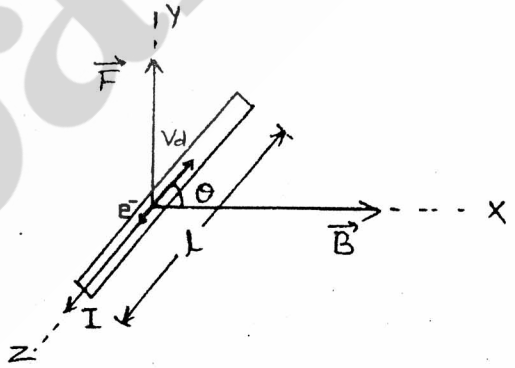
Now $neAV_d$ is the current and neV_d is the current density ($\vec{J}$). Thus -

$$F = [\vec{J}Al] \times \vec{B} \qquad [-V_d \text{ is in the direction of } I$$

or $\boxed{\vec{F} = I\vec{l} \times \vec{B}}$ $\boxed{|\vec{F}| = IBL\sin\theta}$

Here $\vec{l}$ is a vector of magnitude l and with direction towards the current I.

Note- Here $\vec{B}$ is the external magnetic field and not the field produced by the current carrying conductor.



★ Here the direction of the $\vec{F}$ is perpendicular to the plane containing $\vec{B}$ and $\vec{l}$.

(i) If $\theta = 0°$ or $180°$ then $\vec{F} = 0$

(ii) If $\theta = 90°$ then $\vec{F}_m = IBL$

★ The direction of Force can be given by the "Fleming's Left hand Rule".

Motion in a Magnetic Field -

When a moving charge particle moves in parallel or antiparallel direction to the magnetic field then it does not feel any force.

First Condition -

(When charge particle moves at right angle to the magnetic field)

Suppose a particle of mass m and charge q entering in a uniform magnetic field $\vec{B}$ at right angle with velocity $\vec{V}$

So the force $\vec{F}$ on the charge q due to magnetic field is -

$$F = q(\vec{V} \times \vec{B}) = qVB\sin 90^\circ$$

So

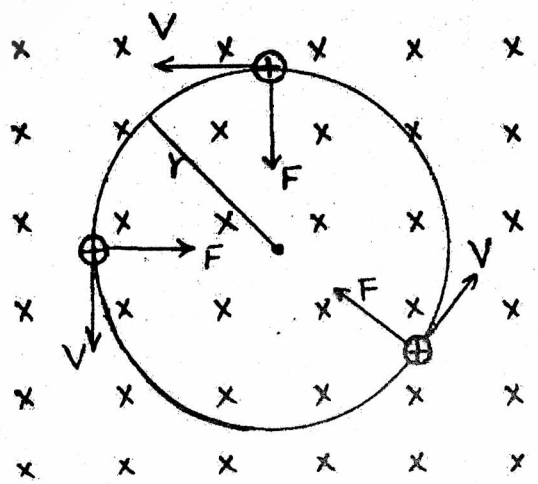
$$F = qVB$$

This force is always perpendicular to the direction of motion and the magnetic field B so the charge will describe a circular path.

Here the force F provides the required centripetal force necessary for motion along a circular path of radius r.

$$qVB = \frac{mv^2}{r}$$

So $$r = \frac{mv^2}{qVB} \Rightarrow \boxed{r = \frac{mv}{qB}}$$



The time period of revolution of the particle is

$$T = \frac{2\pi r}{V} = \frac{2\pi}{V} \times \frac{mV}{qB}$$

So $$\boxed{T = \frac{2\pi m}{qB}}$$

The angular velocity of rotation of the particle

$$\omega = 2\pi\nu = \frac{2\pi}{T} \Rightarrow \boxed{\omega = \frac{qB}{m}}$$

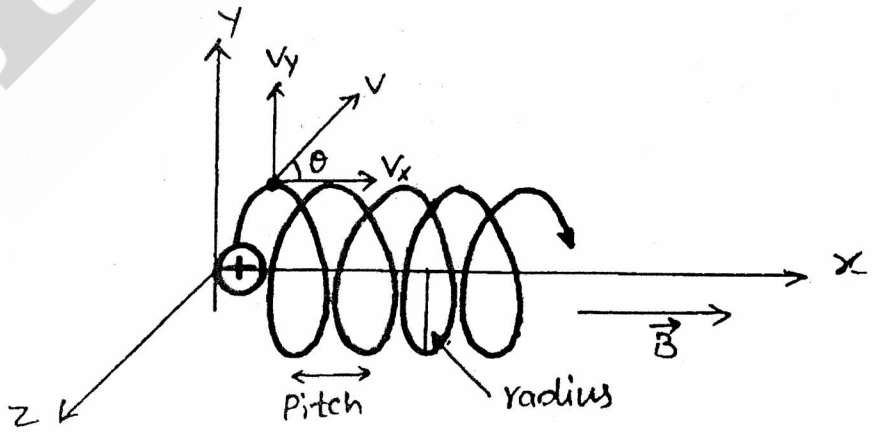
Here we can see that T and ω does not depend upon velocity $\vec{V}$ of the particle. It means all the charged particles having the same specific charge (q/m) but moving with different velocities at a point, will complete their circular paths in the same time.

Second Case -

(When a charge particle at any angle (other than 0°, 90° and 180°) to the magnetic field)

Suppose a particle of mass m and charge q, entering a magnetic field $\vec{B}$ with velocity $\vec{V}$, making an angle θ with the direction of $\vec{B}$.

Now $V_x = V\cos\theta$ is the factor of $\vec{V}$ along the direction of $\vec{B}$ and $V_y = V\sin\theta$ is the factor of $\vec{V}$ in perpendicular direction to $\vec{B}$.



This condition can be used to select charged particles of a particular velocity out of a beam containing charges moving with different speeds. The crossed E and B fields, therefore serve as a velocity selector.

Only the particles with speed E/B pass undeflected through the region of crossed fields.

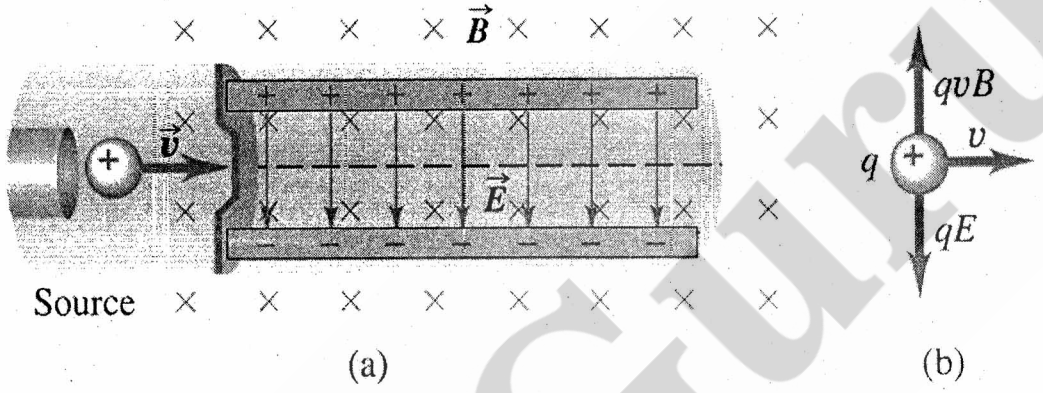


Fig - Velocity Selector

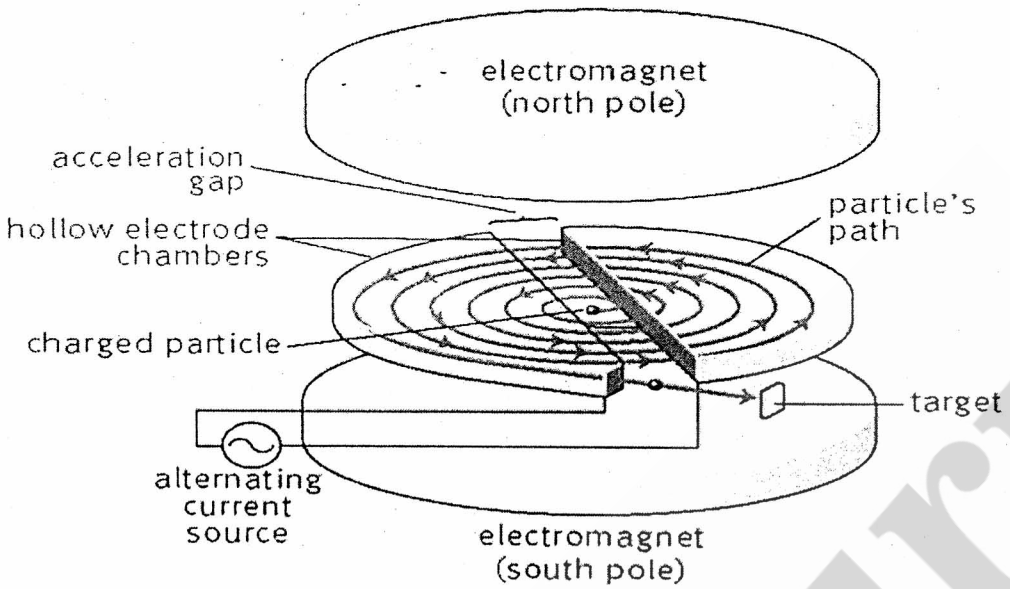
* This method was employed by J.J. Thomson in 1897 to measure the charge to mass ratio (e/m) of an electron.
* The principle is also employed in Mass spectrometer, which is a device that separates charged particles, usually ions, according to their charge to mass ratio.

Cyclotron

The cyclotron is a machine to accelerate charged particles or ions to high energies. It uses both electric and magnetic fields in combination to increase the energy of charged particles.

As the fields are perpendicular to each other so they are called crossed fields.

Cyclotron uses the fact that the frequency of revolution of the charged particle in a magnetic field is independent of its energy. The particles move most of the time inside two semicircular disc-like metal containers D_1 and D_2, which are called dees as they look like the letter D.



Inside the metal boxes the particle is shielded and is not acted on by the electric field. The magnetic field, however acts on the particle and makes it go round in a circular path inside a dee.

Every time the particle moves from one dee to another it is acted upon by the electric field. The sign of the electric field is changed alternately in tune with the circular motion of the particle.

This ensures that the particle is always accelerated by the electric field. Each time the acceleration increases the energy of the particle. As energy increases, the radius of the circular path increases. So the path is a spiral one.

The whole assembly is evacuated to minimise collisons between the ions and the air molecules. A high frequency alternating voltage is applied to the dees.

Positive ions or positively charged particles (e.g. Protons) are released at the centre P. They move in a semicircular path in one of the dees and arrive in the gap between the dees in a time interval T/2; where T is the period or revolution.

$$T = \frac{1}{\nu_c} = \frac{2\pi m}{qB} \quad \text{or} \quad \boxed{\nu_c = \frac{qB}{2\pi m}}$$

This frequency ν_c is called the cyclotron frequency for obvious reasons.

The frequency of the applied voltage is adjusted so that the polarity of the dees is reversed in the same time that it takes the ions to complete one half of the revolution.

The requirement $V_a = V_c$ is called the resonance condition.

The phase of the supply is adjusted so that when positive ions arrive at the edge of D_1, D_2 is at a lower potential and the ions are accelerated across the gap.

Inside the dees the particle travel in a region free of the electric field. The increase in their Kinetic energy is qV each time they cross from one dee to another (V is the applied voltage across the dees at that time)

From equation $r = mv/qB$ it is clear that the radius of their path goes on increasing each time their Kinetic energy increases.

The ions are repeatedly accelerated across the dees until they have the required energy to have a radius approximately that of the dees.

They are then deflected by a magnetic field and leave the system via an exit slit.

So we have $$\boxed{V = \frac{qBR}{m}}$$

where R is the radius of the trajectory at exit and equals the radius of a dee.

Hence the Kinetic energy of the ions is

$$K.E. = \boxed{\frac{1}{2}mv^2 = \frac{q^2B^2R^2}{2m}}$$

Uses -

The cyclotron is used to bombard nuclei with energetic particles, so accelerated by it, and study the resulting nuclear reactions. It is also used to implant ions into solids and modify their properties or even synthesise new materials. It is used in hospitals to produce radiactive substances which can be used in diagonosis and treatment.

Q. A cyclotron's oscillator frequency is 10 MHz. What should be the operating magnetic field for accelerating protons ? If the radius of its 'dees' is 60 cm, what is the Kinetic energy (in Mev). of the proton beam produced by the accelerator.

($e = 1.6 \times 10^{-19} C$, $m_p = 1.67 \times 10^{-27} kg$, $1 Mev = 1.6 \times 10^{-13} J$)

Sol:

The oscillator frequency should be same as proton's cyclotron frequency.

We Know that - $\nu_c = \frac{qB}{2\pi m}$

So $B = \frac{2\pi m \nu_c}{q} = \frac{2\pi \times 1.67 \times 10^{-27} \times 10^7}{1.6 \times 10^{-19}}$

$B = 0.66\ T$

Final Velocity of proton is

$V = r \times 2\pi \nu_c = 0.6 \times 6.3 \times 10^7$

$V = 3.78 \times 10^7\ m/s$

$E = \frac{1}{2} m V^2 = \frac{1}{2} \times \frac{1.67 \times 10^{-27} \times 14.3 \times 10^{14}}{1.6 \times 10^{-13}}$

$E = 7\ Mev.$

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Magnetic Field due to a Current Element

BIOT SAVART LAW -

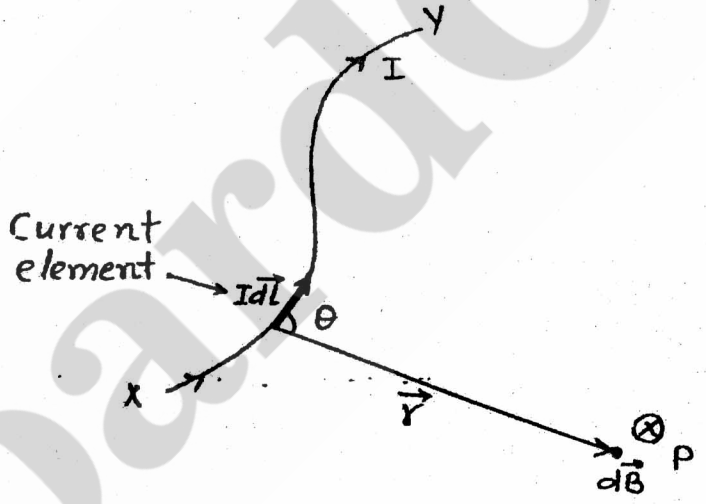
Biot-Savart's Law is an experimental Law predicted by Biot and Savart.

This Law shows the relation between current in a conductor and the magnetic field it produces.

Let us consider a small element of length dl of a conductor XY carrying a current I.

Let $\vec{r}$ be the position vector of the point P from the current element $I\vec{dl}$ and θ be the angle between $\vec{dl}$ and $\vec{r}$.

The current element $I\vec{dl}$ is a vector. Its direction is tangent to the element and is acting in the direction of current flow in the Conductor.



According to the Biot-Savart's Law, the magnitude of the magnetic field $d\vec{B}$ (also called magnetic flux density) at a point P due to current depends upon the following factors.

(i) $dB \propto I$ (ii) $dB \propto dL$

(iii) $dB \propto \sin\theta$ (iv) $dB \propto 1/r^2$

Combining these factors, we get -

$$dB \propto \frac{Idl\sin\theta}{r^2}$$

$$dB = \frac{K\, I dl \sin\theta}{r^2}$$

Here K is a constant whose value depends on the system of units chosen for the measurement of the various quantities and also on the medium between point P and the current element.

When there is free space between current element and point P, then

$K = \frac{\mu_0}{4\pi} = 10^{-7} \rightarrow$ In SI Units

$K = 1 \rightarrow$ In cgs system.

So in SI Units.

$$dB = \frac{\mu_0}{4\pi} \times \frac{I dl \sin\theta}{r^2}$$

In vector form we may write -

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I(\vec{dl} \times \vec{r})}{r^3} = \frac{\mu_0}{4\pi} \frac{I(\vec{dl} \times \hat{r})}{r^2}$$

Here direction of $d\vec{B}$ is represented by 'Right hand Rule'.

* $d\vec{B}$ is perpendicular to the plane containing $\vec{dl}$ and $\vec{r}$ and is directed inwards, if Point P is to the right of the current element.
* If the point P is to the left of the current element, $d\vec{B}$ will be perpendicular to the plane containing $\vec{dl}$ and $\vec{r}$, directed outwards.

Magnetic field at point P due to current through entire wire is

$$B = \int \frac{\mu_0}{4\pi} \frac{I dl \sin\theta}{r^2}$$

Some Important features of Biot Savart's Law -

(i) The direction of $d\vec{B}$ is perpendicular to both $Id\vec{l}$ and $\vec{r}$.

(ii) If $\theta = 0^\circ$ i.e. the point P lies on the axis of the linear conductor carrying current then

$$dB = \frac{\mu_0}{4\pi} \frac{Idl\sin\theta}{r^2} = 0$$

It means there is no magnetic field at any point on the thin linear current carrying conductor.

(iii) If $\theta = 90^\circ$ i.e. the point P lies at a perpendicular position w.r.t. current element then.

$$dB = \frac{\mu_0}{4\pi} \frac{Idl}{r^2}, \text{ which is Maximum.}$$

(iv) If $\theta = 0^\circ$ or 180° then $dB = 0$, which is minimum.

Similarities and Dis-similarities between the Biot-Savart's Law & Coulomb's Law -
(For Magnetic Field & Electrostatic Field)

Similarities -

(i) Both the Laws for fields are long range, since in both the Laws, the field at a point varies inversely as the square of the distance from the source.

(ii) Both the fields obey superposition principle.

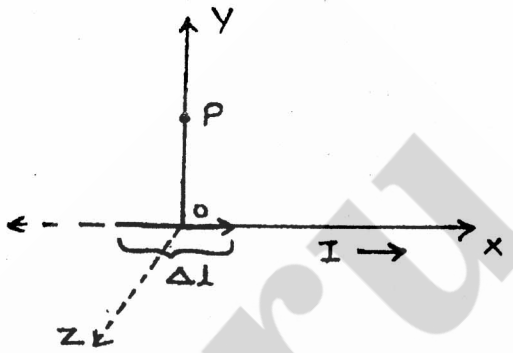
Dis-similarities -

(i) The electrostatic field is acting along the displacement vector while the magnetic field is acting perpendicular to the plane containing the current element Idl and displacement vector $\vec{r}$.

(ii) The Coulomb's Law is independent of angle whereas the Biot-Savart's Law is angle dependent.

(iii) The electrostatic field is produced by a scalar source (q) while magnetic field is produced by a vector source ($Id\vec{l}$)

Q. (CBSE 2009) An element $\Delta l = \Delta x \hat{i}$ is placed at the origin and carries a current $I = 2A$. Find the magnetic field at a point P on the Y axis at a distance of 1 m due to the element $\Delta x = w$ cm. Give also the direction of the field produced.



Sol.

Biot-Savart Law states that -

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl}{r^2} \times \hat{r}$$

Here $Idl = 2 \times w\hat{i}$

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{(2w\hat{i} \times \hat{j})}{(1)^2}$$

Here $(\vec{r} = \hat{j})$ so $(|\vec{r}| = 1)$

$$dB = \frac{\mu_0 w \hat{k}}{2\pi}$$

OR $$|dB| = \frac{\mu_0 w}{2\pi} \quad (\text{along } +z \text{ axis})$$

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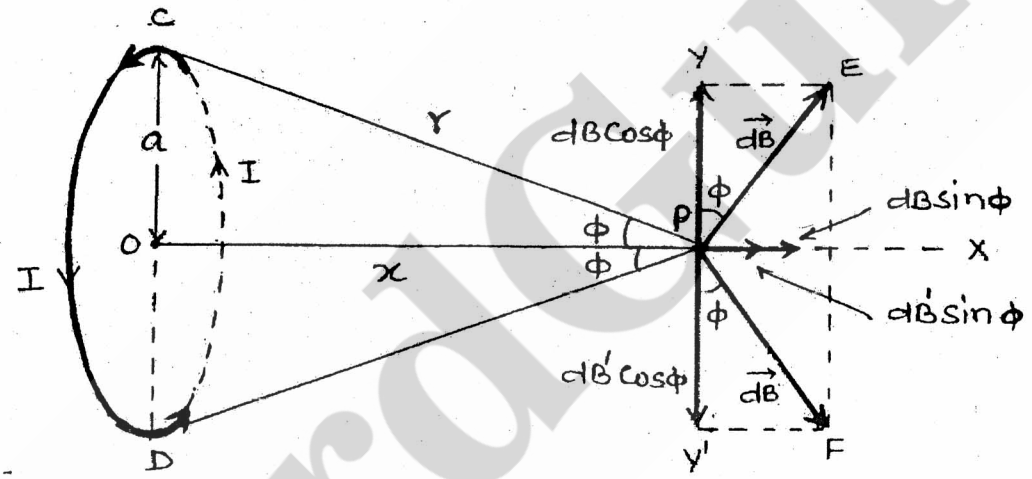
Magnetic Field on the axis of a Circular Current Loop-

Consider a circular coil of radius a with centre O carrying current I. Suppose P is any point on the axis of the circular coil at a distance x from its centre O. (OP = x)

Now consider two small element of the coil each of length dl, at C and D which are situated at diametrically opposite edges.



Here $PC = PD = r = \sqrt{a^2+x^2}$

Let $\angle CPO = \angle DPO = \phi$

According to the Biot-Savart's Law, the magnetic field at P due to current element $I\vec{dl}$ at C is given by-

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin 90^\circ}{r^2}$$

(Here a is small, therefore $\theta = 90^\circ$)

$$\boxed{dB = \frac{\mu_0}{4\pi}\frac{I\,dl}{(a^2+x^2)}} \quad \text{— (1)}$$

The direction of $d\vec{B}$ is in the plane perpendicular to $\vec{dl}$ and $\vec{r}$ and is directed as given by right hand screw rule i.e. acting along PE $\perp$ to CP.

Similarly, the magnitude of magnetic field at P due to current element of length dl at D is given by -

$$dB' = \frac{\mu_0}{4\pi}\frac{I\,dl\sin 90^\circ}{r^2}$$

$$\boxed{dB' = \frac{\mu_0}{4\pi}\frac{I\,dl}{(a^2+x^2)}} \quad \text{— (2)}$$

Its direction is along PF ⊥ DP.

from eq. (1) & (2)

$$\boxed{dB = dB' = \frac{\mu_0 I\,dl}{4\pi(a^2+x^2)}}$$

Since the components of the magnetic fields along PY and PY' are equal and opposite, they cancel each other and the components acting along PX are in same direction so they added up.

The same rule is true for all the diametrically opposite pairs of current elements of the circular coil.

So the total magnetic field at P due to current through whole circular coil is given by -

$$B = \int dB\sin\phi = \int \frac{\mu_0}{4\pi}\frac{I\,dl\sin\phi}{(a^2+x^2)}$$

$$B = \frac{\mu_0 I}{4\pi(a^2+x^2)}\sin\phi\int dl \qquad \left|\ \sin\phi = \frac{a}{\sqrt{a^2+x^2}}\right.$$

$$B = \frac{\mu_0 I}{4\pi(a^2+x^2)} \times \frac{a}{\sqrt{a^2+x^2}} \times 2\pi a \qquad \left|\ \int dl = 2\pi a\right.$$

$$\boxed{B = \frac{\mu_0}{4\pi}\frac{2\pi a^2 I}{(a^2+x^2)^{3/2}} \quad (\text{along PX})}$$

If there are n turns in the coil, then -

$$\boxed{B = \frac{\mu_0}{4\pi}\frac{2\pi n I a^2}{(a^2+x^2)^{3/2}}}$$

Special Case -

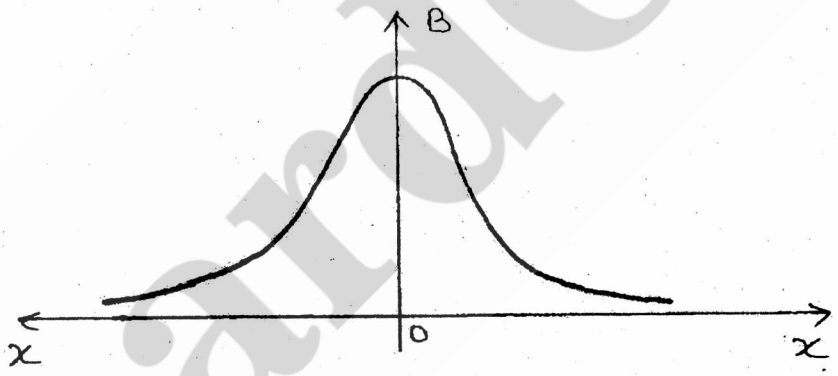
(1) When point P lie at the centre of the circular coil. (then $x = 0$)

$$B = \frac{\mu_0}{4\pi}\frac{2\pi n I a^2}{a^3} = \frac{\mu_0}{4\pi}\frac{2\pi n I}{a} = \frac{\mu_0 n I}{2a}$$

(2) When a point lie on the axis of the coil at a distance equal to the radius of the coil. (then $x = a$)

$$B = \frac{\mu_0}{4\pi}\frac{2\pi n I a^2}{(a^2+a^2)^{3/2}} = \frac{\mu_0 n I}{2^{3/2}a}$$

The variation of magnetic field B and distance x of a point on the axis of a circular coil carrying current is represented by the given graph.



(3) When point P lies far away from the centre of the coil ($x >> a$)

$B = \frac{\mu_0}{4\pi}\frac{2\pi n I a^2}{(a^2+x^2)^{3/2}}$ by neglecting a we get -

$B = \frac{\mu_0}{4\pi}\frac{2nIA}{x^3}$ ($A = \pi a^2$) area of the loop.

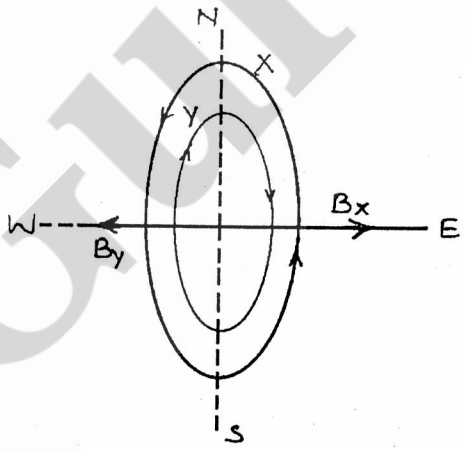
or

$$B = \frac{\mu_0}{4\pi}\frac{2m}{x^3}$$

Here $nIA = m$ is called magnetic dipole moment of the current Loop.

Q. Two concentric circular coils x and y of radius 16 cm and 10 cm respectively lie in the same verticle plane containing the north to south direction. Coil x has 20 turns and carries a current of 16 A. Coil y has 25 turns and carries a current of 18 A. The sense of the current in x is anticlockwise and clockwise in y, for an observer looking at the coils facing west. Give the magnitude and direction of the net magnetic field due to the coils at their centre.

Sol.



Radius of Coil x, $r_1 = 0.16$ m

Radius of Coil y, $r_2 = 0.1$ m

Number of turns of Coil x

$n_1 = 20$

Number of turns of Coil y

$n_2 = 25$

Current in coil x, $I_1 = 16$ A

Current in coil y, $I_2 = 18$ A

Magnetic field due to coil x at their centre is given by-

$$B_1 = \frac{\mu_0 n_1 I_1}{2 r_1}$$

$$B_1 = \frac{4\pi \times 10^{-7} \times 20 \times 16}{2 \times 0.16} = 4\pi \times 10^{-4} \text{ T}$$

(Towards East)

Magnetic field due to coil y at their centre is given by -

$$B_2 = \frac{\mu_0 n_2 I_2}{2 r_2}$$

$$B_2 = \frac{4\pi \times 10^{-7} \times 25 \times 18}{2 \times 0.1} = 9\pi \times 10^{-4} \text{ T}$$

(Towards West)

Hence the net field is -

$$B = B_2 - B_1$$

$$B = 9\pi \times 10^{-4} - 4\pi \times 10^{-4} = 5\pi \times 10^{-4} \text{ T}$$

or $B = 1.57 \times 10^{-3}$ T (Towards West)

Q. CBSE 2008 An electron moves around the nucleus in a hydrogen atom of radius $0.51\,A^\circ$ with a velocity of 2×10^5 m/s. Calculate -

(a) The equivalent current due to orbital motion of electron.

(b) The magnetic field produced at the centre of the nucleus.

(c) The magnetic moment associated with the electron.

Sol: Given that - $r = 0.51\,A^\circ = 0.51\times10^{-10}$

$V = 2\times10^5$ m/s

(a) Time period $T = \frac{2\pi r}{V}$

Electric current $I = \frac{e}{T} = \frac{e}{2\pi r/V} = \frac{eV}{2\pi r}$

$$I = \frac{1.6\times10^{-19}\times2\times10^5}{2\times3.14\times0.51\times10^{-10}}$$

$$I = 9.99\times10^{-5}\ A$$

(b) $B = \frac{\mu_0 I}{2r} = \frac{\mu_0}{2r}\left(\frac{eV}{2\pi r}\right)$

$$B = \frac{4\pi\times10^{-7}}{2\times0.51\times10^{-10}}\times9.99\times10^{-5}$$

or $B = 1.23\ T$

(c) Magnetic Moment $M = IA$

$$M = 9.99\times10^{-5}\times(\pi r^2)$$

$$M = 9.99\times10^{-5}\times3.14\times(0.51\times10^{-10})^2$$

or $M = 8.16\times10^{-25}\ A\cdot m^2$

Q. CBSE 2011, 10 A straight wire of length L is bent into a semi-circular loop. Use Biot-Savart Law to deduce an expression for the magnetic field at its centre due to the current I passing through it.

Sol. Length L is bent into semicircular loop.

Length of wire = Circumference of semicircular wire

$$L = \pi r$$

$$r = \frac{L}{\pi}$$

Considering a small element dl on current loop. The magnetic field dB due to small current element Idl at centre C. Using Biot-Savart Law, we have -

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin 90^\circ}{r^2} \qquad \left(Idl \perp r \;\therefore\; \theta = 90^\circ\right)$$

$$dB = \frac{\mu_0}{4\pi} \frac{Idl}{r^2}$$

Net magnetic field at C due to semicircular loop, -

$$B = \int_{\text{Semicircle}} \frac{\mu_0}{4\pi} \frac{Idl}{r^2}$$

$$B = \frac{\mu_0}{4\pi} \frac{I}{r^2} \int_{\text{Semicircle}} dl = \frac{\mu_0}{4\pi} \cdot \frac{I}{r^2} L$$

But $r = \frac{L}{\pi}$ So that

$$B = \frac{\mu_0}{4\pi} \cdot \frac{IL}{(L/\pi)^2}$$

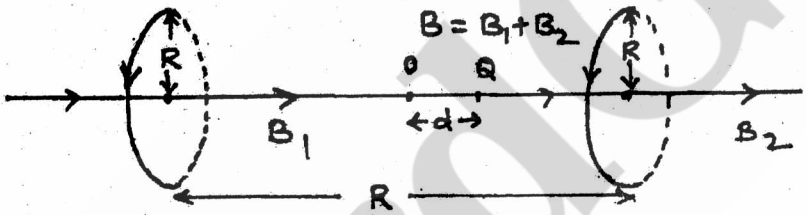
$$\boxed{B = \frac{\mu_0 I \pi}{4L}}$$

Q. Consider two parallel co-axial circular coils of equal radius R and number of turns N, carrying equal currents in the same direction, and separated by a distance R. Show that the field on the axis around the mid-point between the coils is uniform over a distance that is small as compared to R and is given by -

$$B = 0.72\left(\frac{\mu_0 NI}{R}\right) \quad \text{approximately.}$$

[such an arrangement to produce a nearly uniform magnetic field over a small region is known as Helmholtz coils]

Sol. Lets consider point Q at distance d from the centre O.



Then one coil is at a distance of $\left(\frac{R}{2}+d\right)$ from point Q. So the magnetic field at point Q is -

$$B_1 = \frac{\mu_0 NIR^2}{2\left[\left(\frac{R}{2}+d\right)^2+R^2\right]^{3/2}}$$

and the other coil is at distance of $\left(\frac{R}{2}-d\right)$ from point Q. So the magnetic field due to this coil at point Q is -

$$B_2 = \frac{\mu_0 NIR^2}{2\left[\left(\frac{R}{2}-d\right)^2+R^2\right]^{3/2}}$$

So Total magnetic field at point Q is -

$$B = B_1 + B_2$$

$$B = \frac{\mu_0 IR^2 N}{2}\left[\left\{\left(\frac{R}{2}-d\right)^2+R^2\right\}^{-3/2} + \left\{\left(\frac{R}{2}+d\right)^2+R^2\right\}^{-3/2}\right]$$

$$B = \frac{\mu_0 N I R^2}{2}\left[\left(\frac{5R^2}{4} + d^2 - Rd\right)^{-3/2} + \left(\frac{5R^2}{4} + d^2 + Rd\right)^{-3/2}\right]$$

$$B = \frac{\mu_0 N I R^2}{2} \times \left(\frac{5R^2}{4}\right)^{-3/2}\left[\left(1 + \frac{4}{5}\frac{d^2}{R^2} - \frac{4}{5}\frac{d}{R}\right)^{-3/2} + \left(1 + \frac{4}{5}\frac{d^2}{R^2} + \frac{4}{5}\frac{d}{R}\right)^{-3/2}\right]$$

for $d << R$, neglecting the factor $\frac{d^2}{R^2}$, we get-

$$B \cong \frac{\mu_0 N I R^2}{2}\left(\frac{5R^2}{4}\right)^{-3/2}\left[\left(1 - \frac{4}{5}\frac{d}{R}\right)^{-3/2} + \left(1 + \frac{4}{5}\frac{d}{R}\right)^{-3/2}\right]$$

By using Binomial Theorem we get -

$$B \cong \frac{\mu_0 N I R^2}{2R^3}\left(\frac{4}{5}\right)^{3/2}\left[\left(1 - \frac{6d}{5R}\right) + \left(1 + \frac{6d}{5R}\right)\right]$$

$$B \cong \left(\frac{4}{5}\right)^{3/2}\frac{\mu_0 N I}{R}$$

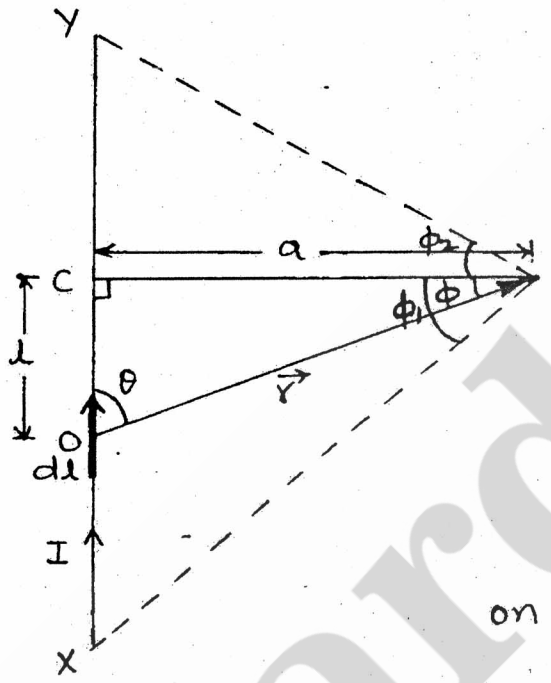
$$B \cong 0.72\left(\frac{\mu_0 N I}{R}\right)$$

Hence it is proved that the field on the axis around the mid-point between the coils is uniform.

Magnetic Field due to a Straight wire carrying Current

Consider a straight wire XY carrying current I. Let P be a point at a perpendicular distance a from the wire. Let the conductor be made of small current elements.

Consider a small current element $I\vec{dl}$ of the wire at O. Here $\vec{r}$ is the position vector of P w.r.t. current element and θ be the angle between $I\vec{dl}$ and $\vec{r}$. Let CO = l.



$\theta = 90^\circ - \phi$

$\sin\theta = \sin(90^\circ - \phi)$

$\sin\theta = \cos\phi$

$\cos\phi = \frac{a}{r}$ or $r = \frac{a}{\cos\phi}$

$l = a\tan\phi$

on differentiating it $dl = a\sec^2\phi\, d\phi$

According to Biot-Savart's Law the magnetic field $\vec{B}$ at point P is (due to current element Idl)

$$dB = \frac{\mu_0}{4\pi} \times \frac{I\,dl\sin\theta}{r^2}$$

Or $$dB = \frac{\mu_0}{4\pi} \frac{I(a\sec^2\phi\, d\phi)\cos\phi}{(a^2/\cos^2\phi)}$$

Or $$dB = \frac{\mu_0}{4\pi} \frac{I}{a}\cos\phi\, d\phi$$

The total magnetic field at point P due to current through the whole straight conductor XY can be obtained by integrating dB within the limits $-\phi_1$ and $+\phi_2$.

$$\int_{-\phi_1}^{\phi_2} dB = \int_{-\phi_1}^{\phi_2} \frac{\mu_0 I}{4\pi a} \cos\phi \, d\phi$$

$$\boxed{B = \frac{\mu_0 I}{4\pi a} (\sin\phi_1 + \sin\phi_2)}$$

Special Cases -

(i) When the conductor XY is of infinite length and point P lies near the centre of the conductor ($\phi_1 = \phi_2 = 90^\circ$)

So $B = \frac{\mu_0 I}{4\pi a} [\sin 90^\circ + \sin 90^\circ]$

$$\boxed{B = \frac{\mu_0}{4\pi} \frac{2I}{a}}$$

(ii) When the conductor XY is of infinite length but the point P lies near the end Y (or X) ($\phi_1 \cong 90^\circ$ and $\phi_2 \cong 0^\circ$)

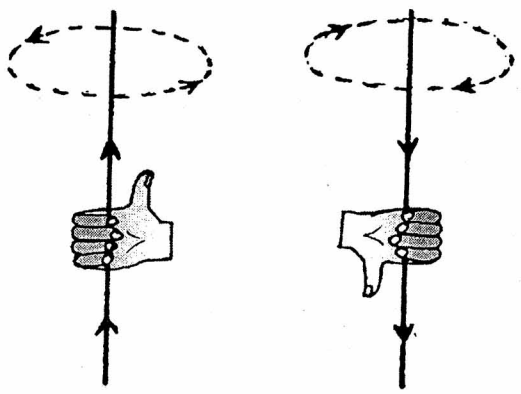
So $B = \frac{\mu_0 I}{4\pi a} [\sin 90^\circ + \sin 0^\circ]$

$$\boxed{B = \frac{\mu_0 I}{4\pi a}}$$

Direction of Magnetic Field

Right Hand Thumb Rule -

If we imagine the linear wire conductor to be held in the grip of the right hand so that the thumb points in the direction of current, then the curvature of the fingres around the conductor will represent the direction of magnetic field Lines.



Ampere's Circuital Law

It states that the line integral of magnetic field $\vec{B}$ around a closed path in vacuum is equal to μ_0 times the total current I threading the closed path.

$$\oint \vec{B}.\vec{dl} = \mu_0 I$$

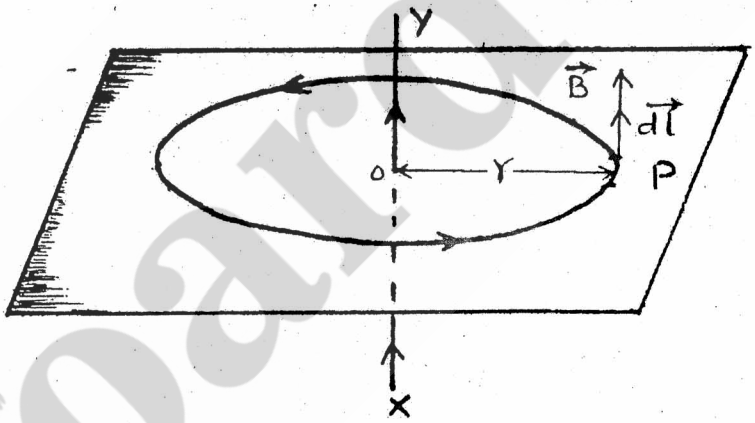
Proof -

Consider a long thin straight wire XY carrying current I. The magnetic field lines around the conductor are concentric circles in the plane perpendicular to the wire XY.

The magnitude of magnetic field produced at a point P at distance r from the conductor is.

$$B = \frac{\mu_0}{4\pi}\frac{2I}{r} = \frac{\mu_0 I}{2\pi r}$$

The direction of $\vec{B}$ is along the tangent to the magnetic field line at that point.



Consider a circular closed path of radius r whose centre O lie on the straight conductor.

Take a small element of length dL of the closed path at P. The direction of $\vec{B}$ and $\vec{dl}$ is the same.

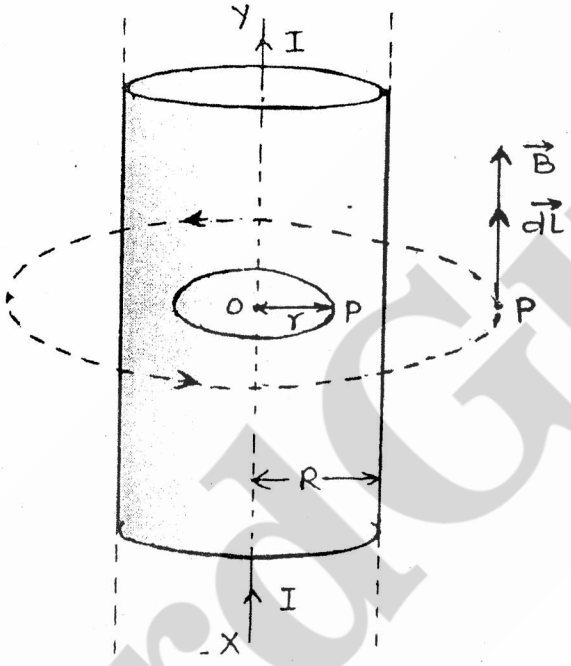
Therefore line integral of $\vec{B}$ around the closed path of radius r is

$$\oint \vec{B}.\vec{dl} = \oint B\,dl\cos 0^\circ = \oint B\,dL = B\oint dL$$

or $\oint \vec{B}.\vec{dl} = \frac{\mu_0 I}{2\pi r} \times 2\pi r$ so $\boxed{\oint \vec{B}.\vec{dl} = \mu_0 I}$ Hence Proved.

Magnetic Field due to Current through a very Long Circular Cylinder -

Consider an infinite Long cylinder of radius R with axis XY. Let current I be the current passing through the cylinder. The magnetic field lines of force are perpendicular to the length of cylinder.



Case I - (Point P is lying outside the cylinder)

Let r be the perpendicular distance of point P from the axis of the cylinder (where $r > R$).

The magnetic field $\vec{B}$ is acting tangential to the magnetic lines of force at P.

Here $\vec{B}$ and $\vec{dl}$ are acting in the same direction.

Applying Ampere circuital Law we have -

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

$$B\, 2\pi r = \mu_0 I \qquad \oint dl = 2\pi r$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}} \quad \text{i.e.} \quad \boxed{B \propto \frac{1}{r}}$$

Case II- (Point P is lying inside cylinder) - ($r < R$)

We may have two possibilities.

(i) If the current is only along the surface of cylinder which is so if the conductor is a cylinderical sheet of metal, then current through the closed path L is zero.

Using Ampere circuital Law $\boxed{B = 0}$

(ii) If the current is uniformly distributed throughout the cross-section of the conductor, then the current through closed path L is given by-

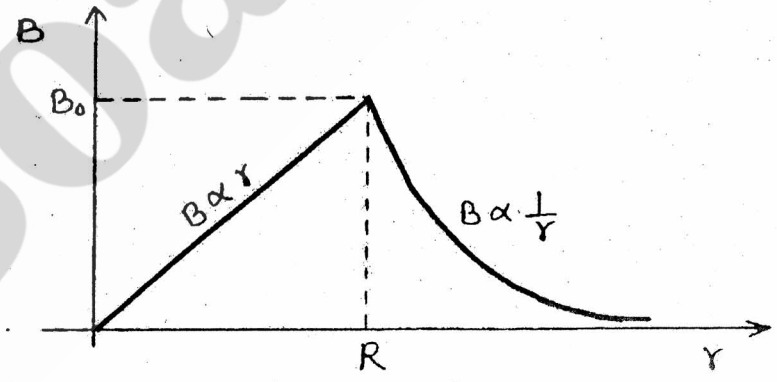
$$I' = \frac{I}{\pi R^2} \times \pi r^2 = \frac{I r^2}{R^2}$$

Applying Ampere's circuital Law, we have

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I'$$

$$B(2\pi r) = \mu_0 \frac{I r^2}{R^2}$$

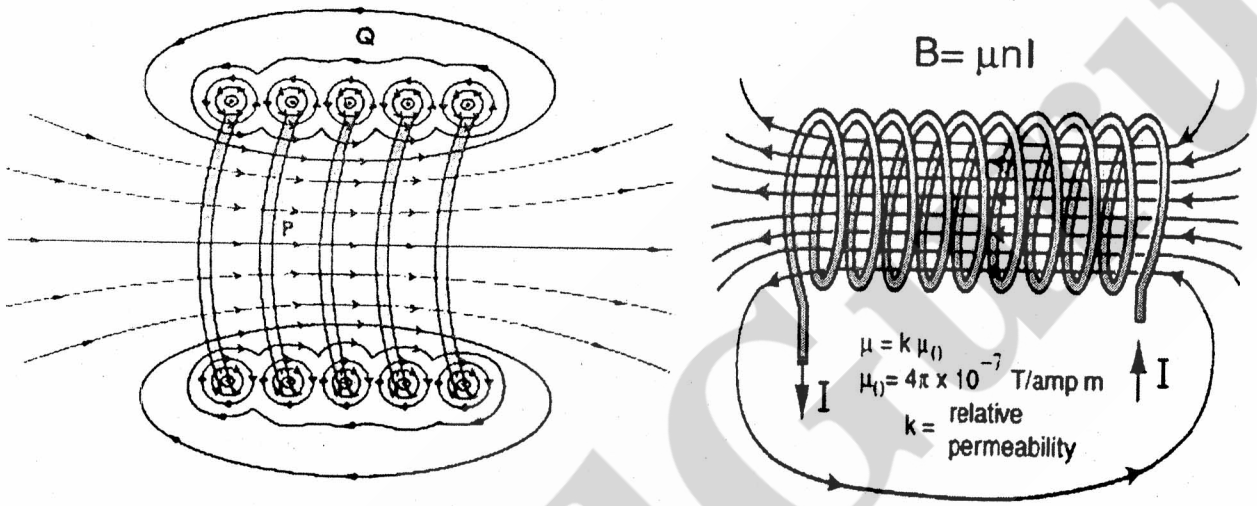
$$\boxed{B = \frac{\mu_0 I r}{2\pi R^2}} \qquad \boxed{B \propto r} \quad \text{for } r < R$$



* So the magnetic field is maximum for a point on the surface of solid cylinder carrying current and is zero for a point on the axis of cylinder.

The SOLENOID -

A solenoid consist of an insulating long wire closely wound in the form of a helix. Its length is very large as compared to its diameter.



Magnetic Field due to a Solenoid -

Consider a long straight solenoid of circular cross section. Each two turns of the solenoid are insulated from each other. When current is passed through the solenoid, then each turn of the solenoid can be considered as a circular loop carrying current and thus will be producing a magnetic field.

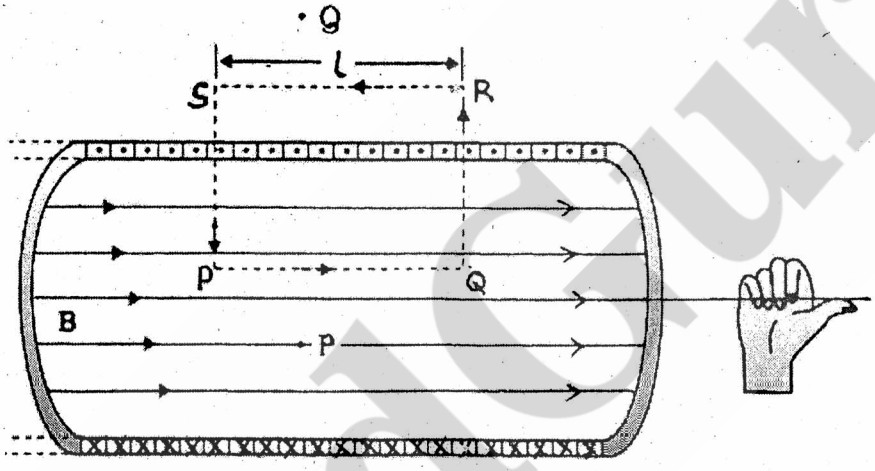
The total magnetic field is the vector sum of the magnetic fields due to currents through all the turns in the solenoid.

At a point outside the solenoid, the magnetic fields due to neighbouring loops oppose each other and at a point inside the solenoid, the magnetic fields are in the same direction.

As a result of it the effective magnetic field outside the solenoid becomes weak, whereas the magnetic field inside the solenoid becomes strong and uniform and acting along the axis of solenoid.

When the solenoid is more and more tightly packed, the magnetic field due to current through the solenoid will become zero for a point outside the solenoid and is strong and uniform at a point inside the solenoid.

Let n be the number of turns per unit length of solenoid and I be the current flowing through the solenoid and the turns of the solenoid be closely packed.



Now consider a rectangular amperian loop PQRS near the middle of solenoid, where PQ = L.

Let the magnetic field along the path PQ be B and is zero along RS. As the path QR and SP are perpendicular to the axis of solenoid, the magnetic field component along these paths is zero.

Total number of turns in length L are = nL.

The line integral of magnetic field induction $\vec{B}$ over the closed path PQRS is

$$\oint_{PQRS} \vec{B}\cdot d\vec{l} = \int_P^Q \vec{B}\cdot d\vec{l} + \int_Q^R \vec{B}\cdot d\vec{l} + \int_R^S \vec{B}\cdot d\vec{l} + \int_S^P \vec{B}\cdot d\vec{l}$$

$$\oint \vec{B}\cdot d\vec{l} = \int_P^Q B\,dl\cos 0^\circ + \int_Q^R B\,dl\cos 90^\circ + \int_R^S B\cdot dl + \int B\,dl\cos 90^\circ$$

$$= BL + 0 + 0 + 0$$

So $\oint_{PQRS} \vec{B}.d\vec{l} = BL$

From Ampere's Circuital Law-

$\oint_{PQRS} \vec{B}.d\vec{l}$ = μ_0 x total current through the rectangle PQRS

= μ_0 x no. of turns in rectangle x Current

$BL = \mu_0 n L I$ [N = Total no. of turns in solenoid

$$\boxed{B = \mu_0 n I} \qquad \boxed{B = \frac{\mu_0 N I}{l}}$$

It is the magnetic field inside the solenoid.

* At a point near the end of a solenoid, the magnetic field is found to be $\frac{\mu_0 n I}{2}$
* The Linear solenoid carrying current is equivalent to a bar magnet.

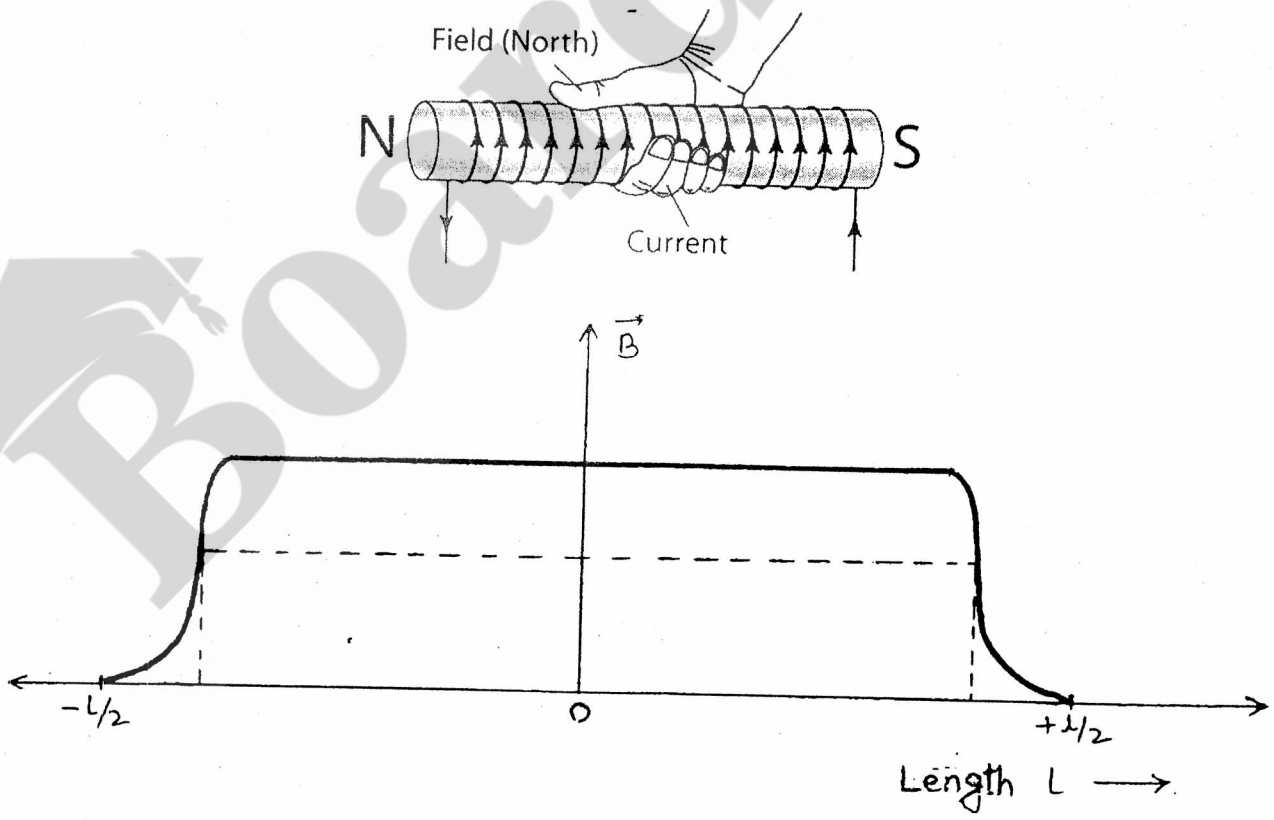


Fig:- Change in magnetic field with distance along the axis of the solenoid

Q. Delhi 2008 A solenoid of length 1 m has a radius of 1 cm. and has a total 1000 turns wound on it. It carries a current of 5 A. Calculate the magnitude of the axial magnetic field inside the solenoid. If an electron was to move with a speed of 10^4 m/s along the axis of this current carrying solenoid, what would be the force experienced by this electron ?

Sol. In a solenoid the magnetic field is along its axis, so it is called axial magnetic field

Given - $L = 1\ m$, $r = 1\ cm = 10^{-2}\ m$.
$N = 1000$, $I = 5\ A$

∴ Magnetic field B inside the solenoid -

$$B = \mu_0 n I = \mu_0 \frac{N}{L} I$$

$$B = 4\pi \times 10^{-7} \times \frac{1000}{1} \times 5$$

$$B = 2\pi \times 10^{-3}\ T$$

The direction of B is along the axis of solenoid.

Now $q = -e$, $V = 10^4$ m/s

and the angle between B and V is 0° because the electron moves along the direction of the magnetic field.

∴ Magnetic Lorentz force

$$F_B = qVB \sin 0^\circ = qVB \times 0 = 0$$

So $F_B = 0$

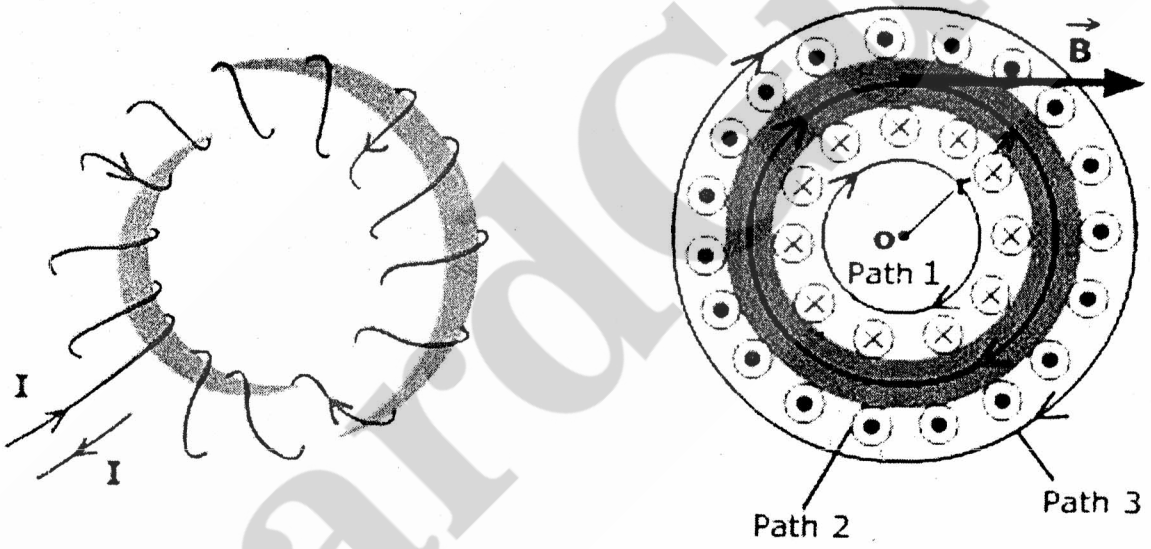
Hence No magnetic force experienced by the electron.

The TOROID

The toroid is a hollow circular ring on which a large number of insulated wire turns are closely wound. Infact a toroid is an endless solenoid in the form of a ring.

Magnetic Field due to current in ideal toroid.

Let n be the number of turns per unit length of toroid and I be the current flowing through it. In case of ideal toroid, the coil turns are circular and closely wound.



The direction of the magnetic field at a point is given by the tangent to the magnetic field line at that point.

We draw three circular amperian loop 1, 2 and 3 of radius r_1, r_2 and r_3 to be traversed in clockwise direction, so that the points P, S and Q may lie on them.

So the magnetic field along loop 1 is.

$$\oint_{\text{loop 1}} \vec{B}_1 \cdot d\vec{l} = \mu_0 I$$

Here $I = 0$ for loop1 So $\boxed{B_1 = 0}$

. The current enclosed by loop 3 is zero so the magnetic field along the loop 3 -

$$\oint_{\text{loop}3} \vec{B}_3 \cdot d\vec{l} = \mu_0 \times 0$$

$$\boxed{B_3 = 0}$$

Let B is the magnetic field along the loop 2.

Current enclosed By the loop 2 =

= number of turns x Current in each turn

$= 2\pi r_2 n \times I$

So $\oint_{\text{loop}2} \vec{B} \cdot d\vec{l} = \mu_0 \times 2\pi r_2 n I$

$B \times 2\pi r_2 = \mu_0 \times 2\pi r_2 n I$ $\quad \left[\oint_{\text{loop}2} dL = 2\pi r_2\right]$

$$\boxed{B = \mu_0 n I}$$

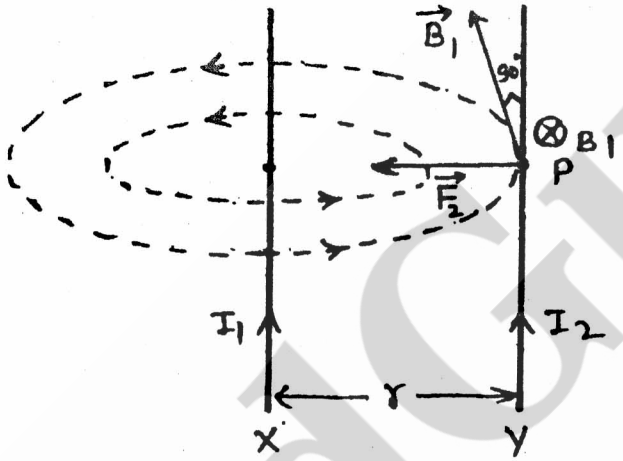
This result is similar to that of solenoid.

Note - In reality the turns of the toroid form a helix and not circle as in the case for ideal toroid.
There is also a small magnetic field in the external portion of the toriod.

Force between two parallel Current Carrying Conductors

Consider two infinite long straight conductors carrying currents I_1 and I_2 in the same direction. They are held parallel to each other at a distance r apart.

Since magnetic field is produced due to current through each conductor, therefore each conductor experiences a force.



Magnetic field at a point P on conductor Y due to current I_1 passing through X is given by -

$$B_1 = \frac{\mu_0}{4\pi} \frac{2I_1}{r}$$

According to right hand Rule, the direction of magnetic field $\vec{B}_1$ is perpendicular to the conductor Y, directed inwards.

As the current carrying conductor Y lies in the magnetic field $\vec{B}_1$, therefore the unit length of Y will experience a force given by-

$$F_2 = B_1 I_2 \sin\theta \qquad [\ L = 1 \text{ (unit length)}$$

$$F_2 = B_1 I_2 \times 1 = B_1 I_2$$

Putting the value of B_1, we have -

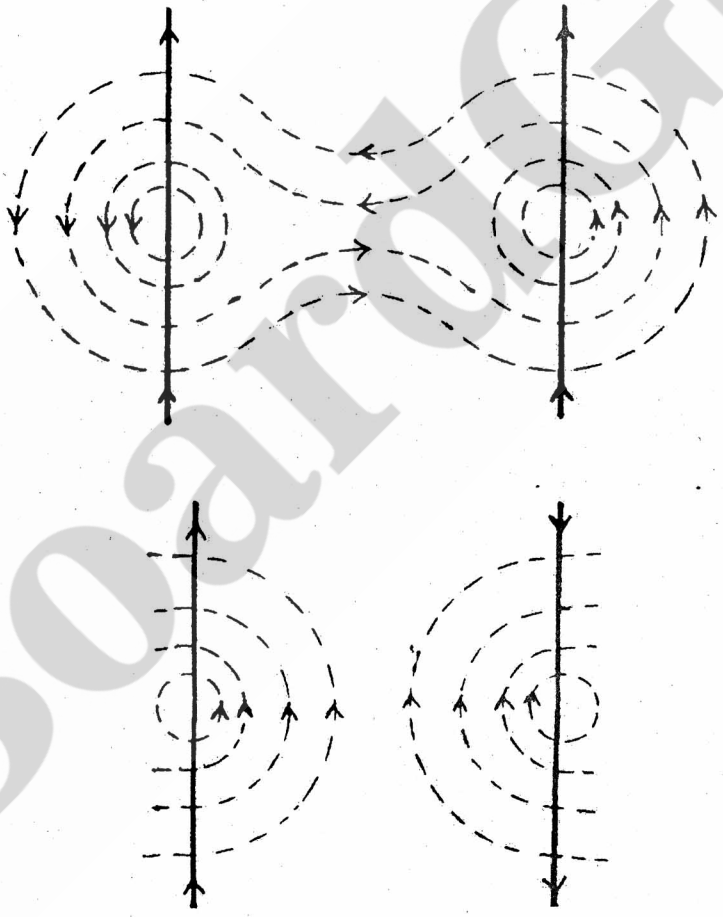
$$\boxed{F_2 = \frac{\mu_0}{4\pi} \times \frac{2I_1 I_2}{r}}$$

According to Fleming's Left Hand Rule, force on conductor Y acts perpendicular to Y, directed towards X.

Similarly the conductor X also experiences a force $F_1 = F_2$, which acts perpendicular to X and directed towards Y. Hence X and Y attract each other.

* Two parallel conductors carrying currents in the same direction attract each other.

* If the currents in conductor X and Y are in opposite directions then they repel each other.



* If two long straight conductors carrying current in the same direction then the magnetic field at the half distance between the conductors

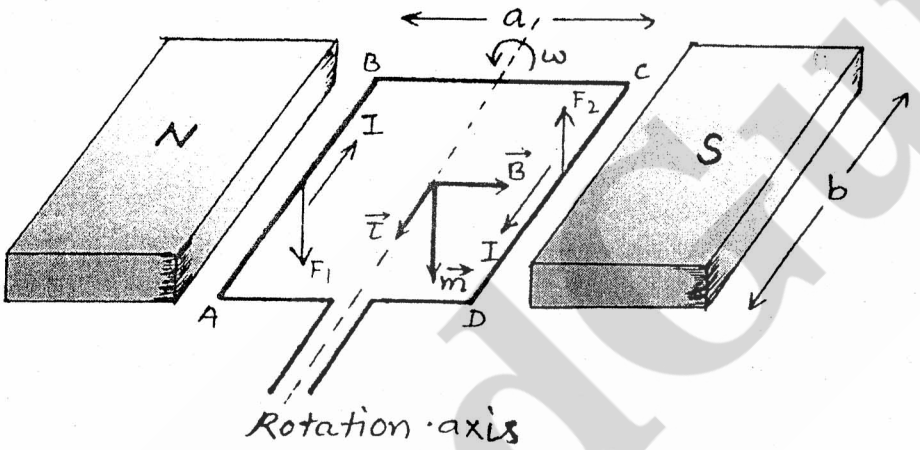
$$B = B_1 - B_2 = 0$$

* If the current is in opposite direction then $B = B_1 + B_2 = 2\left(\frac{\mu_0 2I}{4\pi r}\right)$

Torque on a current carrying Coil in a Magnetic Field -

When a rectangular current carrying coil is placed in a uniform magnetic field then it experiences a torque. It does not experience a net Force.

We first consider the simple case when the rectangular loop is placed such that the uniform magnetic field B is in the plane of the loop.

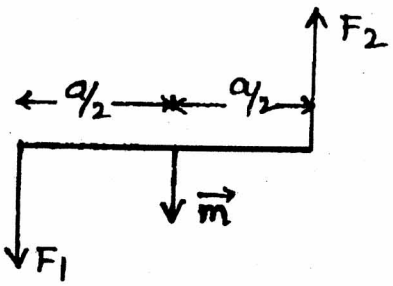


The field exerts no force on the two arms AD and BC of the loop.

The magnetic field is perpendicular to the arm AB of the loop and exerts a force F_1 on it, which is directed into the plane of the loop.

$$F_1 = IbB$$

Similarly the magnetic field exerts a force F_2 on the arm CD, which is directed out of the plane of the loop.



$$F_2 = IbB = F_1$$

Thus the net force on the loop is zero.

There is a torque on the loop due to the pair of forces F_1 and F_2.

Fig. Shows that the torque on the loop tends to rotate it anti-clockwise.

This torque is -

$$\tau = F_1 \times \frac{a}{2} + F_2 \times \frac{a}{2}$$

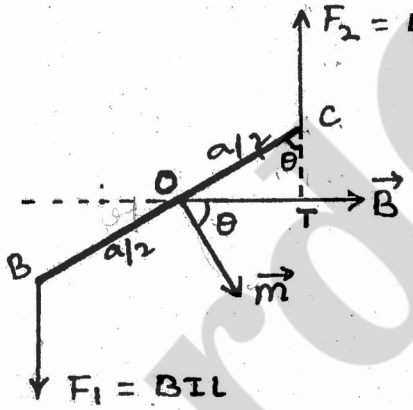
$$\tau = IbB\frac{a}{2} + IbB\frac{a}{2}$$

$$\boxed{\tau = Iab B = IAB}$$

Here A is the area of the rectangle.

Now consider the case when the plane of the loop, is not along the magnetic field and makes an angle with it.

Let the angle between the field and the normal to the coil be angle θ.



$$\sin\theta = \frac{OT}{a/2}$$

$$OT = \frac{a}{2}\sin\theta$$

The force on arms AB and CD are F_1 and F_2.

$$F_1 = F_2 = IBb.$$

Here the forces F_1 and F_2 results a torque which is however less than the earlier case when plane of loop was along the magnetic field. This is because the perpendicular distance between the forces of the couple has decreased.

So the magnitude of the torque on the loop is

$$\tau = F_1 \frac{a}{2}\sin\theta + F_2 \frac{a}{2}\sin\theta$$

$$\boxed{\tau = Iab B\sin\theta = IAB\sin\theta}$$

$$\vec{\tau} = I(\vec{A} \times \vec{B})$$

The torques can be expressed as vector product of the magnetic moment of the coil and the magnetic field.

$$\vec{\tau} = \vec{m} \times \vec{B}$$

$\theta \rightarrow$ angle between $\vec{m}$ and $\vec{B}$

$\vec{m}$ = magnetic moment of the current loop.

$$\vec{m} = I\vec{A}$$

SI Unit of magnetic moment is $A\text{-}m^2$.

* This torque tends to rotate the coil about its own axis. Its value changes with angle between plane of coil and direction of magnetic field.

If the loop has N closely wound turns, then the torque will be -

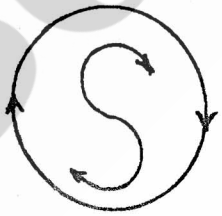
and

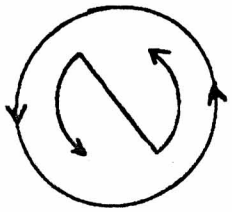
$$\vec{\tau} = NI(\vec{A} \times \vec{B})$$

$$\vec{m} = NI\vec{A}$$

Polarity of the magnetic dipole moment -

Look at one face of the coil. If the direction of current through the coil is clockwise then the face has south polarity.





If the direction of current is anticlockwise, then that face has north polarity.

Q. Delhi 2010 A square coil of side 10 cm. has 20 turns and carries a current of 12 A. The coil is suspended vertically and the normal to the plane of the coil, makes an angle θ with the direction of a uniform horizontal magnetic field of 0.8 T. If the torque, experienced by the coil equals 0.96 N-m, find the value of θ.

Sol. Here area of the coil $= 10 \times 10 = 100\ cm^2$
$= 10^{-2}\ m^2$

Number of turns N = 20

Current = 12 A

Magnetic field B = 0.8 T

Torque = 0.96 N-m.

Torque experienced by current carrying coil in the magnetic field is

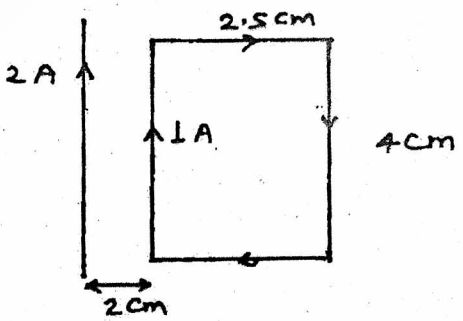
$$\tau = NIAB\sin\theta$$

$$0.96 = 20 \times 12 \times 10^{-2} \times 0.8 \times \sin\theta$$

$$\sin\theta = \frac{0.96}{1.92} = \frac{1}{2}$$

So $\theta = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ rad.

Q. CBSE 2012 A rectangular loop of wire of size 2.5 cm × 4 cm. carries steady current of 1 A. A straight wire carrying 2 A current is kept near the loop as shown. If the loop and the wire are coplanar, find the (i) torque acting on the loop and (ii) the magnitude and direction of force on the loop due to the current carrying wire.



Sol: There will be force of attraction between the straight wire and the 4 cm long arm of loop nearer to straight conductor.

$$F_1 = \frac{\mu_0}{4\pi} \frac{2 \times 2 \times 1}{(2 \times 10^{-2})} \times 4 \times 10^{-2}$$

$$F_1 = 8 \times 10^{-7} \text{ N} \quad \text{(towards straight wire)}$$

Similarly the force on other 4 cm arm of loop away from the straight conductor

$$F_2 = \frac{\mu_0}{4\pi} \frac{2 \times 2 \times 1}{(4.5 \times 10^{-2})} \times 4 \times 10^{-2}$$

$$F_2 = 3.55 \times 10^{-7} \text{ N} \quad \text{(away from straight wire)}$$

(i) F_1 and F_2 are of different magnitude therefore do not form couple and hence

Torque $\tau = 0$

(ii) Net force on loop

$$F = F_1 - F_2 \quad \text{(towards straight wire)}$$

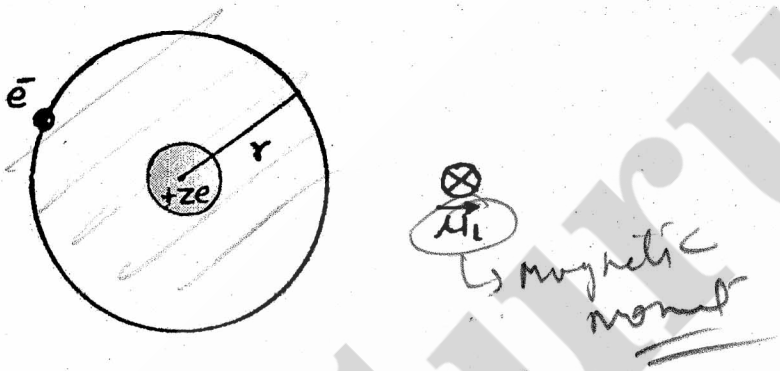
$$F = 8 \times 10^{-7} - 3.55 \times 10^{-7}$$

$$F = 4.45 \times 10^{-7} \text{ N}$$

The force on two branches of loop are equal in magnitude, opposite in direction, hence they balance each other.

The Magnetic dipole moment of a revolving electron -

In Bohr model a negatively charged particle (electron) revolves around a positively charged nucleus.



The electron ($e = 1.6 \times 10^{-19} C$) performs uniform circular motion around a stationary heavy nucleus of charge +Ze.

This constitutes a current I as -

$$I = \frac{e}{T} \qquad T \rightarrow \text{Time period of revolution}$$

Let r be the orbital radius of the electron, and v is the orbital speed. Then -

$$T = \frac{2\pi r}{v} \quad \text{then} \quad I = \frac{ev}{2\pi r}$$

There will be a magnetic moment, denoted by μ_l, associated with this circulating current.

where $$\mu_l = I \times \pi r^2 = \frac{evr}{2}$$

$\mu_l = \frac{ev}{2\pi r} \times \pi r^2$

The direction of this magnetic moment is into the plane of papers. where we assume that the electron is moving anti-clockwise, leading a clockwise current.

We can write $$\mu_l = \frac{e}{2m_e}(m_e v r) = \frac{e}{2m_e} \times L$$

↳ mass of electron

Here L is the magnitude of the angular momentum of the electron about the central nucleus.

Here the ratio $\boxed{\frac{\mu_l}{l} = \frac{e}{2m_e}}$ is called the gyromagnetic ratio and is a constant.

Its value is 8.8×10^{10} C/kg for an electron.

Bohr assume that the angular momentum have a discrete set of values as.

$$l = \frac{nh}{2\pi} \qquad n = 1, 2, 3 \ldots .$$

h is planck's Constant. ($h = 6.63 \times 10^{-34}$ Js)

This condition of discreteness is called the "Bohr quantisation condition".

Take the value of $n = 1$, we have.

$$(\mu_l)_{min} = \frac{eh}{4\pi m_e}$$

$$= \frac{1.6 \times 10^{-19} \times 6.63 \times 10^{-34}}{4 \times 3.14 \times 9.11 \times 10^{-31}}$$

$$(\mu_l)_{min} = 9.27 \times 10^{-24} \text{ Am}^2$$

This value is called the "Bohr magneton".

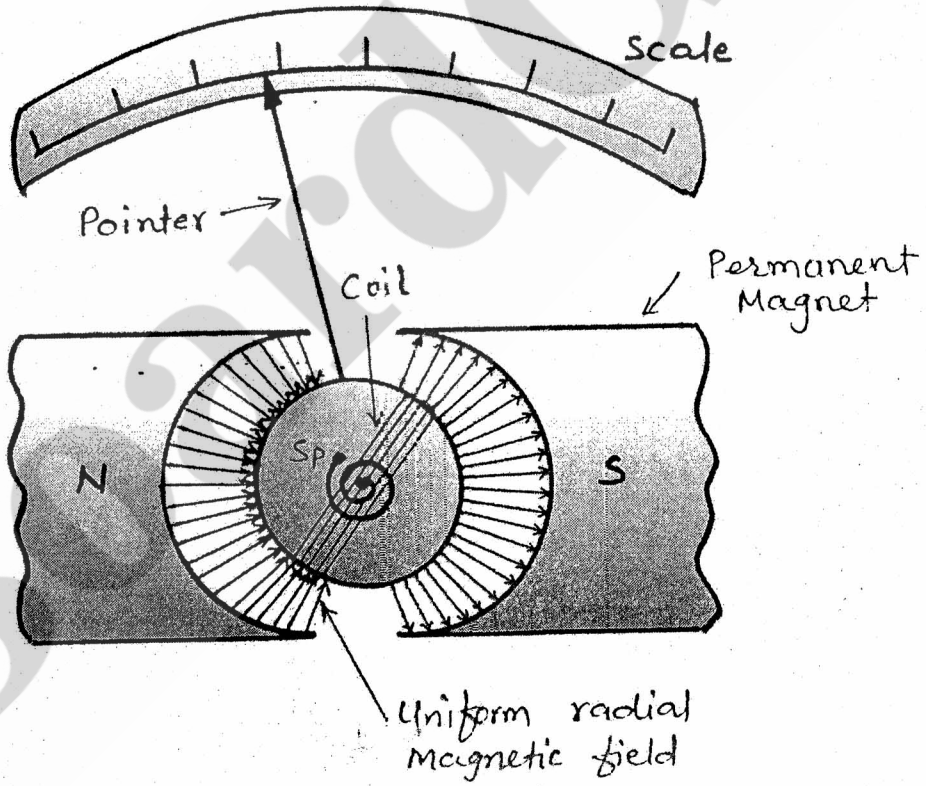
The Moving Coil GALVANOMETER -

Moving coil galvanometer is an instrument used for detection and measurement of small electric current and voltage.

Principle - Its working is based on the fact that when a carrying coil is placed in a magnetic field, it experiences a torque.

Construction - The galvanometer consist of a coil with many turns, which is free to rotate about a fixed axis in a uniform radial magnetic field.

There is a cylinderical soft iron core, which increases the strength of the magnetic field and makes the field radial.



Working :- When the current flows through the coil, a torque acts on it. Which can be given as -

$$\tau = NIAB \qquad (\sin\theta = 1)$$

Since the field is radial by design, so we have taken $\sin\theta = 1$.
This magnetic torque tends to rotate the coil.

The spring Sp provides a counter torque $K\phi$ that balance the magnetic torque NIAB and results in steady angular deflection ϕ.
In equilibrium -

$$K\phi = NIAB$$

Where K is the torsional constant of the spring i.e. restoring torque per unit twist.

The deflection ϕ is indicated on the scale by a pointer attached to the spring.

We have $$\boxed{\phi = \left(\frac{NBA}{K}\right) I}$$

Hence the deflection produced is proportional to the current flowing through the galvanometer.

Current Senitivity -

It is defined as the deflection produced in the galvanometer when a unit current flows through it.

$$\boxed{\text{Current sensitivity } I_s = \frac{\phi}{I} = \frac{NBA}{K}}$$

The unit of current sensitivity is rad./A or division/A.

Voltage Sensitivity -

It is defined as the deflection produced in the galvanometer when a unit voltage is applied across the two terminals of the galvanometer.

$$\boxed{\text{Voltage Sensitivity } V_s = \frac{\phi}{IR} = \frac{NBA}{KR} = \frac{I_s}{R}}$$

The unit of voltage sensitivity is rad/V or division/V.

Q. (All India 2008) Why should the spring/suspension wire in a moving coil galvanometer have low torsional constant?

Sol. Low torsional constant facilitate greater deflection (α) in coil for given value of current and hence sensitivity of galvanometer increases.

Q. (CBSE 2006) Write two factors by which current sensitivity of a galvanometer can be increased.

Sol. Current sensitivity of galvanometer can be increased by -

(i) Increasing the number of turns of coil

(ii) Increasing the magnetic field.

Q. (CBSE 2008) Write two factors by which Voltage sensitivity of a moving coil galvanometer can be increased.

Sol. Voltage sensitivity of galvanomer can be increased by (i) Increasing the magnetic field

(ii) Decreasing the value of torsional Constant.

Q. (RBSE 2005) The coils in certain galvanometer, have a fixed core made of a non-magnetic metallic material. Why does the oscillating coil come to rest so quickly in such a core?

Sol. Due to eddy currents produced in core which opposes the cause (deflection of coil), that produces it.

Ammeter -

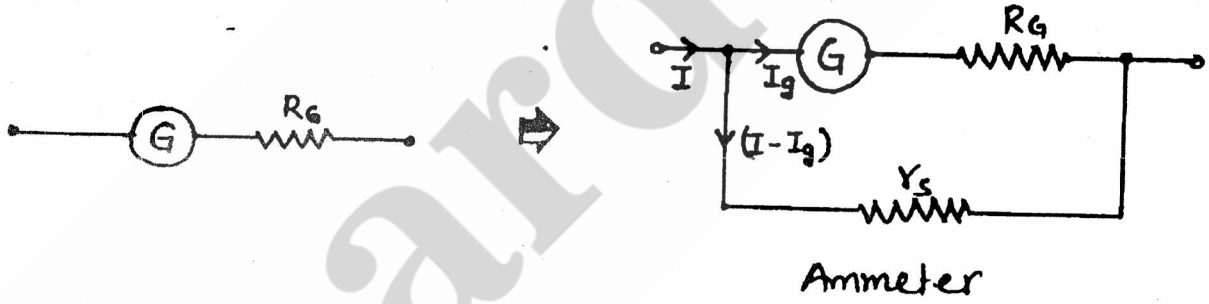
An ammeter is a low resistance galvanometer It is used to measure the current in a circuit in amperes.

Galvanometer is a very sensitive device, it gives a full scale deflection for a current of the order of μA. Also for measuring currents, the galvanometer has to be connected in series and as it has a large resistance, this will change the value of the current in the circuit.

To overcome these difficulties a low resistance wire r_s, called shunt resistance is connected in parallel with the galvanometer coil, so that most of the current passes through the shunt.

The resistance of the arrangement is -

$$\frac{R_G r_s}{(R_G + r_s)} \simeq r_s \quad (\text{if } R_G >> r_s)$$



Ammeter

* An ideal ammeter has zero resistance.

Effective resistance of converted galvanometer into ameter -

$$R_p = \frac{R_G r_s}{R_G + r_s}$$

Also Here - $I_g R_G = (I - I_g) r_s$

So $$r_s = \frac{I_g}{(I - I_g)} R_G$$

Voltmeter -

A voltmeter is a high resistance galvanometer. It is used to measure the potential difference between two points of a circuit in volts.

A galvanometer can be converted into a voltmeter by connecting a high resistance in series with the galvanometer. So that it draws a small current, otherwise the voltage measurement will disturb the original set up by an amount which is very large.

The resistance of the arrangement is -

$R_G + R \approx R$ (if $R_G << R$)

* The resistance of an ideal voltmeter is infinity.

Effective resistance of converted galvanometer into Voltmeter -

$$R_s = R_G + R$$

Also from ohm's Law - $I_g = \frac{V}{R_G + R}$

$$R = \frac{V}{I_g} - R_G$$

Q. All India 2007 A galvanometer has a resistance of 30 Ω. It gives full scale deflection with a current of 2 mA. Calculate the value of the resistance needed to convert it into an ammeter of range 0 - 0.3 A

Sol.

Resistance of galvanometer $R_G = 30\ \Omega$

Range of galvanometer $I_g = 2mA = 2 \times 10^{-3}\ A$

Range of ammeter $I = 0.3\ A$

A galvanometer of range I_g and resistance R_G can be converted into an ammeter by connecting a very low resistance r_s in parallel with the galvanometer where value is given by -

$$r_s = \frac{I_g R_G}{I - I_g}$$

$$r_s = \frac{2 \times 10^{-3} \times 30}{(0.3 - 2 \times 10^{-3})} = \frac{6 \times 10^{-2}}{0.298}$$

$$r_s = 0.2\ \Omega$$

Q. CBSE 2006 Which one of the two an ammeter or a milliammeter, has higher resistance and why?

Sol. The milliammeter has higher resistance.

∵ Higher the range of ammeter, smaller the value of shunt resistance. The equivalent resistance of parallel combination of the galvanometer and shunt resistance is smaller than the shunt resistance.

Ammeter has got higher range, therefore smaller value of shunt resistance and smaller equivalent resistance.

VERY IMPORTANT QUESTIONS

1 Mark Questions -

Q.1. Two particles A and B of masses m and 2m have charges q and 2q respectively, both these particles moving with velocities v_1 and v_2 respectively in the same direction enter the same magnetic field $\vec{B}$ acting normally to their direction of motion. If the two forces F_A and F_B acting on them are in the ratio of 1:2, find the ratio of their velocities.

Answer = 1 : 1 (Delhi 2011)

Q.2. A beam of α particles projected along +x axis, experiences a force due to a magnetic field along the +Y-axis. What is the direction of the magnetic field?

Answer = along -Z axis. (All India 2010)

Q.3. An electron does not suffer any deflection while passing through a region of uniform magnetic field. What is the direction of the magnetic field?

Answer = Along or opposite to the motion of electron. (All India 2009)

Q.4. Write two properties of a material used as a suspension wire in a moving coil galvanometer. (CBSE 2006)

Q.5. A circular coil of closely wound N turns and radius r carries a current I. Write the expression for the following:

(i) The magnetic field at its centre.

(ii) the magnetic moment of this coil. (All India 2012)

2 Marks Questions -

Q.1 A particle of charge q and mass m is moving with velocity v. It is subjected to a uniform magnetic field B directed perpendicular to its velocity. Show that it describes a circular path. Write the expression for its radius.

(CBSE 2012)

Q.2. A long solenoid of length L having N turns carries a current I. deduce the expression for the magnetic field in the interior of the solenoid.

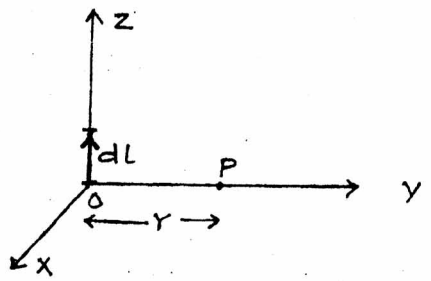
(CBSE 2011,08)

Q.3. Obtain with the help of necessary diagram, the expression for the magnetic field in the interior of a toroid carrying current.

(CBSE 2006)

Q.4. State Biot - Savart - Law.
A current I flows in a conductor placed perpendicular to the plane of the paper. Indicate the direction of the magnetic field due to a small element dl at a point P situated at a distance r from the element as shown in figure.

(CBSE 2009)



Q.5. A circular coil of radius R carries a current I. Write the expression for the magnetic field due to this coil at its centre. Find out the direction of the field.

(CBSE 2008,06)

Q.6. Using Biot - Savart Law, derive the expression for the magnetic field at a distance r along the axis from the centre of a current carrying current loop.

(Delhi 2006)

Q.7. An electron and proton moving with the same speed enter the same magnetic field region at right angle to the direction of the field. Show the trajectory followed by the two particles in the magnetic field. Find the ratio of the radius of the circular paths which the particles may describe.

(CBSE 2010)

Q.8. A cyclotron, when being used to accelerate positive ions (mass = 6.7×10^{-27} kg, charge = 3.2×10^{-19} c) has a magnetic field of $\pi/2$ T. What must be the value of the frequency of the applied alternating electric field to be used in it?

(All India 2009)

Answer = 1.2×10^{7} cycle/sec.

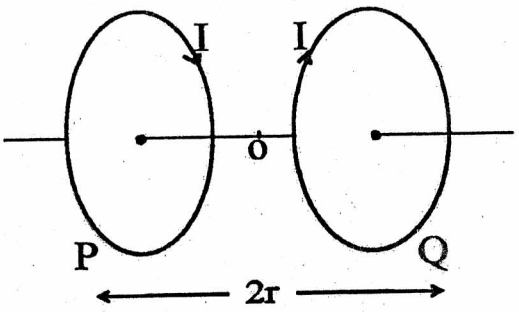
Q.9. Write the relation for the force F acting on a charge q moving with a velocity v through a magnetic field B in vector notation. Using this relation, derive the conditions under which this force will be (a) Maximum and (b) minimum.

(Delhi 2007)

Q.10. State the principle of working of a cyclotron. Write two use of this machine.

(Delhi 2007)

Q.11 Two identical circular loops P and Q each of radius r and carrying equal currents are kept in the parallel planes having a common axis passing through O. The direction of current in P is clockwise and in Q is anti-clockwise as seen from O which is equidistant from the loops P and Q. Find the magnitude of the net magnetic field at O.



(Delhi 2012)

Answer = $B = \frac{\mu_0 I}{2^{3/2} r}$

Q.12. A steady current (I_1) flows through a long straight wire. Another wire carrying steady current (I_2) in the same direction is kept close and parallel to the first wire. Show with the help of a diagram how the magnetic field due to the current I_1 exert a magnetic force on the second wire. Derive the expression for this force.

(All India 2011, 07)

Q.13. How is a moving coil galvanometer converted into a voltmeter? Explain giving the necessary circuit diagram and the required mathematical relation used.

(RBSE 2011)

Q.14 A galvanometer having a coil resistance of 100 Ω gives full scale deflection with a current of 1 mA. Calculate the value of the resistance needed to convert it into an ammeter of range 0 - 1 A.

(CBSE 2007)

Answer = $r_s = 0.1\ \Omega$

Q.15. A galvanometer coil of 50 Ω resistance shows full scale deflection for a current of 5 mA. How will you convert this galvanometer into a voltmeter of range 0 to 15 V?

(CBSE 2011)

3 Marks Questions -

Q.1. A long straight wire of a circular cross-section of radius 'a' carries a steady current I. The current is uniformly distributed across the cross-section. Apply ampere's circuital Law to calculate the magnetic field at a point in the region for (a) $r < a$ and (b) $r > a$

(CBSE 2010, 11)

Q.2. State Ampere's circuital Law. Write the expression for the magnetic field at the centre of a circular coil of radius R carrying a current I. Draw the magnetic field lines due to this coil.

(CBSE 2007)

Q.3. Draw a schematic sketch of the cyclotron. State its working principle. Show that the cyclotron frequency is independent of the velocity of the charged particles.

(Delhi 2011, 09)

Q.4. Draw a labelled diagram of a moving coil galvanometer and explain its working. What is the function of radial magnetic field inside the coil?

(CBSE 2012)

Q.5. A moving coil galvanometer of resistance R_G gives its full scale deflection when a current I_g flows through its coil. It can be converted into a ammeter of range (0 to I) where $(I > I_g)$, when a shunt of resistance r_s is is connected into an ammeter of range 0 to I, find the expression for the shunt required in terms of I_g and R_G.

(Delhi 2010)

Q.6. Derive the expression for force per unit length between two long straight parallel current carrying conductors. Hence define one ampere.

(Delhi 2009)

Q.7. A circular coil of 200 turns and radius 10 cm is placed in a uniform magnetic field of 0.5 T, Normal to the plane of the coil. If the current in the coil is 3 A, calculate the
(a) Total torque on the coil
(b) Total force on the coil
(c) average Force on each electron in the coil, due to the magnetic field. Assume the area of cross-section of the wire to be 10^{-5} m^2 and the free electron density is $10^{29}/m^3$.

Answer = (a) $\tau = 9.42$ N-m (All India 2008)
(b) Zero (c) $F = e\left(\frac{I}{neA}\right)B = 1.5 \times 10^{-24}$ N

Q.8. Deduce the expression for the torque acting on a rectangular current loop placed in a uniform magnetic field. If m represents the magnetic moment of the current loop and B the magnetic field, show that the torque τ can be expressed as - $\vec{\tau} = \vec{m} \times \vec{B}$

(Delhi 2009)

5 Marks Questions-

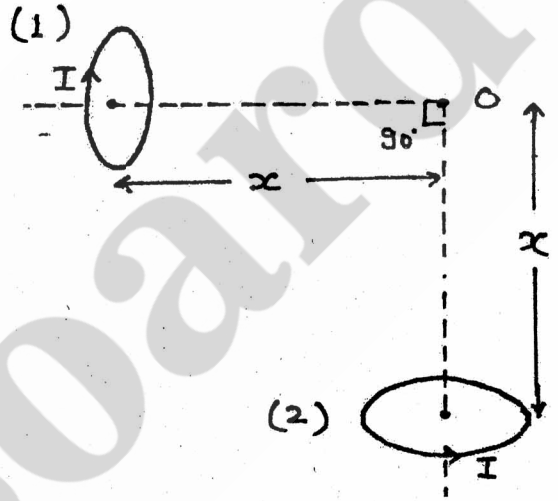
Q. 1(a) What is a toroid? Show that for an ideal toroid of closely wound turns, the magnetic field (i) Inside the toroid is constant and (ii) In the open space inside an exterior to the toroid is zero.

(b) Sketch the magnetic field lines for a finite solenoid. How are these field lines different from the electric field lines from an electric dipole?

(CBSE 2010)

Q.2(a) Two small identical circular coils marked 1, 2 carry equal currents and are placed with their geometric axes perpendicular to each other as shown in fig. Derive expression for resultant magnetic field at O.

Answer = $B = \frac{\sqrt{2}\mu_0 I R^2}{2(x^2+R^2)^{3/2}}$



(b) State Biot-Savart Law and give the expression for it.

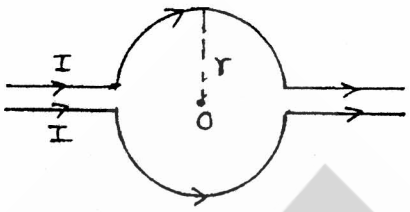
(CBSE 2007)

Q.3. With the help of a neat diagram, explain the principle and working of a moving coil galvanometer. What is the function of-

(i) Uniform radial field (Delhi 2007)

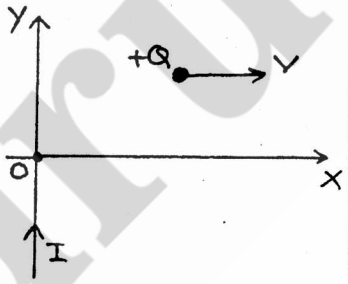
(ii) Soft iron core

Q.1. What is the value of magnetic field at point O due to current flowing in the wires?



Ans. Zero

Q.2. A long straight wire carries a steady current I along the positive Y-axis in a coordinate system. A particle of charge +Q is moving with a velocity $\vec{v}$ along the X-axis. In which direction will the particle experience a force?



Ans. along Y-axis

Q.3. In a certain region of space electric field $\vec{E}$ and magnetic field $\vec{B}$ are perpendicular to each other. An electron enters in the region perpendicular to the direction of both $\vec{B}$ and $\vec{E}$ and moves undeflected. Find the velocity of the electron.

Ans. $V = \frac{E}{B}$

Q.4. A narrow beam of protons and deutrons, each having the same momentum, enters a region of uniform magnetic field directed perpendicular to the direction of their momentum. What would be the ratio of the circular paths described by them?

Ans. $r_p : r_d = 1:1$

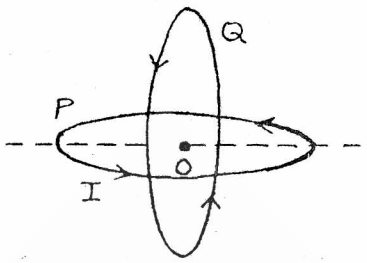
Q.5. Which one have the minimum frequency of revolution, when projected with the same velocity (V) perpendicular to the magnetic field (B) (i) α-particle and (ii) β-Particle.

Ans. - β-particle

Q.6. How will be magnetic field intensity at the centre of a circular coil carrying current change if the current through the coil is doubled and the radius of the coil is halved?

Ans. $B' = 4B$

Q.7. Two identical circular loops P and Q each of radius **R** and carrying current '**I**' are kept in perpendicular planes such that they have a common centre as shown in fig. Find the magnitude and direction of the net magnetic field at the common centre of the coils.



Ans. $B = \frac{\mu_0 I}{\sqrt{2}R}$

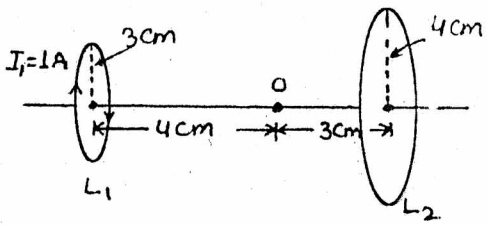
Q.8. A wire AB is carrying a steady current of 12A and is lying on the table. Another wire CD carrying 5A is held directly above AB at a height of 1mm. Find the mass per unit length of the wire CD so that it remains suspended at its position when left free. Give the direction of the current flowing in CD with respect to that in AB.

Ans. 1.2×10^{-3} Kg/m.

Q.9. A uniform conducting wire of length 12a is wound up as a current carrying coil in the shape of (i) an equilateral triangle of side 'a' (ii) a square of side 'a' (iii) a regular hexagon of side 'a'. The coil is connected to a voltage source. Find the magnetic moment of the coils in each case.

Ans. (i) $\sqrt{3}a^2I$ (ii) $3a^2I$ (iii) $3\sqrt{3}a^2I$

Q.10. Two co-axial circular loops L_1 and L_2 of radius 3cm. and 4cm. are placed as shown. What should be the magnitude and direction of the current in the loop L_2 so that the net magnetic field at the point 'O' be zero ?



Ans. $I_2 = \frac{9}{16}$ A

Q.11. A loop of irregular shape carrying current is located in an external magnetic field. If the wire is flexible, why does it change to a circular shape?

Ans- To minimize the potential energy.

PHYSICS

Unit - 5. Magnetism and Matter

Class - 12th

BoardGuru

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5. Magnetism & Matter

Introduction -

A Greek philosopher Thales had observed that a naturally occuring ore of iron attracted small pieces of iron towards it. This ore was found in an island in Greece called Magnesia. Hence that ore was named Magnetite.

Magnetism -

The phenomena of attraction of small bits of iron, steel, cobalt, nickel etc. towards the ore is called magnetism. The structural formula of this natural ore was Fe_3O_4.

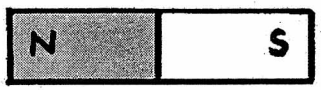
The Chinese discovered that a piece of magnetite when suspended freely, it always points out roughly in the north south direction. Thus a natural magnet has attractive and directive properties.

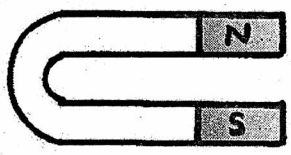
A magnetic compass based on directive property of magnets was used by navigators to find their way in steering the ships. That is why magnetite was also called 'Load stone' in the sense of leading stone.

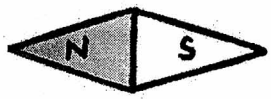
Artificial Magnets -

The natural magnets have often irregular shape and they are weak. It is found that a piece of iron or steel can acquire magnetic properties on rubbing with a magnet. Such magnets made out of iron and steel are called artificial magnets.

A bar magnet, a horse shoe magnet, magnetic needle etc. all are artificial magnets.

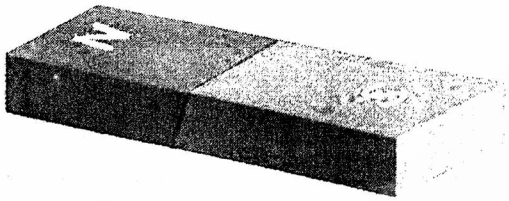




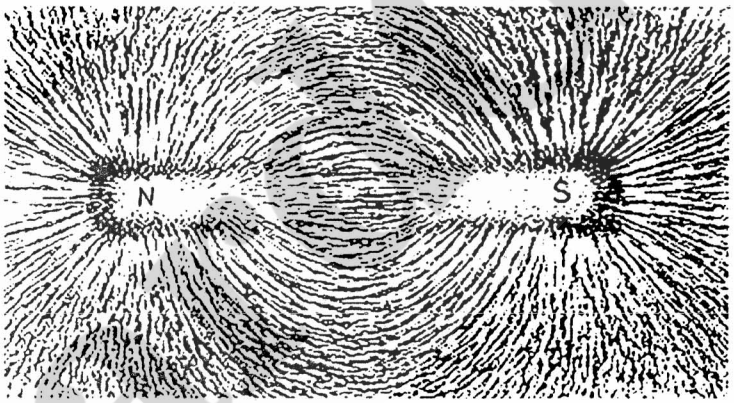


Bar Magnet -

It is the most commonly used form of an artificial magnet which have north pole and south pole of same strength at some distance.



When we hold a sheet of glass over a short bar magnet and sprinkle some iron filings on the sheet then the iron filings rearrange themselves as shown in figure. The pattern suggests that attraction is maximum at the two ends of the bar magnet. These ends are called poles of the magnet.



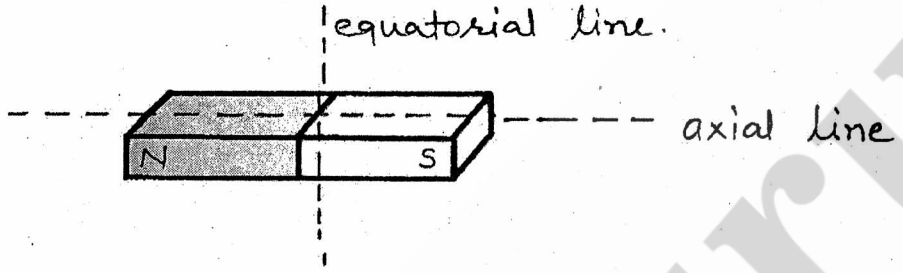
Basic properties of Bar Magnet -

(1.) Every magnet attracts small piece of magnetic substances like iron, cobalt, nickel and steel towards it. The attraction is maximum at the two ends of the magnet. These ends are called poles.

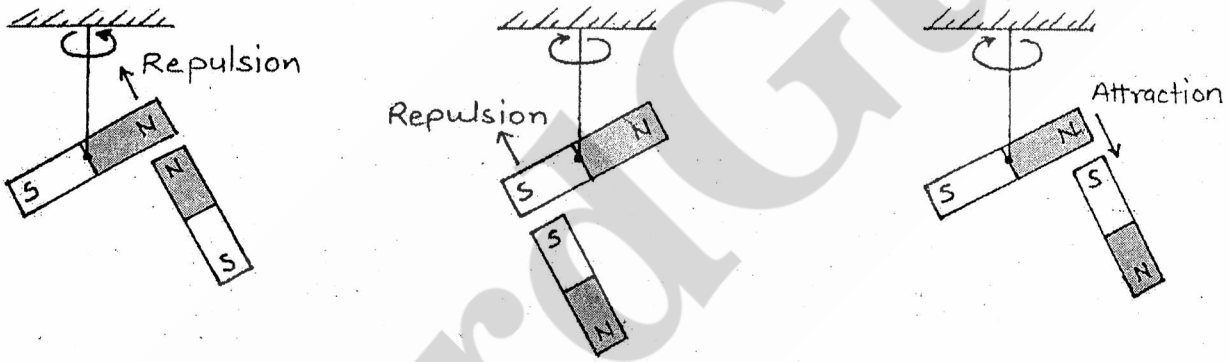
(2) When a magnet is suspended freely with the help of a thread, then it comes to rest along the north south direction.

The pole of the magnet which points towards geographic north is called north pole and the pole which points towards geographic south is called south pole.

(3) The straight line passing through north and south pole of magnet is called axial line. and the line passing through centre of the magnet in a direction perpendicular to the length of the magnet is called equatorial line of the magnet.



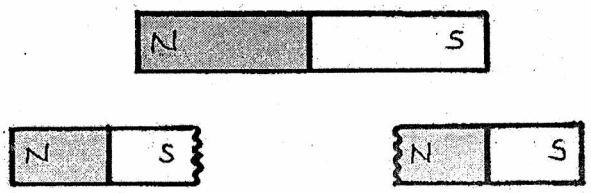
(4) Like poles repel and unlike poles attract each other.



(5) **Inductive property-**

When a piece of a magnetic material like soft iron, cobalt, nickel etc. is placed near a bar magnet, it acquires magnetism. The magnetism so acquired is called induced magnetism and this property of magnetism is called inductive property.

(6) The magnetic poles always exist in pairs. The poles of a magnet can never be separated i.e. magnetic monopole do not exists.



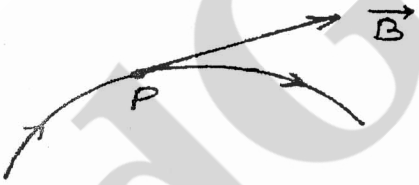
Magnetic Field -

It is the space around a magnet or a current carrying conductor in which its magnetic effect can be felt.

In order to visualise a magnetic field graphically, Faraday introduced the concept of the magnetic field lines.

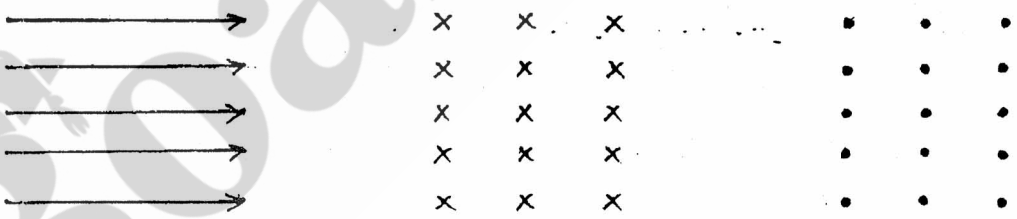
Magnetic Field Lines -

It is an imaginary curve, the tangent to which at any point gives us the direction of magnetic field $\vec{B}$ at that point.



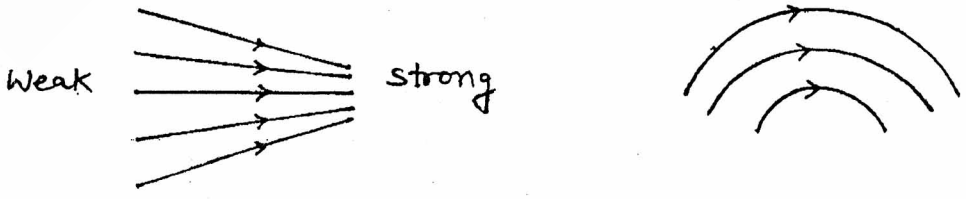
There are basically two types of magnetic field -

(i) Uniform Magnetic Field -



Examples- Earth's magnetic Field, Magnetic field inside a solenoid carrying current etc.

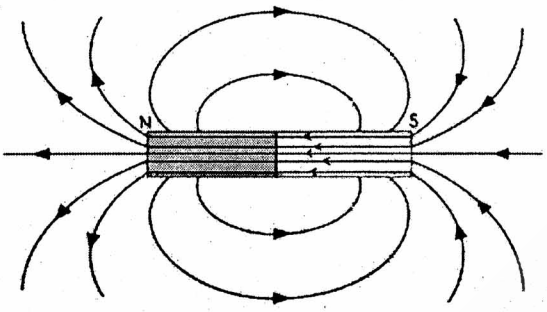
(ii) Non- Uniform Magnetic Field -

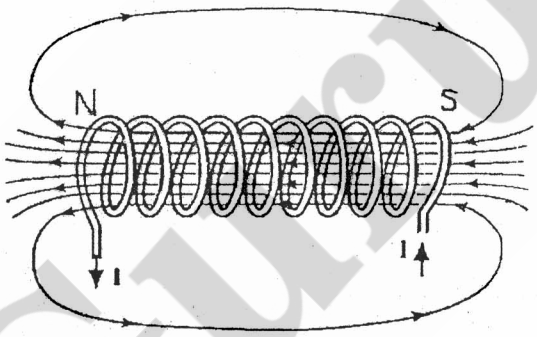


Example - Magnetic Field due to a bar Magnet.

Properties of Magnetic Field Lines -

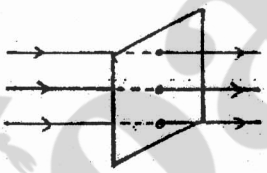
(1.) The magnetic field lines of a magnet (or of a solenoid) form closed continuous loops. The magnetic field lines extending through the body of the magnet / solenoid from south pole to north pole.



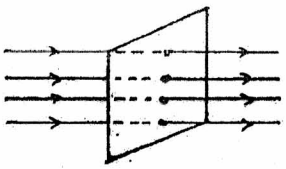


(2) Outside the body of the magnet the direction of magnetic field lines is from north pole to south pole.

(3) The magnitude of magnetic field at any point is represented by the number of magnetic field lines passing normally through unit area around that point.

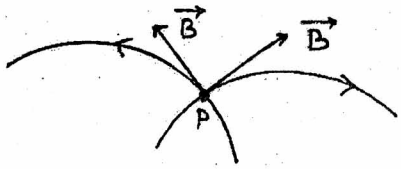


Weak Magnetic Field



Strong Magnetic field

(4) Two magnetic field lines can not intersect each other. Because at the point of intersection we get two directions of magnetic field which is not possible.



(5) Magnetic field lines exists inside every magnetised material.

Note -
The electric field lines continuous curves which are not closed, whereas magnetic field lines are closed continuous curves.

Magnetic Dipole -

A magnetic dipole consist of two unlike poles of equal strength and separated by a small distance.

Examples - Bar magnet, a compass needle etc.

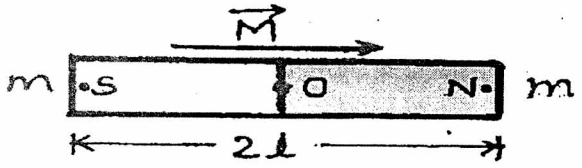
The distance between the two poles of a magnetic dipole is called the magnetic length of the magnet. It is a vector directed from south pole to north pole of the magnetic dipole. It is represented by $2\vec{l}$.

Magnetic Dipole Moment ($\vec{M}$) -

It is the product of strength of either pole (m) and the magnetic length ($2\vec{l}$) of the magnet. It is represented by $\vec{M}$.

$$\vec{M} = m(2\vec{l})$$

* Magnetic dipole moment is a vector quantity and its direction is from south to north pole of the magnet.



* SI Unit of $\vec{M}$ is Jule/Tesla
* SI Unit of pole strength m is A-m.

Magnetic Field Strength at a point due to Bar Magnet

The strength of magnetic field at any point is defined as the force experienced by a hypothetical unit north pole placed at that point.

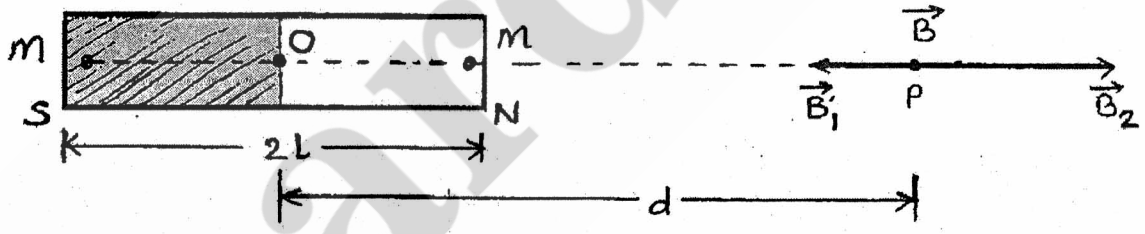
It is a vector quantity. The direction of magnetic field is along which hypothetical unit north pole would tend to move.

(A) When a point lies on axial Line of bar magnet -

Let 2l be the magnetic length of a bar magnet with centre O. M is the magnetic dipole moment of the magnet.

where $\boxed{M = m \times 2l}$

Here OP = d is the distance of the point P on the axial line from the centre of the magnet.



If m is the strength of each pole, then magnetic field strength at point P due to S pole of magnet is -

$$B_1 = \frac{\mu_0}{4\pi} \times \frac{m \times 1}{SP^2} = \frac{\mu_0}{4\pi} \frac{m}{(d+l)^2}$$

Magnetic field strength at P due to N pole of magnet -

$$B_2 = \frac{\mu_0}{4\pi} \times \frac{m \times 1}{NP^2} = \frac{\mu_0}{4\pi} \frac{m}{(d-l)^2}$$

So the Magnetic field strength at P due to the bar magnet is -

$$\boxed{B = B_2 - B_1}$$

$$B = \frac{\mu_0}{4\pi}\frac{m}{(d-l)^2} - \frac{\mu_0}{4\pi}\times\frac{m}{(d+l)^2}$$

$$= \frac{\mu_0}{4\pi} m\left[\frac{1}{(d-l)^2} - \frac{1}{(d+l)^2}\right]$$

$$= \frac{\mu_0}{4\pi} m\left[\frac{(d+l)^2-(d-l)^2}{(d^2-l^2)^2}\right]$$

$$\boxed{B = \frac{\mu_0}{4\pi} m\cdot\frac{4ld}{(d^2-l^2)^2} = \frac{\mu_0}{4\pi}\frac{2Md}{(d^2-l^2)^2}}$$

When the magnet is short ($l^2 << d^2$) so -

$$B = \frac{\mu_0}{4\pi}\frac{2Md}{d^4} \quad \text{or} \quad \boxed{\vec{B} = \frac{\mu_0}{4\pi}\frac{2\vec{M}}{d^3}} \quad (\text{Along NP})$$

* At any point on axial line of a dipole, magnetic field strength $\vec{B}$, is along $\vec{M}$

(B) When point Lies on equatorial line of bar Magnet

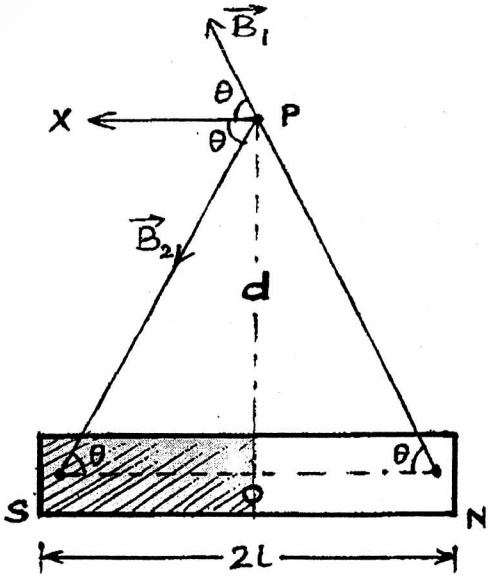
Let here OP = d. and point P lies on equatorial line of the bar magnet.

Magnetic field strength at P due to North pole of magnet -

$$B_1 = \frac{\mu_0}{4\pi}\times\frac{m\times 1}{NP^2} = \frac{\mu_0}{4\pi}\frac{m}{(d^2+l^2)}$$

Magnetic field strength at P due to south pole of magnet -

$$B_2 = \frac{\mu_0}{4\pi}\times\frac{m\times 1}{SP^2} = \frac{\mu_0}{4\pi}\frac{m}{(d^2+l^2)}$$



As $B_1 = B_2$ in magnitude, so their vertical components $B_1\sin\theta$ and $B_2\sin\theta$ Cance each other.

However, Components along PX (which is parallel to NS) add up.

Therefore Magnetic field strength at P due to the bar magnet is -

$$B = B_1 \cos\theta + B_2 \cos\theta = 2B_1 \cos\theta \quad (\text{along } PX)$$

so $$B = \frac{2\mu_0}{4\pi} \frac{m}{(d^2+l^2)} \times \frac{l}{\sqrt{(d^2+l^2)}} \qquad \left[\cos\theta = \frac{l}{\sqrt{d^2+l^2}}\right.$$

$$\boxed{B = \frac{\mu_0}{4\pi} \frac{M}{(d^2+l^2)^{3/2}}}$$

If the magnet is short ($l^2 << d^2$) then

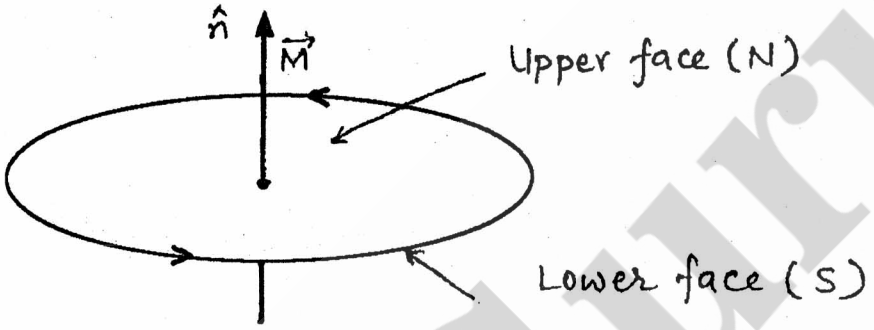
$$B = \frac{\mu_0}{4\pi} \frac{M}{d^3} \qquad \text{or} \qquad \boxed{\vec{B} = -\frac{\mu_0}{4\pi} \frac{\vec{M}}{d^3}}$$

★ At any point on equatorial line, magnetic field strength is in the direction opposite to $\vec{M}$.

Note-
Magnetic field due to a short magnet at any point on axial line of magnet is twice the magnetic field at a point at the same distance on the equatorial line of the magnet.

Current Loop as a Magnetic Dipole -

Consider a plane loop of wire carrying current in anticlockwise direction. Therefore, it has a north polarity. Looking at the lower face of the loop current is clockwise, therefore it has a south polarity



Thus the current carrying loop behaves as a magnetic dipole.

The magnetic dipole moment of the current loop (M) is directly proportional to

(i) strength of current through the loop (I)
(ii) Area (A) enclosed by the loop.

$$M \propto I \quad \& \quad M \propto A$$

So $M = KIA$ (Here $K = 1$ .....)

$$M = IA$$

for N turns $\boxed{\vec{M} = NI\vec{A}}$

SI Unit - ampere meter^2 (Am^2)

1 ampere meter^2 - It is the magnetic moment of one turn loop of area one square meter carrying a current of one ampere.

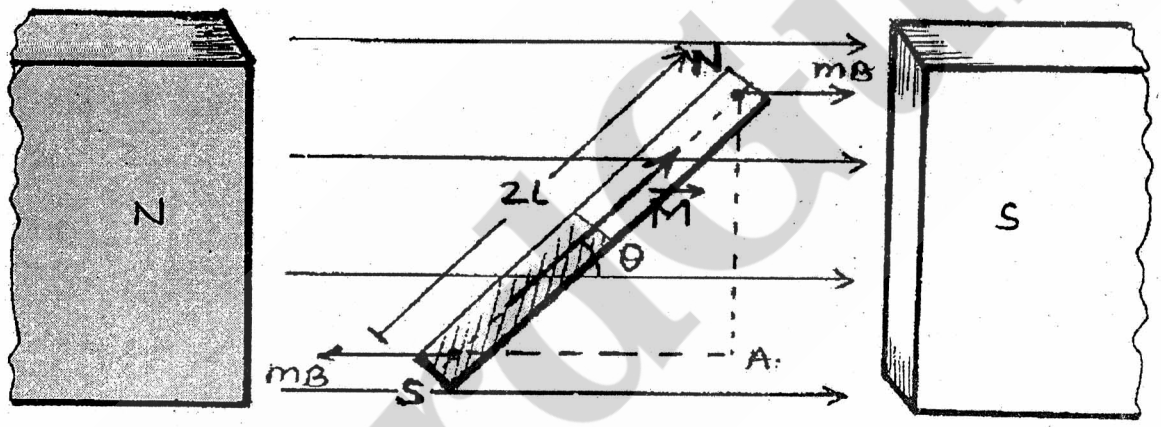
Torque on a Bar Magnet in a Magnetic field -

Let NS is a bar magnet of length 2l and strength of each pole m is placed in a uniform magnetic field $\vec{B}$. The magnet is held at angle θ with the direction of $\vec{B}$.

Force on north pole = mB (along $\vec{B}$)

Force on South pole = mB (opposite to $\vec{B}$)

These forces are equal and tends to rotate the magnet clockwise.



Torque acting on the bar magnet is -

Torque (τ) = Force x perpendicular distance

= mB x NA

In Δ NAS $\sin\theta = \frac{NA}{NS} = \frac{NA}{2l}$

$\therefore$ $NA = 2l\sin\theta$

So $\tau = mB \times 2l \sin\theta$

$$\boxed{\tau = MB\sin\theta} \qquad (M = m \times 2l)$$

$$\boxed{\vec{\tau} = \vec{M} \times \vec{B}}$$

The direction of $\vec{\tau}$ is perpendicular to the plane containing $\vec{M}$ and $\vec{B}$ and is given by the right hand rule.

When $B = 1$ and $\theta = 90^\circ$ then ...

$$\tau = M \times 1 \times \sin 90^\circ \Rightarrow \boxed{\tau = M}$$

Magnetic dipole moment is the torque acting on a dipole held perpendicular to a uniform magnetic field of unit strength.

SI Unit of $\vec{M}$ = Joule per Tesla (JT^{-1})

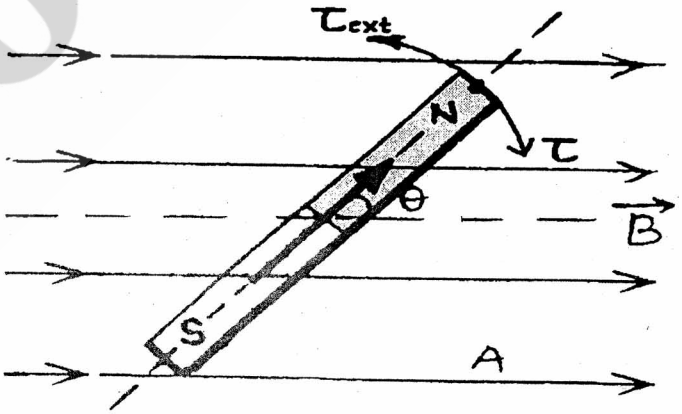
Potential Energy of a Magnetic Dipole in a Magnetic Field

The energy possessed by the dipole due to its particular position in the field is the potential energy of a magnetic dipole.

When a magnetic dipole of moment $\vec{M}$ is held at an angle θ with the direction of a uniform magnetic field $\vec{B}$, so the magnitude of the torque acting on the dipole is -

$$\boxed{\tau = MB\sin\theta}$$

This torque tend to align the dipole in the direction of the field. Work has to be done in rotating the dipole against the action of the torque. This work done is stored in the magnetic dipole as potential energy of dipole.



Now, small amount of work done in rotating the dipole through a small angle $d\theta$ against the rotating torque is -

$$dW = \tau d\theta = MB\sin\theta\, d\theta$$

Total work done in rotating the dipole from θ_1 to θ_2 is

$$W = \int_{\theta_1}^{\theta_2} MB\sin\theta\, d\theta = MB\left[-\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$W = -MB\left[\cos\theta_2 - \cos\theta_1\right]$$

So the potential energy of the dipole is -

$$\boxed{\Delta U = W = -MB\left(\cos\theta_2 - \cos\theta_1\right)}$$

When $\theta_1 = 90^\circ$ and $\theta_2 = \theta$ then -

$$\boxed{\begin{aligned} U &= W = -MB\cos\theta \\ U &= -\vec{M}\cdot\vec{B} \end{aligned}}$$

* Special Cases -

(i) When $\theta = 90^\circ$ then

$$U = -MB\cos 90^\circ = 0$$

When the dipole is perpendicular to magnetic field its potential energy is zero.

(ii) When $\theta = 0^\circ$ then

$$U = -MB\cos 0^\circ = -MB$$

It is minimum. This is the position of stable equilibrium.

So when the magnetic dipole is aligned along the magnetic field, it is in stable equilibrium having minimum P.E.

(iii) When $\theta = 180^\circ$ then

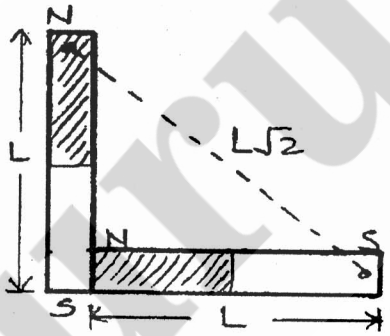
$$U = -MB\cos 180^\circ = MB$$

It is maximum. This is the position of unstable equilibrium.

Q.1 Two identical thin bar magnets, each of length L and pole strength m are placed at right angles to each other, with the N-pole of one touching the S-pole of the other. Find the magnetic moment of the system.

Sol:

Here the system behaves as a magnet whose pole stength is m and length is $L\sqrt{2}$



So the magnetic moment of the system is.

$$M = \text{Pole strength} \times L\sqrt{2}$$

$$M = \sqrt{2}\, mL$$

Q. What is the magnitude of the equatorial and axial fields due to a bar magnet of length 5 cm at a distance of 50 cm. from its mid point? . The magnetic moment of the bar magnet is $0.4\ Am^2$.

Sol. on axial line the magnetic field is

$$B_1 = \frac{\mu_0}{4\pi}\frac{2Md}{(d^2-l^2)^2} = \frac{\mu_0}{4\pi}\frac{2M}{d^3} \qquad (l \ll d)$$

$$B_1 = \frac{10^{-7} \times 2 \times 0.4}{(0.5)^3} = 6.4 \times 10^{-7}\ T$$

On equatorial line the magnetic field is.

$$B_2 = \frac{\mu_0}{4\pi} \times \frac{M}{(d^2+l^2)^{3/2}} = \frac{\mu_0}{4\pi}\frac{M}{d^3} \qquad (l \ll d)$$

$$B_2 = 10^{-7} \times \frac{0.4}{(0.5)^3} = 3.2 \times 10^{-7}\ T$$

Q. A bar magnet of magnetic moment $1.5\ JT^{-1}$ lies aligned with the direction of a uniform magnetic field of 0.22 T.

(a) What is the amount of work done to turn the magnet so as to align its magnetic moment
(i) Normal to the field direction
(ii) Opposite to the field direction ?

(b) What is the torque on the magnet in cases (i) and (ii) ?

Sol. Here $M = 1.5\ JT^{-1}$, $B = 0.22\ T$

(a) (i) Here $\theta_1 = 0^\circ$ (along the fied)
$\theta_2 = 90^\circ$ ($\perp$ to the field)

As $W = -MB(\cos\theta_2 - \cos\theta_1)$

$\therefore W = -1.5 \times 0.22(\cos 90^\circ - \cos 0^\circ)$
$= -0.33(0-1) = 0.33\ J$

(ii) Here $\theta_1 = 0^\circ$, $\theta_2 = 180^\circ$

$W = -1.5 \times 0.22(\cos 180^\circ - \cos 0^\circ)$
$= -0.33(-1-1) = 0.66\ J$

(b) Torque $\tau = MB\sin\theta$

(i) Here $\theta = 90^\circ$

So $\tau = 1.5 \times 0.22 \sin 90^\circ = 0.33\ Nm$

(ii) Here $\theta = 180^\circ$

So $\tau = 1.5 \times 0.22 \sin 180^\circ = 0$

Q. A circular coil of 16 turns and radius 10 Cm. carrying a current of 0.75 A rests with its plane normal to an external field of magnitude 5×10^{-2} T. The coil is free to turn about an axis in its plane perpendicular to the field direction. When the coil is turned slightly and released, it oscillates about its stable equilibrium with a frequency of $2\ s^{-1}$. What is the moment of inertia of the coil about its axis of rotation?

Sol: Here $N = 16$, $r = 10\ cm = 0.1\ m$.

$I = 0.75\ A$, $B = 5 \times 10^{-2}\ T$, $\nu = 2\ s^{-1}$

So that $M = NIA = NI\pi r^2$

$M = 16 \times 0.75 \times 3.14 \times (0.1)^2$

$M = 0.377\ J\,T^{-1}$

As $\nu = \frac{1}{2\pi}\sqrt{\frac{MB}{I}}$ where I is the moment of inertia of the coil.

$\therefore \quad I = \frac{MB}{4\pi^2\nu^2} = \frac{0.377 \times 5 \times 10^{-2}}{4 \times (3.14)^2 \times 2^2}$

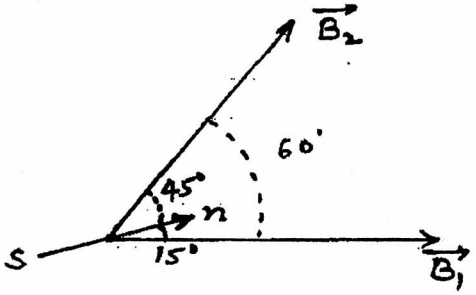
$I = 1.2 \times 10^{-4}\ Kg\,m^2$.

Q. A magnetic dipole is under the influence of two magnetic fields. The angle between the field directions is 60° and one of the fields has a magnitude of 1.2×10^{-2} T. If the dipole comes to stable equilibrium at an angle of 15° with this field, what is the magnitude of the other field?

Sol: Here $\theta = 60°$

$B_1 = 1.2 \times 10^{-2}\ T$

$\theta_1 = 15°$, $\theta_2 = 60° - 15° = 45°$



In equilibrium, torques due to two fields must balance.

$$\tau_1 = \tau_2$$

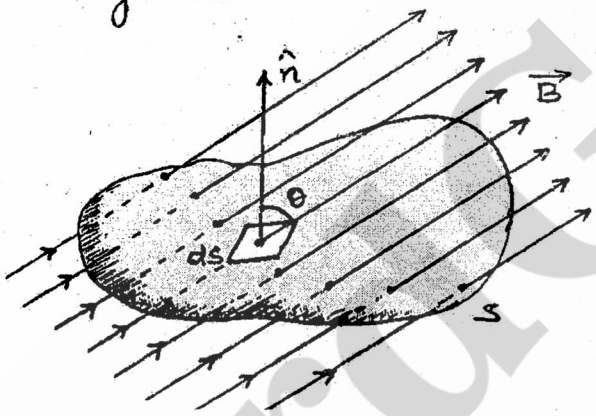
$$MB_1 \sin\theta_1 = MB_2 \sin\theta_2$$

$$B_2 = \frac{B_1 \sin\theta_1}{\sin\theta_2} = \frac{1.2 \times 10^{-2} \times \sin 15^\circ}{\sin 45^\circ}$$

$$B_2 = \frac{1.2 \times 10^{-2} \times 0.2588}{0.7071} = 4.4 \times 10^{-3}\ T$$

Gauss's Law for Magnetism -

According to Gauss's Law for magnetism, the net magnetic flux (Φ_B) through any closed surface is always zero.



Consider a small vector area element $d\vec{s}$ of this surface. Magnetic flux through this area element is defined as

$$d\Phi_B = \vec{B} \cdot d\vec{s} \quad \ldots\ldots$$

Summing over all small area elements of the surface we obtain according to Gauss's Law for magnetism.

$$\Phi_B = \oint_S \vec{B} \cdot d\vec{s} = 0$$

* The law implies that the number of magnetic field lines leaving any closed surface is always equal to the number of magnetic field lines entering it.
* The fact that $\Phi_B = 0$ indicates that the simplest magnetic element is a dipole or current loop. The isolated magnetic poles (called magnetic monopoles) are not known to exist.

Magnetic Field of Earth -

Sir William Gilbert was the first to suggest in the year 1600, that earth itself is a huge magnet.

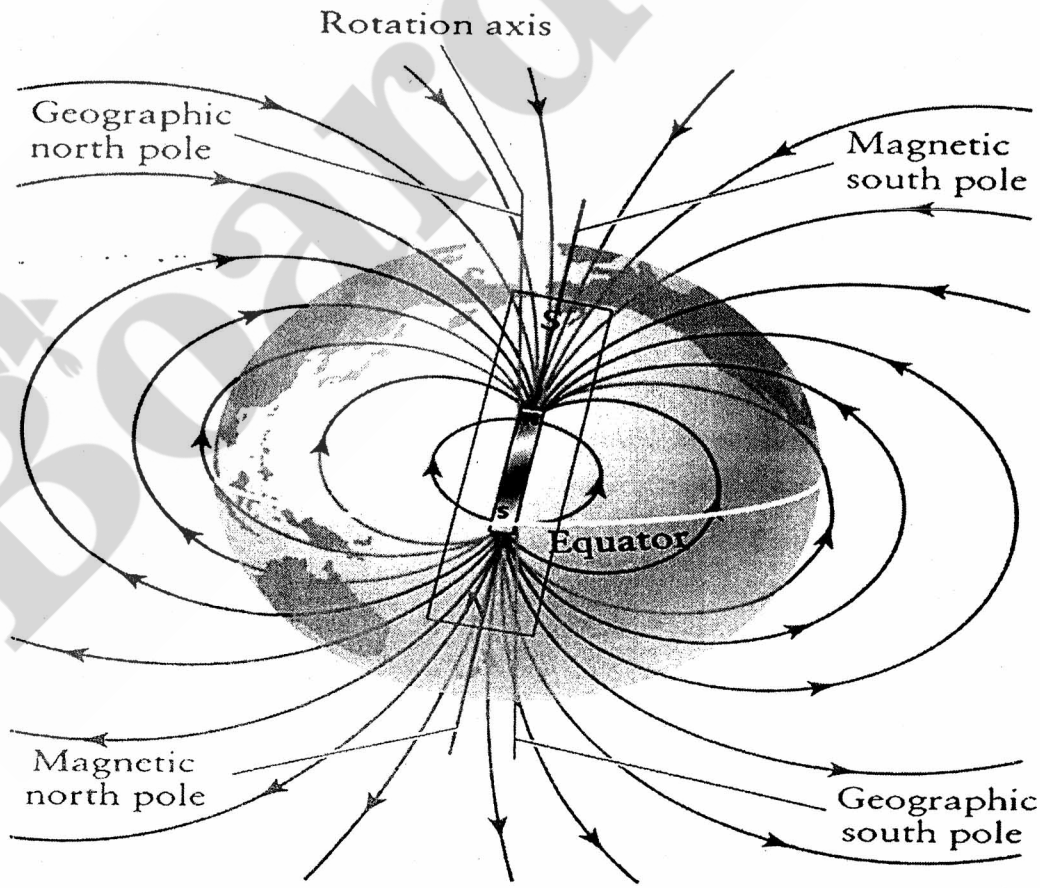
A magnet suspended from a thread and free to rotate in the horizontal plane comes to rest along the north-south direction.

This is like a huge bar magnet is placed approximately along the axis of rotation of earth and deep in its interior.

The north pole of this imaginary magnet must be towards geographic south so as to attract south pole of the suspended magnet and vice-versa.

The imaginary magnetic axis of earth makes an angle of 20° with the geographic, rotational axis.

Magnetic poles are approximately 2000 Km. away from the geographic poles.



Cause of Earth's Magnetism

The exact causes of earth's magnetism is not yet known fully. However, some important postulates in this respect are as follows -

(i) Earth's magnetism may be due to rotation of earth about its axis. This is because every substance is made up of charged particles (protons & electrons). Therefore a substance rotating about an axis is equivalent to circulating currents, which are responsible for magnetism.

(ii) In the outer layers of earth's atmosphere, gases are in the ionised state, primarily on account of cosmic rays. As earth rotates, strong electric currents are set up due to movement of (charged) ions. These currents might be magnetising the earth.

(iii) The earth's core is extremely hot and molten. circulating ions of iron and nickel in the highly conducting liquid region of the earth's core might be forming current loops and producing earth's magnetic field.

Magnetic Elements of Earth -

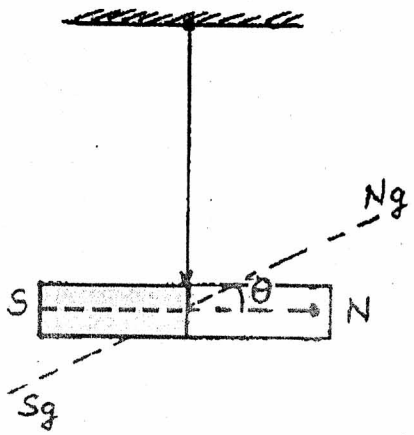
Magnetic elements of earth at a place are the quantities which describe completely in magnitude as well as direction the magnetic field of earth at that place.

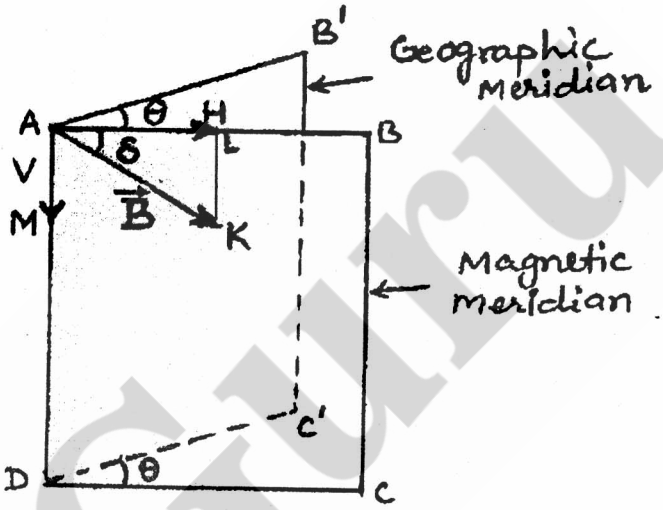
Magnetic axis :- The line joining the north and south poles of a freely suspended magnet is called magnetic axis.

Geographic axis :- The axis about which earth rotates is the geographic axis.

(i) Magnetic declination (θ)

The small angle between magnetic axis and geographic axis at a place is defined as the magnetic declination at the place.





Magnetic Meridian:-

A verticle plane passing through N-S line of a freely suspended magnet is called magnetic meridian.

Geographic Meridian:-

The verticle plane passing through the geographic north-south direction is called geographic meridian.

Here in figure CD represents the magnetic axis and C'D represents the geographic axis.

The verticle plane CDAB is the magnetic meridian and verticle plane C'DAB' is the **geographic meridian.**

BoardGuru

(ii) Magnetic Dip or Magnetic Inclination (δ)

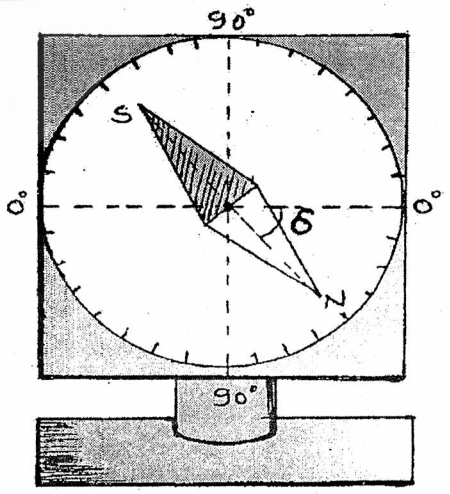
It is defined as the angle, which the direction of total strength of earth's magnetic field makes with a horizontal line in magnetic meridian.

At different places on the surface of earth, δ is different. The angle of dip at two poles is 90°. On the magnetic equator, the angle of dip is zero. At other places, the value of δ lies between 0° and 90°.

Note -

* The magnetic declination θ is determined by a compass needle free to rotate in the horizontal plane about the verticle axis.
* The magnetic dip angle δ is measured by allowing the compass needle to rotate freely in the verticle plane of magnetic meridian about the horizontal axis.

The value of magnetic dip at a place can be measured using an instrument called 'dip circle'.



(iii) Horizontal Component (H) -

It is the Component of the total intensity of earth's magnetic field in the horizontal direction in magnetic meridian.

The resultant intensity B along AK is resolved into two rectangular components -

Horizontal Component along AB $\quad H = B\cos\delta$

Verticle Component along AD $\quad V = B\sin\delta$

So $\boxed{B = \sqrt{H^2 + V^2}}$

and $\boxed{\tan\delta = \frac{V}{H}}$

The value of horizontal Component (H) is different at different places.

(i) At the Magnetic poles ($\delta = 90^\circ$) $\boxed{H = \text{zero}}$

(ii) At the Magnetic equator ($\delta = 0^\circ$) $\boxed{H = R}$

* The value of H at a place on the surface of earth is of the order of 3.2×10^{-5} T.
* The direction of the Horizontal Component H of the earth's magnetic field is from geographic south to geographic north above the surface of earth.

Q. The vertical component of earth's magnetic field at a place is $0.16\sqrt{3} \times 10^{-4}$ T. Calculate the value of horizontal component of earth's magnetic field, if angle of dip at that place is 30°.

Sol. Here $V = 0.16\sqrt{3} \times 10^{-4}$ T , $\delta = 30^\circ$

From $\tan\delta = \frac{V}{H}$

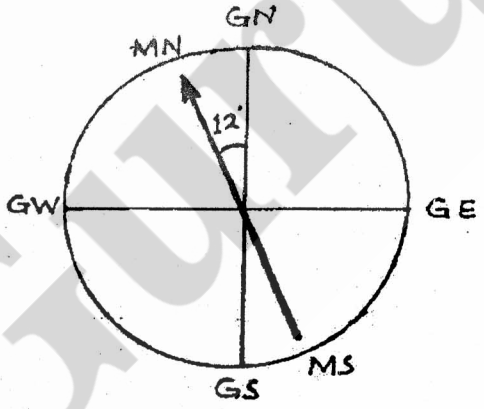
$$H = \frac{V}{\tan\delta} = \frac{0.16\sqrt{3} \times 10^{-4}}{\tan 30^\circ}$$

$$H = \frac{0.16\sqrt{3} \times 10^{-4}}{1/\sqrt{3}} = 0.48 \times 10^{-4}\ T$$

Q. At a certain location in Africa, compass points 12° west of geographic north. The north tip of magnetic needle of a dip circle placed in the plane of magnetic meridian points 60° above the horizontal. The horizontal component of earth's field is measured to be 0.16 G. Specify the direction and magnitude of the earth's field at the location.

Sol. declination $\theta = 12°$ West,
dip $\delta = 60°$, $H = 0.16\ G$

$1\ G = 10^{-4}\ T$



As $H = R\cos\delta$

$$\therefore \quad R = \frac{H}{\cos\delta} = \frac{0.16 \times 10^{-4}}{\cos 60°}$$

$$R = \frac{0.16 \times 10^{-4}}{1/2}$$

$$R = 0.32 \times 10^{-4}\ T$$

The earth's field lies in a vertical plane 12° west of geographic meridian at an angle of 60° above the horizontal.

Q. CBSE 2012 The horizontal component of earth's magnetic field at a place is B and angle of dip is 60°. What is the value of vertical component of earth's magnetic field at equator?

Sol. Horizontal Component

$$H = R\cos 60° = B \quad \text{(given)}$$

$$R = \frac{B}{\cos 60°} = \frac{B}{1/2} = 2B$$

Vertical Component

$$V = R\sin 60°$$

$$= 2B \times \frac{\sqrt{3}}{2} = \sqrt{3}\,B$$

Q. CBSE 2010 The horizontal Component of the earth's magnetic field at a place is $\sqrt{3}$ times its vertical component there. Find the value of the angle of dip at that place. What is the ratio of the horizontal component to the total magnetic field of the earth at that place ?

Sol. Here $H = \sqrt{3}V$, where H and V are the horizontal and vertical components of earth's magnetic field.

If angle of dip at that place is δ, then

$$\tan\delta = \frac{V}{H} = \frac{V}{\sqrt{3}V} = \frac{1}{\sqrt{3}}$$

so that $\delta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

Horizontal component of earth's magnetic field

$H = R\cos\delta$ ($R \rightarrow$ earth's magnetic field)

$$\frac{H}{R} = \cos\delta = \cos\frac{\pi}{6}$$

so $$\frac{H}{R} = \frac{\sqrt{3}}{2}$$

Some Important Terms used in Magnetism

(i) Magnetic Permeability (μ)

It is the ability of a material to permit the passes of magnetic lines of force through it.

The degree or extent to which magnetic field can penetrate or permeate a material is called relative magnetic permeability of the material. It is represented by μ_r.

Relative magnetic permeability of a material

The relative magnetic permeability of a material may also be defined as the ratio of magnetic permeability of the material (μ) and magnetic permeability of free space (μ_0).

$$\boxed{\mu_r = \frac{\mu}{\mu_0}} \quad \text{or} \quad \boxed{\mu_r = \frac{B}{B_0}}$$

μ_r has no dimensions. Its value is 1 for vacuum.

$$\mu_0 = 4\pi \times 10^{-7} \text{ Wb A}^{-1} \text{m}^{-1}$$

(ii) Magnetising Force or Magnetic Intensity

The degree to which a magnetic field can magnetise a material is represented in terms of magnetising force or magnetic intensity (H).

Lets consider a toroidal solenoid with n turns per unit length carrying a current I wound round a ring of a magnetic material.

The magnetic field produced in the material is -

$$B = \mu n I$$

The product nI is called the magnetising force or magnetic intensity H.

$$\boxed{H = nI} \qquad \text{so} \qquad \boxed{B = \mu H}$$

If inside the toroidal solenoid, there is free space, then magnetic field.

$$\boxed{B_0 = \mu_0 H}$$

The SI unit of H is Ampere turn/m (Am^{-1})

(iii) Magnetisation Intensity (I)

The Magnetisation intensity of a magnetic material is defined as the magnetic moment per unit volume of the material.

$$\boxed{I = \frac{\text{Magnetic moment}}{\text{Volume}} = \frac{M}{V}}$$

$$\boxed{I = \frac{m \times 2l}{a \times 2L} = \frac{m}{a}}$$

Hence the Magnetisation intensity of a material is also defined as the pole strength per unit area of cross-section of the material.

$$I = \frac{\text{Magnetic moment}}{\text{Volume}} = \frac{\text{Amp} \cdot \text{metre}^2}{\text{metre}^3} = A m^{-1}$$

So the SI unit of I is $A m^{-1}$

(iv) Magnetic Susceptibility (χ_m)

It is a property which determine how easily a material can be magnetised.

Susceptibility of a magnetic material is defined as the ratio of the intensity of magnetisation (I) induced in the material to the magnetising force (H) applied on it.

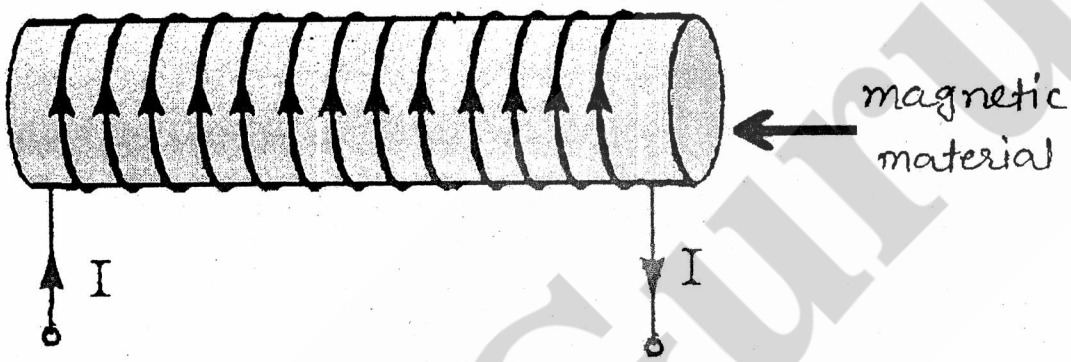
Magnetic susceptibility is represented by χ_m.

$$\boxed{\chi_m = \frac{I}{H}}$$

As unit of I and H are same ($A m^{-1}$), therefore (χ_m) has no units and no dimensions.

Relation between Magnetic Permeability and Magnetic Susceptibility -

When a magnetic material is placed in a magnetic field of Magnetic Intensity H, the material get magnetised.



The total magnetic field B in the material is the sum of the magnetic field B_0 in vacuum produced by the magnetic intensity and magnetic field B_m, due to magnetisation of the material.

$$\boxed{B = B_0 + B_m}$$

But $B_0 = \mu_0 H$ and $B_m = \mu_0 I$

So $B = \mu_0 H + \mu_0 I$

$\therefore \quad B = \mu_0 (H + I)$

Also as $\chi_m = I/H$ $\quad \therefore \quad I = \chi_m H$

So $B = \mu_0 (H + \chi_m H)$

$B = \mu_0 H (1 + \chi_m)$

$\therefore \quad \mu H = \mu_0 H (1 + \chi_m)$ as $B = \mu H$

or $\frac{\mu}{\mu_0} = 1 + \chi_m$

$$\boxed{\mu_r = 1 + \chi_m}$$

This is the relation between the relative magnetic permeability and magnetic susceptibility of the material.

Some Important terms used in Magnetism

	Physical Quantity	Symbol	Nature	Units	Remarks
1.	Magnetic field, Magnetic flux density, Magnetic Induction,	$\vec{B}$	Vector	T	$1T = 10^4 G$ G → Gauss
2.	Magnetic Intensity Magnetising force	$\vec{H}$	Vector	Am^{-1}	
3.	Magnetisation Intensity	$\vec{I}$	Vector	Am^{-1}	$\vec{I} = \frac{\vec{M}}{V}$
4.	Magnetic Moment	$\vec{M}$	Vector	Am^2	
5.	Magnetic Permeability	μ	Scalar	TmA^{-1}	
6.	Magnetic Permeability of free space	μ_0	Scalar	TmA^{-1}	$\frac{\mu_0}{4\pi} = 10^{-7}$
7.	Relative Magnetic Permeability	μ_r	Scalar	No	$\mu_r = \frac{\mu}{\mu_0}$
8.	Magnetic Susceptibility	χ_m	Scalar	No	$\mu_r = 1 + \chi_m$

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Classification of Magnetic Materials

Faraday classified the substances on the basis of their magnetic properties, into the following categories.

(i) Diamagnetic Substances
(ii) Paramagnetic Substances
(iii) Ferromagnetic Substances

(i) Diamagnetic Substances -

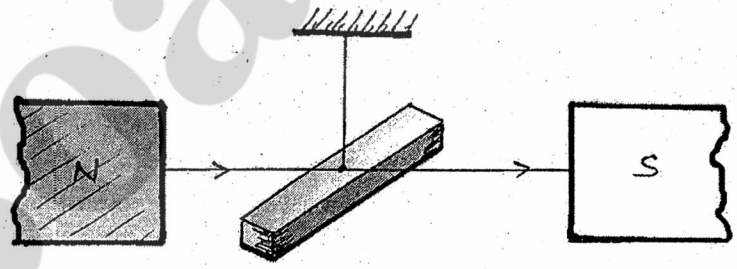
The diamagnetic substances are those in which the individual atoms / molecules / ions do not possess any net magnetic moment on their own.

When such substances are placed in an external magnetic field they get weekly magnetised in a direction opposite to the magnetising field.

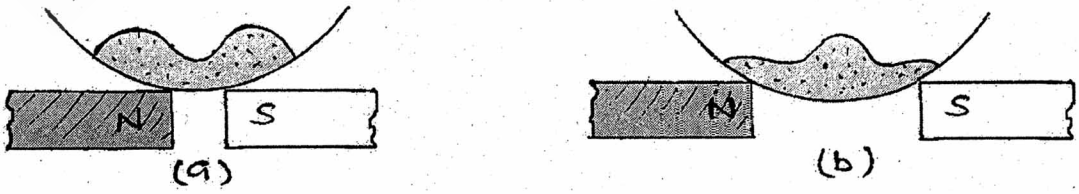
Examples- Antimony, Bismuth, Copper, Lead, Quartz, Water, Mercury, Hydrogen, inert gases etc.

Properties -

(i) When diamagnetic substances are suspended in a uniform magnetic field, they set their longest axis at right angle to the direction of the field.

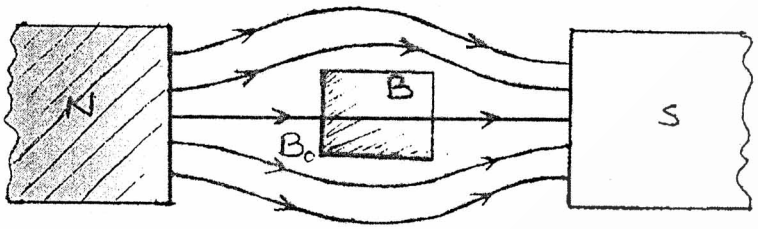


(ii) When diamagnetic substances are placed in a non-uniform magnetic field, these substances have a tendency to move from stronger parts of the field to the weaker parts.



(iii) When a piece of diamagnetic material is placed in a magnetising field, the magnetic field lines prefer not to pass through the material.

It implies that the magnetic field lines are repelled and the field inside the material is reduced slightly.



$B < B_0$

(iv) Relative magnetic permeability of diamagnetic substances is always less than unity.

$$\mu_r = \frac{B}{B_0} \qquad \text{as} \quad B < B_0$$

So $\boxed{\mu_r < 1}$

(v) As $\mu_r = 1 + \chi_m$

and $\mu_r < 1$ so **χ_m is negative**

Hence susceptibility of diamagnetic material has a small negative value.

(vi) Susceptibility of diamagnetics does not change with temperature.

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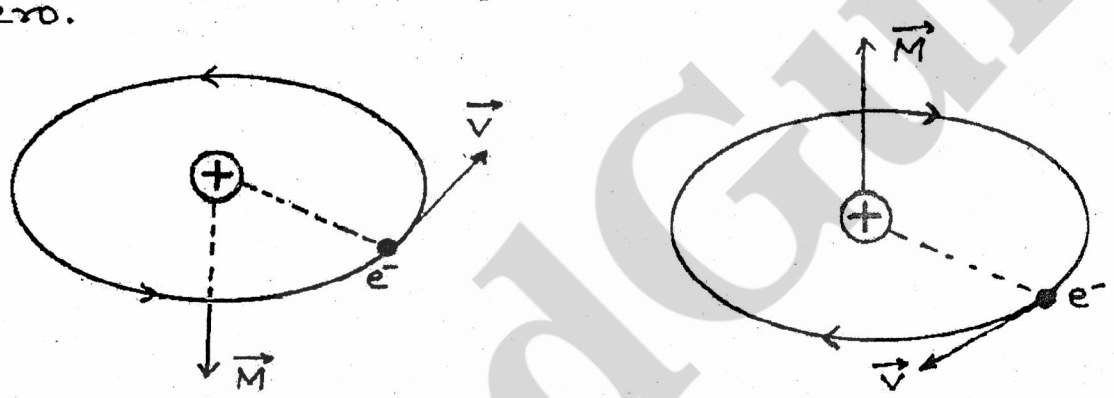
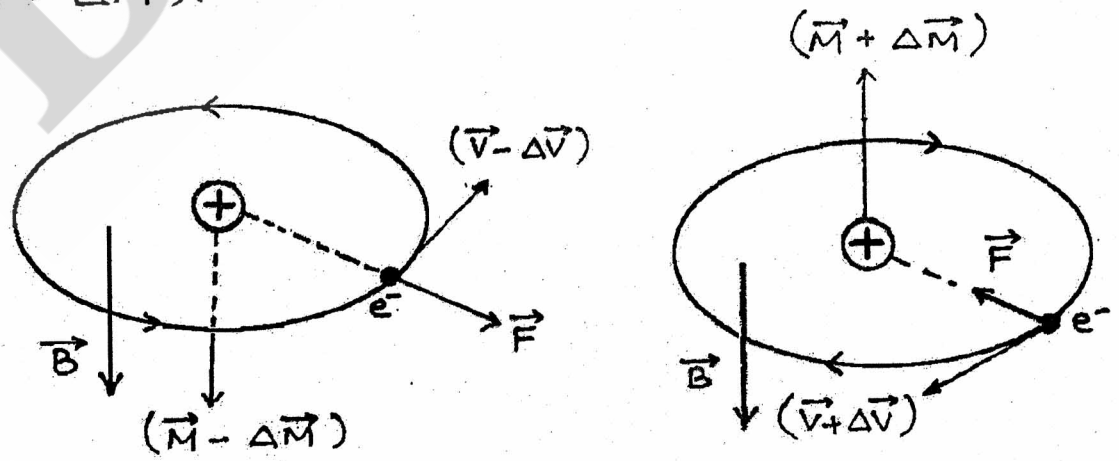
Explanation of Diamagnetism -

In a diamagnetic substance the motion of all the electrons in an atom is assumed as the motion of two electrons revolving with same angular velocity in a circular orbit of same radius, but in opposite sense.

In the absence of any external magnetic field the magnetic moment of two electrons being equal and opposite and cancel each other for every atom, so that net magnetic moment of each atom is zero.



When a uniform magnetic field $\vec{B}$ be applied then each electron experiences a magnetic Lorentz force $F = evB$. According to Fleming's Left hand rule, the Lorentz force F acts radially outwards on the electron revolving anti-clockwise. It tends to decrease the centripetal force and hence decrease the velocity of electron to $(\vec{v} - \Delta\vec{v})$. As a result, the magnetic moment of electron decreases to $(\vec{M} - \Delta\vec{M})$.



On the electron revolving clockwise, the Lorentz force acts radially inwards, tending to increase the centripetal force and hence the velocity of electron to $(\vec{v} + \Delta\vec{v})$. Therefore, the magnetic moment of electron increases to $(\vec{M} + \Delta\vec{M})$.

The vector addition of these two magnetic moments gives rise to a net dipole magnetic moment $2\Delta\vec{M}$, directed upward i.e. opposite to external field $\vec{B}$.

The net magnetic moment $2\Delta\vec{M}$ so developed is called induced magnetic moment. The magnetic moment induced in different atoms add vertically to give a net magnetization of the material in a direction opposite to $\vec{B}$. This accounts for the diamagnetic behaviour of material.

* The appearence of induced magnetic moment in atoms is not affected by the thermal motion of atoms, therefore, magnetic susceptibility of such materials does not depends on temperature of the materials.

(ii) Paramagnetic Substances -

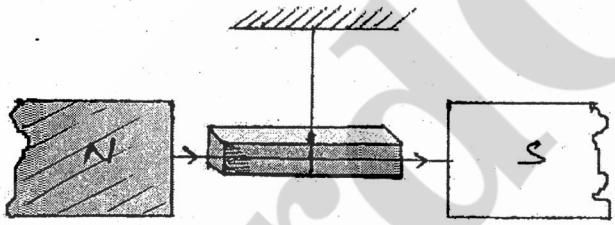
Paramagnetic substances are those in which each individual atom/molecule/ion has a net non-zero magnetic moment of its own.

When such substances are placed in an external magnetic field they get weakly magnetised in the direction of the magnetising field.

Examples - Aluminium, chromium, manganese, Lithium, Platinum, Tungsten, Sodium, Oxygen etc.

Properties -

(i) When a paramagnetic substance is suspended in a uniform magnetic field, they rotate so as to bring their longest axis along the direction of the field.

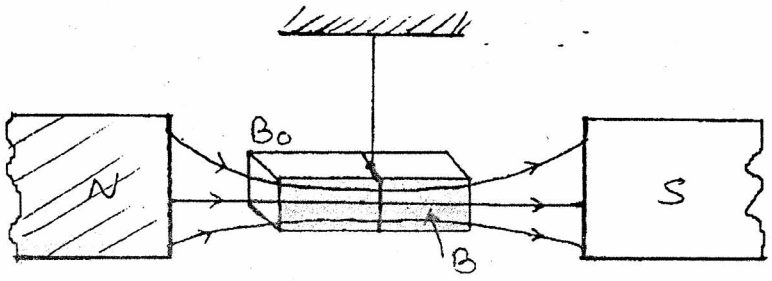


(ii) When the paramagnetic materials are placed in a non-uniform magnetic field they tend to move from weaker parts of the field to the stronger parts.

It implies that paramagnetic substances get weakly attracted to the magnet.

(iii) When a piece of a paramagnetic substance is placed in a magnetising field, the magnetic field lines prefer to pass through the piece rather than through air.

Thus magnetic field B inside the piece is more than the magnetic field B_0 outside the piece. $B > B_0$



$B > B_0$

(iv) Relative magnetic permeability of paramagnetic substance is always more than unity.

As $\mu_r = \frac{B}{B_0}$ and $B > B_0$

So $\boxed{\mu_r > 1}$

It means paramagnetic substances have a tendency to pull in magnetic field lines when placed in a magnetising field.

(v) As $\mu_r = 1 + \chi_m$ and $\mu_r > 1$

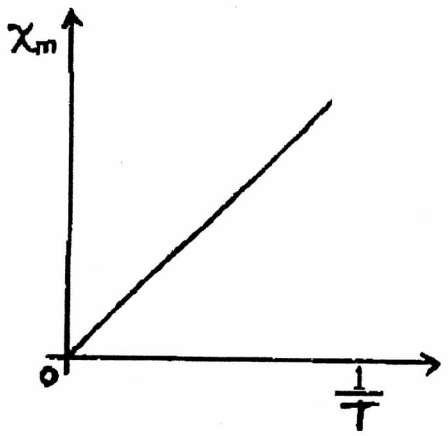
therefore χ_m must be positive.

Hence susceptibility of paramagnetic substance is positive with small value.

(vi) Susceptibility of paramagnetic substances varies inversely as the temperature of the substance.

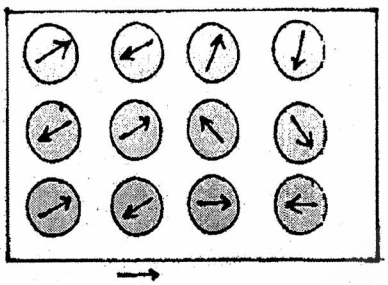
$$\boxed{\chi_m \propto \frac{1}{T}}$$

It means they loose their magnetic character with rise in temperature.

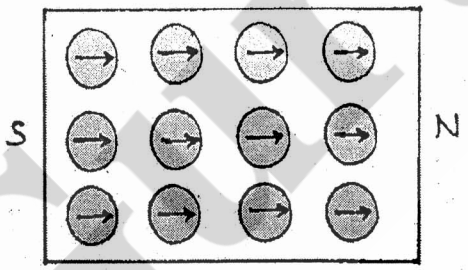


Explanation of paramagnetism -

In paramagnetic materials, every atom has some permanent magnetic dipole moment. In the absence of external magnetic field, the atomic dipoles are randomly oriented so that average magnetic moment per unit volume of the material is zero. Therefore a paramagnetic material does not behave as a magnet in the absence of an external magnetic field.



$\vec{H} = 0$



$\vec{H}$

When an external magnetic field is applied, it tries to align the atomic magnetic dipole in the direction of the field. That is why the material gets magnetised weakly in the direction of the field. This is paramagnetism.

* When we raise the temperature of the material, the atomic magnetic dipoles acquires some Kinetic energy. This tend to disorient the dipoles. That is why magnetic susceptibility of the paramagnetic materials decreases with rise in temperature.

(iii) Ferromagnetic Substances

Ferromagnetic substances are those in which each individual atom / molecule / ion has a non zero magnetic moment, as in a paramagnetic substances. When such substances are placed in an external magnetising field, they get strongly magnetised in the direction of the field.

Examples - Iron, Cobalt, Nickel and a number of their alloys.

Properties -

(i) They are strongly magnetised in the direction of external field in which they are placed.

(ii) They have a strong tendency to move from a region of weak magnetic field to the region of strong magnetic field. Hence they get strongly attracted to a magnet.

(iii) Relative magnetic permeability of feromagnetic materials is very large. ($\simeq 10^3$ to 10^5)

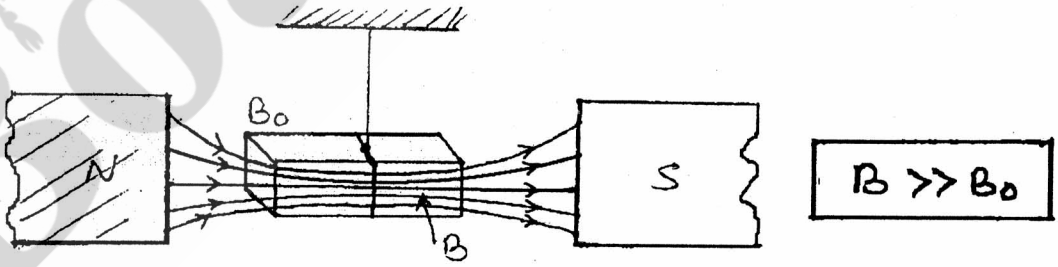
$$\mu_r \gg 1$$

(iv) The susceptibility of ferromagnetic materials is very large. That is why they can be magnetised easily and strongly.

$$\chi_m = \mu_r - 1 \qquad \chi_m = \text{large positive value}$$

(v) With rise in temperature, susceptibility of ferromagnetic materials decreases. At a certain temperature ferromagnetics change over to paramagnetics. This transition temperature is called curie temperature. For example, curie temperature of iron is about 1000 K.

(vi) When a piece of a ferromagnetic substance is placed in a magnetising field, the magnetic field lines prefer to pass through the piece rather than through air.



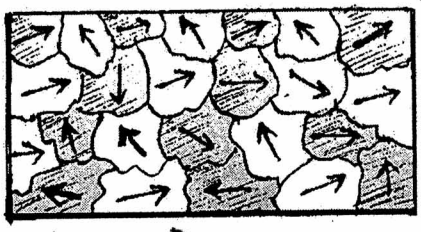
Explanation :-

Each atom of ferromagnetic substance is a tiny magnetic dipole having permanent dipole moment.

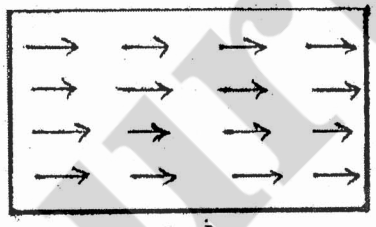
In ferromagnetic substances, atoms form a very large number of small effective regions called domains. Each domain is a strong magnet without any external magnetic field.

Inspite of this, a ferromagnetic substance does not behave as a magnet, because in the absence of external magnetic field the magnetic moments of different domains are randomly oriented so that their resultant magnetic moment in any direction is zero.

When an external magnetic field is applied on the ferromagnetic substance it gets strongly magnetised.



$\vec{B} = 0$



$\vec{B}$

The domains starts to rotate till their magnetic moments are aligned in the direction of the applied magnetic field. and arranged to form a single giant domain. This is how ferromagnetic materials gets strongly magnetised in the direction of the applied field.

Soft Ferromagnets :-

When external magnetic field is removed, magnetisation disappears in some of the materials like soft iron. These materials are called soft ferromagnets.

Hard Ferromagnets :-

In materials like Alnico (an alloy of iron, aluminium, nickel, cobalt and copper) the magnetisation persists even after removal of external magnetic field. Such materials are called hard ferromagnets.

* When ferromagnetic materials is heated, its magnetisation decreases gradually. This is because the domain structure disturbs with rise in temperature.

χ_m, μ_r and μ of magnetic substances

	substance	χ_m	μ_r	μ
1.	Diamagnetic	$-1 \leq \chi_m < 0$	$0 \leq \mu_r < 1$	$\mu < \mu_0$
2.	paramagnetic	$0 < \chi_m < 1$	$1 < \mu_r < (1+\delta)$	$\mu > \mu_0$
3.	Ferromagnetic	$\chi_m >> 1$	$\mu_r >> 1$	$\mu >> \mu_0$

* Here δ is a small positive number.

Q. A magnetic field of $1600\ Am^{-1}$ produces a magnetic flux of 2.4×10^{-5} weber in a bar of iron of cross section $0.2\ cm^2$. Calculate permeability and susceptibility of the bar.

Sol. Here $H = 1600\ Am^{-1}$, $\phi = 2.4 \times 10^{-5}$ Wb

$a = 0.2\ cm^2 = 0.2 \times 10^{-4}\ m^2$

So $B = \frac{\phi}{a} = \frac{2.4 \times 10^{-5}}{0.2 \times 10^{-4}} = 1.2$ Weber$/m^2$

$\mu = \frac{B}{H} = \frac{1.2}{1600} = 7.5 \times 10^{-4}\ T A^{-1} m.$

As $\mu = \mu_0 (1 + \chi_m)$

$\therefore \chi_m = \frac{\mu}{\mu_0} - 1$

$\chi_m = \frac{7.5 \times 10^{-4}}{4\pi \times 10^{-7}} - 1$

$\chi_m = 596.1$

Curie Law in Magnetism :-

Intensity of magnetisation (I) of a magnetic material is -

(i) directly proportional to magnetic field (B), and

(ii) inversely proportional to the temperature (T) of a material.

$$I \propto B \quad \text{and} \quad I \propto \frac{1}{T}$$

Combining these factors we get -

$$I \propto \frac{B}{T}$$

Also $B \propto H$

So magnetisation Intensity $I \propto \frac{H}{T}$

$$\text{or} \quad \frac{I}{H} \propto \frac{1}{T}$$

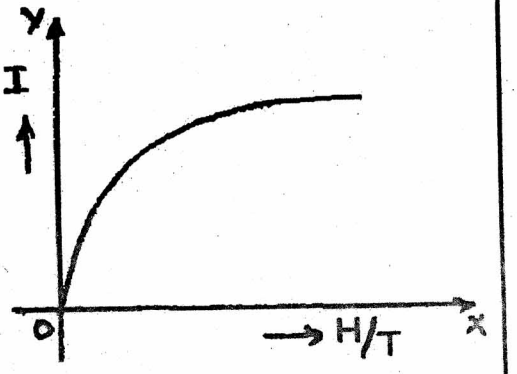
But $\frac{I}{H} = \chi_m$

$$\therefore \quad \chi_m \propto \frac{1}{T} \quad \text{or} \quad \boxed{\chi_m = \frac{C}{T}}$$

Here C is a constant called **curie constant.**

* Thus for a paramagnetic material both χ_m and μ_r depends not only on the material, but also on the sample temperature.

* At very low temperatures, the magnetisation approaches its maximum value, when all atomic dipole moments are aligned. This is called the saturation magnetisation



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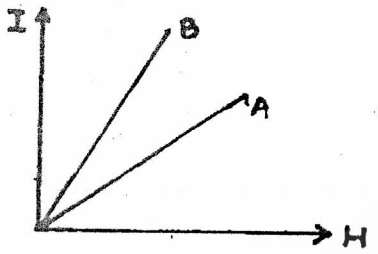
Q. CBSE 2008 The given figure shows the variation of intensity of magnetisation versus the applied magnetic field intensity H for two magnetic materials A and B.



(a) Identify the materials A and B.

(b) Why does the material B have a larger susceptibility than A, for a given field at constant temperature?

Sol:- (a) The slope of I-H curve gives magnetic susceptibility $\chi_m = \frac{I}{H}$

Material A is paramagnetic substance because χ_m is small but positive.

Material B is Ferromagnetic substance because χ_m is large and positive.

(b) The susceptibility of material B is Larger than A at given magnetic field because ferromagnetic material gets strongly magnetised and hence produces larger intensity of magnetisation in comparison to paramagnetic substance, therefore it is strongly magnetized.

Q. CBSE 2006 Why does a paramagnetic substance display greater magnetization for the same magnetizing field when cooled? How does diamagnetic substance respond to similar temperature change?

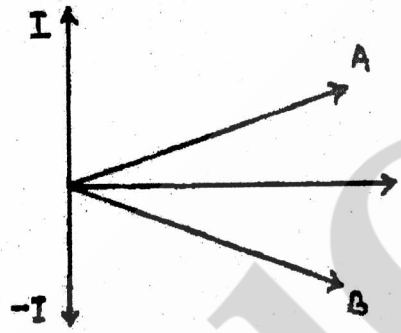
Sol. Since intensity of magnetization of paramagnetic substance varies as -

$$I \propto \frac{H}{T} \qquad \text{(Curie Law)}$$

Smaller the temperature at a given magnetizing field H, Higher the intensity of magnetization.

The susceptibility of diamagnetic substance is independent of temperature and magnetizing field.

Q. CBSE 2008 The given figure shows the variation of intensity of magnetization versus the applied magnetic field intensity H for two magnetic materials A and B.



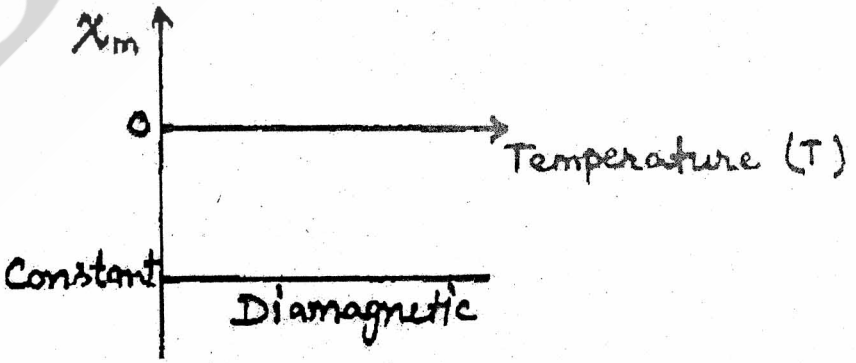
(a) Identify the material A and B.
(b) Draw the variation of susceptibility (χ_m) with temperature for B.

Sol:

(a) Material A is paramagnetic substance because χ_m is small and positive.

Material B is Diamagnetic substance because χ_m is small but negative.

(b) The susceptibility of Diamagnetic substance is independent of temperature and magnetizing field.



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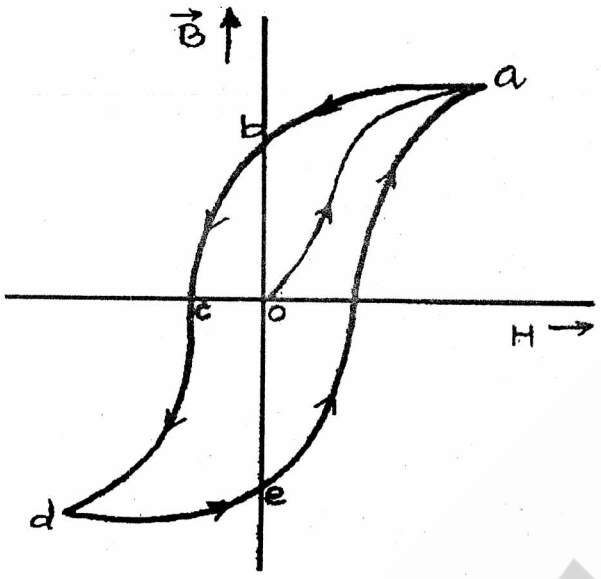
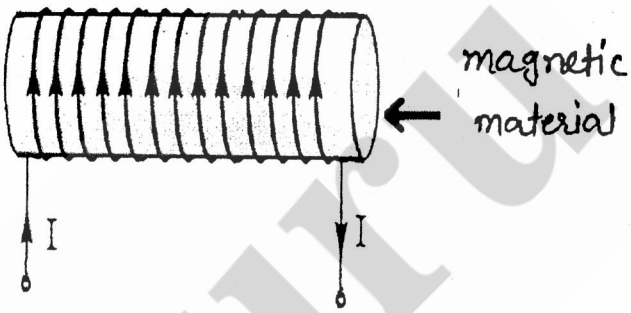
Hysteresis Curve

The hysteresis curve represents the relation between magnetic field ($\vec{B}$) of a ferromagnetic material with magnetising force ($\vec{H}$).





Suppose the material is unmagnetised initially i.e. $B=0$ and $H=0$. This state is represented by the origin O.

Now we place the material in a solenoid and increase the current through the solenoid gradually. The magnetising force $\vec{H}$ increases.

The magnetic field increases and saturates as shown in the curve Oa. This behaviour represents alignment of the domain of ferromagnetic material until no further increase in $\vec{B}$ is possible. Therefore there is no use of increase in current of solenoid and hence magnetic intensity beyond this.

Now we decrease the solenoid current and hence magnetic intensity $\vec{H}$ till it reduce to zero. The curve follow the path ab, which is showing that $H=0$ but $B\neq 0$. Thus some magnetism is left in the material.

The value of magnetic field left in the material when the magnetic intensity ($\vec{H}$) is reduced to zero is called **Retentity** or **Residual magnetism** of the material.

Next, the current in the solenoid is reversed and increased slowly. Certain domains are flipped until the net magnetic field inside is reduced to zero. This is represented by the curve bc.

It means to reduce the residual magnetism or retentivity to zero, we have to apply a magnetising force or magnetic intensity which is equal to oc in opposite direction. This value of magnetising force is called **Coercivity** of the material.

As the reverse current in solenoid is increased in magnitude, we once again obtain saturation in the reverse direction at d. The variation is represented by the curve cd.

Next the solenoid current is reduced curve (de), reversed, and increased (curve ea) The cycle repeats itself.

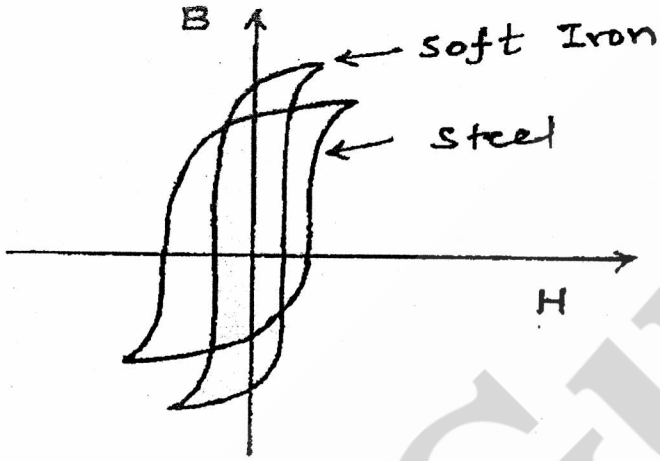
Thus it is clear that when a piece of a magnetic material is taken through a cycle of magnetisation, the intensity of magnetization (I) and magnetic field (B) lag behind the magnetising force (H). Thus

The phenomenon of Lagging of I or B behind H when a specimen of a magnetic material is subjected to a cycle of magnetisation is called **hysteresis.**

★ We can plot I-H Loop in the same way as we have plotted B-H Loop of a ferromagnetic material. The shapes of I-H loop and B-H loop for a given material are identical.

Hysteresis Loop for soft iron and steel -

Hysteresis loop for soft iron is narrow and large whereas the loop for steel is wide and short.



The hysteresis loops of soft iron and steel reveal that -

(i) The retentivity of soft iron is greater than the retentivity of steel.

(ii) Soft iron is more strongly magnetised than steel.

(iii) Coercivity of soft iron is less than the steel. It means soft iron looses its magnetism more rapidly then steel does.

(iv) As the area of B-H loop for soft iron is smaller than the area of B-H loop for steel therefore hysteresis loss in case of soft iron is smaller than the steel.

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Use of Ferromagnetics

(i) Permanent Magnets-

Permanent magnets are the materials which retain their ferromagnetic properties for a long time. The material chosen should have-

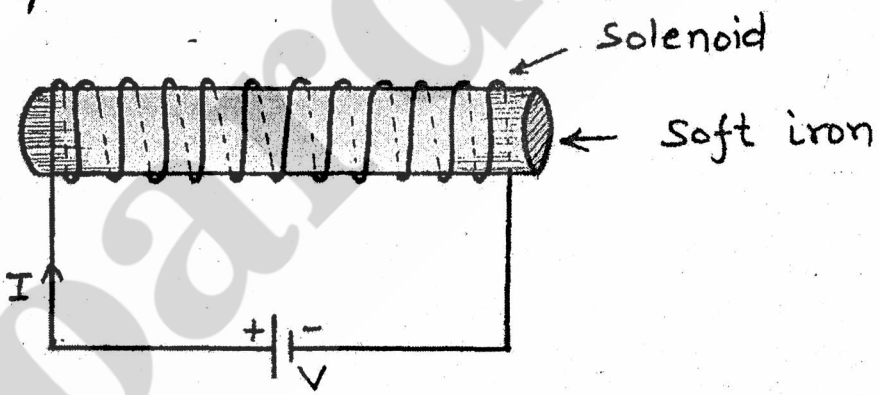
(i) High retentivity so that the magnet is strong.

(ii) High coercivity so that the magnetisation is not erased by little magnetic fields, temperature changes or mechanical damage due to rough handling.

(iii) Heigh permeability so that it can be magnetised easily.

(ii) Elertromagnets -

The core of electromagnets are made of ferromagnetic materials, which have high permeability and low retentivity. Soft iron is suitable material for this purpose.



Use -

Electromagnets are used in electric bells, loudspeakers, telephone diaphragm and to lift machinery etc.

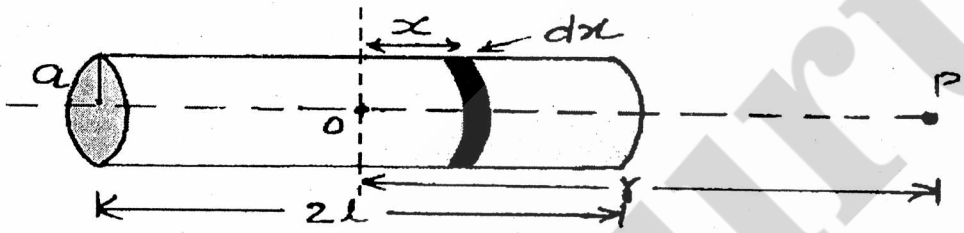
(iii) Transformer Core -

Soft iron is used for making transformer cores Because hysteresis loss for this is minimum and it has high resistivity to decrease energy losses due to eddy currents.

Bar Magnet As an Equivalent Solenoid –

Consider a solenoid of length $2l$ and radius a carrying a current I. Number of turns per unit length of solenoid are n.

Now consider a small element of thickness dx at a distance x from centre of the solenoid O. Number of turns in the element are ndx.



Magnetic field at a point P (where $OP = r$) due to this element is –

$$dB = \frac{\mu_0 I (ndx) a^2}{2[a^2 + (r-x^2]^{3/2}}$$

If r is very large, i.e. $r >> a$, $r >> x$ and $r >> a$ then –

$$dB = \frac{\mu_0 n I a^2 dx}{2r^3}$$

Hence the magnetic field at P due to complete solenoid –

$$B = \int_{-l}^{l} \frac{\mu_0 n I a^2 dx}{2r^3} = \frac{\mu_0 n I a^2}{2r^3} \int_{-l}^{l} dx$$

$$B = \frac{\mu_0 n I a^2}{2r^3} [x]_{-l}^{l} = \frac{\mu_0 n I a^2 \times 2l}{2r^3}$$

or $$B = \frac{\mu_0}{4\pi} \frac{(n \cdot 2l) I (\pi a^2)}{r^3} \times 2$$

$$B = \frac{\mu_0}{4\pi} \frac{N I A \times 2}{r^3} \qquad [\text{where } N = n \cdot 2l, \; A = \pi a^2]$$

or $$B = \frac{\mu_0}{4\pi} \frac{2M}{r^3} \qquad [\because M = NIA]$$

Thus the axial magnetic field of a current carrying solenoid is same as that of a bar magnet. Hence a current carrying solenoid is equivalent to a bar magnet.

Very Important Questions

1 Mark Questions -

Q.1 What is the angle of dip at a place where the horizontal and vertical components of the earth's magnetic field are equal ?
(CBSE 2012)

Q.2. Where on the surface of earth is the angle of dip 90° ?
Answer - at poles. (CBSE 2010)

Q.3. Where on the surface of earth is the vertical component of earth's magnetic field zero ?
Answer - at equator. (CBSE 2011)

Q.4. The permeability of a magnetic material is $\mu_r = 0.9983$. Name the type of magnetic material it represents.
(Delhi 2011)

Q.5. The susceptibility of a magnetic material is 1.9×10^{-5}. Name the type of magnetic material it represents.
(RBSE 2011)

Q.6. The susceptibility of a magnetic material is -4.2×10^{-6}. Name the type of magnetic material it represents.
(Delhi 2008)

Q.7. What is the characteristic property of a diamagnetic material ?
(CBSE 2006)

Q.8. Define the term **magnetic declination**.
(CBSE 2009)

Q.9. Steel is preferred for making permanent magnets whereas soft iron is preferred for making electromagnets. Give one reason.
(CBSE 2005,07)

2 Mark Questions -

Q.1. A circular coil of N turns and diameter d carries a current I. It is unwound and rewound to make another coil of diameter 2d, current I remaining the same. Calculate the ratio of the magnetic moments of the new coil and the original coil.

Answer - $M_1/M_2 = 1/2$

(CBSE 2005, 12)

Q.2. The relative magnetic permeability of a magnetic material is 800. Identify the nature of magnetic material and state its two properties.

(CBSE 2007, 12)

Q.3. A magnetic needle free to rotate in a verticle plane parallel to the magnetic meridian has its north tip down at 60° with the horizontal. The horizontal component of the earth's magnetic field at the place is 0.4 G. Determine the magnitude of the earth's magnetic field at the place.

Answer - $B_e = 0.8$ G.

(CBSE 2005, 09)

Q.4. (a) Write two characteristics of a material used for making permanent magnets?

(b) Why is core of an electromagnet made of ferromagnetic materials?

(CBSE 2010)

Q.5. Out of the following identify the materials which can be classified as (i) Paramagnetic (ii) diamagnetic

(a) Aluminium (b) Bismuth
(c) Copper (d) Sodium

write one property to distinguish between paramagnetic and diamagnetic materials.

(CBSE 2009, 06)

Q.6. Define magnetic susceptibility of a material. Name two elements, one having positive susceptibility and the other having negative susceptibility. What does negative susceptibility signify?

(CBSE 2008)

3 Marks Questions.

Q.1. Three identical specimens of a magnetic materials Nickel, Antimony, Aluminium are kept in a non-uniform magnetic field. Draw the modification in the field lines in each case.

(CBSE 2011)

Q.2. (a) What happens when a diamagnetic substance is placed in a varying magnetic field ?

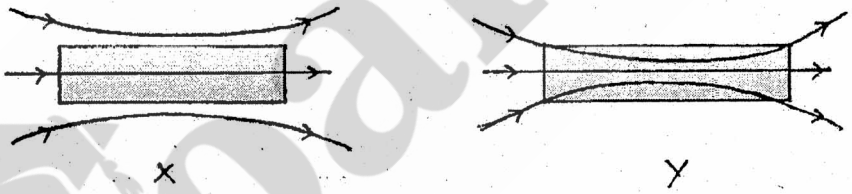
(b) Define the term magnetic permeability of a magnetic material. Write any two characteristics of a magnetic substance if it is to be used to make a parmanent magnet.

(CBSE 2007, 09)

Q.3. (a) How does angle of dip change as line goes from magnetic pole to magnetic equator of the earth ?

(b) A uniform magnetic field gets modified as shown below two specimens X and Y are placed in it. Identify whether specimens X and Y are diamagnetic, paramagnetic or ferromagnetic.

(CBSE 2009)



Q.4. If χ_m stands for the magnetic susceptibility of a given material, identify the class of materials for which - (i) $-1 \leq \chi_m < 0$

(ii) $0 < \chi_m < \epsilon$ (ϵ stands for a small positive number)

(a) Write the range of relative magnetic permeability of these materials.

(b) Draw the pattern of the magnetic field lines when these materials are placed in an external magnetic field.

(CBSE 2006, 08)

Q.5. Name the three elements required to specify the earth's magnetic field at a given place. Draw a labeled diagram to define these elements. Explain briefly how these elements are determined to find out the magnetic field at a given place on the surface of earth?

(CBSE 2008, 10)

Q.6. Distinguish the magnetic properties of dia, para and ferromagnetic substances in terms of - (a) susceptibility (b) Coercitivity. (c) magnetic permeability.
Give one example of each of these materials. Draw the field lines due to an external magnetic field near a (a) diamagnetic and (b) paramagnetic substance.

(CBSE 2007, 09)

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Q.1. How does the (i) Pole strength and (ii) Magnetic moment of each part of a bar magnet change if it is cut into two equal pieces transverse to length?

Ans. (i) Unchanged (ii) $M_1 = 2M_2$

Q.2. What is the angle of dip at a place where the horizontal and verticle components of the earth's magnetic field are equal?

Ans. $\delta = 45°$

Q.3. Which of the following substances are diamagnetic?
Bi, Al, Na, Cu, Ca and Ni

Ans. Bi, Cu.

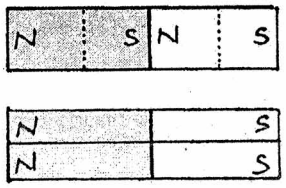
Q.4. What should be the orientation of a magnetic dipole in a uniform magnetic field so that its potential energy is maximum?

Ans. $\theta = 180°$

Q.5. The susceptibility of a magnetic material is 2.6×10^{-5}. Identify the type of magnetic material and state its two properties.

Q.6. Why are the field lines repelled (expelled) when a diamagnetic material is placed in an external uniform magnetic field?

Q.7. What happens if a bar magnet is cut into two parts
(i) transverse to its length
(ii) along the length



Q.8. A small magnet is pivoted to move freely in the magnetic meridian. At what place on the surface of the earth will the magnet be verticle?

Ans. at poles.

Q.9. What is curie Law in magnetism ?

Q.10. A short bar magnet placed with its axis at 30° with a uniform external magnetic field is 0.25 T experiences a torque of magnitude equal to 4.5×10^{-2} N.m. What is the magnitude of magnetic moment of the magnet ?

Ans. $M = 0.36\ Am^2$

Q.11 A bar magnet of magnetic moment $1.5\ JT^{-1}$ Lies aligned with the direction of a uniform magnetic field of 0.22 T. Calculate the amount of work done to turn the magnet so as to align its magnetic moment (i) Normal to field direction and (ii) opposite to field direction.

Ans. (i) 0.33J (ii) 0.66J

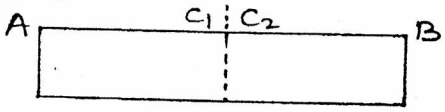
Q.12. When current is passed through a spring, it contracts, why ?

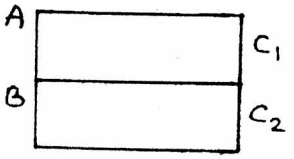
Q.13. Draw the pattern of the magnetic field lines when the materials for which (i) $-1 \leq \chi < 0$ (ii) $0 < \chi < \varepsilon$ (ε is a small positive number) are placed in an strong magnetic field.

Q.14. A bar magnet has magnetic moment M. It is divided into n-equal parts. Will each part bea magnetic dipole? What will be the magnetic moment of each part ?

Ans. (i) Yes (ii) $M' = \frac{M}{n}$

Q.15. A hypothetical bar magnet (AB) is cut into two equal parts. One part is now Kept over the other, so that the p C_2 is above C_1. If M is the magnetic moment of the original magnet, what would be the magnetic moment of the combination, so formed ?





PHYSICS

Unit - 6. Electromagnetic Induction

Class - 12th

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6. Electromagnetic Induction

Electromagnetic Induction (EMI)

The phenomena of generating current and e.m.f. by changing magnetic fields is called Electromagnetic Induction (EMI).

Induced e.m.f. & Induced Current

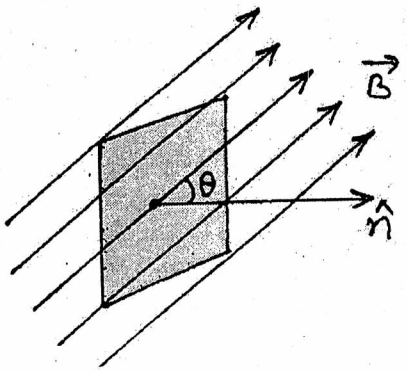
The e.m.f. so developed is called induced e.m.f. If the conductor is in the form of a closed circuit then a current flow in the circuit. This is called induced current.

* The phenomena of EMI is the basis of working of power generators, dynamos, transformers etc.

Magnetic Flux (ϕ)

The magnetic flux (ϕ) through any surface held in a magnetic field $\vec{B}$ is measured by the total number of magnetic lines of force crossing the surface normally.

$$\phi = \vec{B}.\vec{A} = BA\cos\theta$$



If the coil has N turns then total amount of magnetic flux linked with the coil is

$$\phi = N(\vec{B}.\vec{A}) = NBA\cos\theta$$

* when $\theta = 90^\circ$ then $\phi = 0$
* when $\theta = 0^\circ$ then $\phi = BA$ (Max. value)

SI Unit of magnetic flux is weber (Wb).

1 Weber (Wb) –

One weber is the amount of magnetic flux over an area of 1 metre^2 held normal to a uniform magnetic field of 1 Tesla.

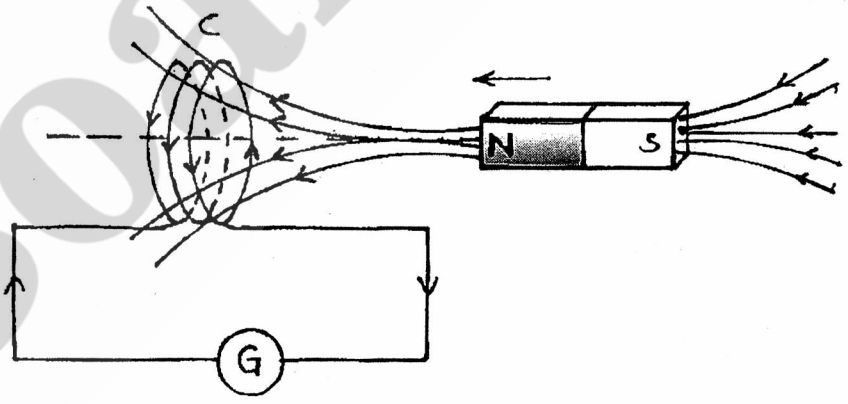
$$1 \text{ weber} = 1 \text{ tesla} \times 1 \text{ m}^2$$

* Magnetic flux (ϕ) is a scalar quantity.
* When a body is present in a uniform or non-uniform magnetic field, outward flux is taken to be positive, while inward flux is taken as negative.

Experiments of Faraday and Henry

Experiment - 1 (Current induced by a magnet)

Fig shows a coil or loop C of a few turns of conducting material insulated from each other. It is connected to a sensitive galvanometer G. Faraday and Henry observed that –



(i) When north pole of a bar magnet is pushed towards the coil, the galvanometer shows a sudden deflection, which indicates that current is induced in the coil.

(ii) There is no deflection when the bar magnet is held stationary anywhere.

(iii) When the magnet is moved away from the coil, the galvanometer shows deflection in the opposite direction, which indicates reversal in the direction of induced current.

(iv) When south pole of bar magnet is moved towards or away from the coil, the galvanometer deflections are opposite to those observed with the north pole for similar movement.

(v) The galvanometer deflection (and hence induced current) is found to be larger when magnet is pushed towards or pulled away from the coil faster.

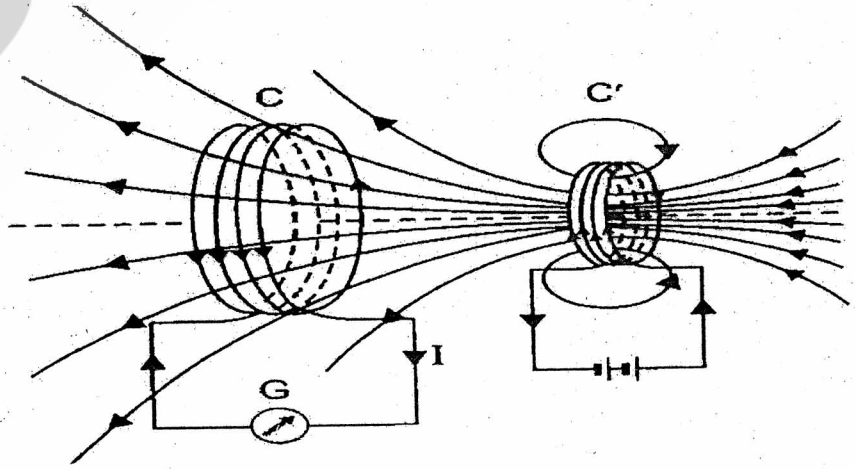
(vi) When the bar magnet is held stationary and coil C is moved towards or away from the magnet, the same effects are observed.

It shows relative motion between the coil and the magnet is responsible for induction of electric current in the coil.

Experiment 2 - (Current induced by current)

Fig shows a coil or loop C of a few turns of conducting material insulated one another and connected to a galvanometer. G.

C' is another such coil connected to a battery. A steady current through coil C' produces a uniform magnetic field along its axis.



(i) When coil C' is moved towards the coil C, the galvanometer shows a sudden deflection. This indicates that electric current is induced in the coil C.

(ii) When the coil C' is moved away from the coil C, the galvanometer shows a deflection in the opposite direction. This indicates that direction of current induced in coil C is reversed.

(iii) There is no deflection when there is no relative motion between the two coils.

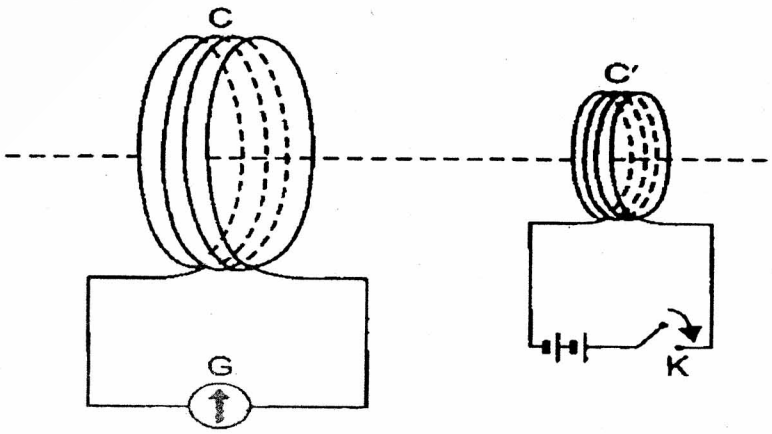
(iv) The galvanometer deflection (and hence induced current) is found to be larger when coils are moved faster towards / away from each other.

* Here the coil C' carrying current behaves as a bar magnet.

Experiment 3 - (Current induced by changing Current)

From the experiment 3 Faraday showed that relative motion is not an absolute requirement. Current can be induced without relative motion.

Fig. shows two coils C and C' held stationary coil C is connected to a sensitive galvanometer G and coil C' is connected to a battery through a Key. It is observed that -



(i) When Key K is pressed, galvanometer in coil C shows a momentary deflection, indicating that current is induced in coil C.

(ii) When the Key is pressed continuously, there is no deflection in the galvanometer.

(iii) When the Key is released the galvanometer shows again a momentary deflection but in the opposite direction.

(iv) The galvanometer deflection increases, when an iron rod is inserted into the coils along their axis., and the Key is pressed / released.

Cause of Induced E.M.F.

All experiment shows that induced e.m.f. appears in a coil whenever the amount of magnetic flux linked with the coil changes with time.

Note- Only the presence of magnetic flux is not enough. The amount of magnetic flux linked with the coil must change in order to produce any induced em.f. in the coil.

Faraday's Law of Electromagnetic Induction

First Law- Whenever the amount of magnetic flux linked with a circuit changes, an e.m.f is induced in the circuit. The induced e.m.f. exists so long as the change in magnetic flux continues.

Second Law- The magnitude of e.m.f. induced in a circuit is directly proportional to the rate of change of magnetic flux linked with the circuit.

$$\text{induced e.m.f } e \propto \frac{(\phi_2 - \phi_1)}{t}$$

$$\text{or} \quad e = K\frac{(\phi_2 - \phi_1)}{t}$$

Here $K = 1$ in all system of Units.

So $$\boxed{e = \frac{\phi_2 - \phi_1}{t}}$$

If $d\phi$ is the small change in magnetic flux in a small time dt then we can rewrite-

$$\boxed{e = -\frac{d\phi}{dt}}$$

* Negative sign is taken because induced e.m.f. always apposes any change in magnetic flux associated with the circuit.

In case of a closely wound coil of N turns, change in magnetic flux associated with each turn is the same. Therefore total induced e.m.f. is given by-

$$\boxed{e = -N\frac{d\phi}{dt}}$$

So by increasing the number of turns N in the coil we can increase the induced e.m.f.

Q. A circular coil of radius 10 cm, 500 turns and resistance 2 Ω is placed with its plane perpendicular to the horizontal component of the earth's magnetic field. It is rotated about its vertical diameter through 180° in 0.25 s. Estimate the magnitude of the e.m.f. and current induced in the coil. Horizontal component of earth's magnetic Field is 3×10^{-5} T.

Sol: Here - $r = 10\text{ cm} = 10^{-1}$ m., $N = 500$
$R = 2\ \Omega$, $\theta_1 = 0°$, $\theta_2 = 180°$
$dt = 0.25$ s

$$e = -N\frac{(\phi_2 - \phi_1)}{dt} = -\frac{NBA(\cos\theta_2 - \cos\theta_1)}{dt}$$

$$e = \frac{-500 \times 3\times10^{5} \times 3.14\times10^{-2}(\cos 180° - \cos 0°)}{0.25}$$

$$e = 3.8\times10^{-3}\ V$$

$$I = \frac{e}{R} = \frac{3.8\times10^{-3}}{2} = 1.9\times10^{-3}\ A$$

Q. The magnetic flux through a coil perpendicular to its plane and directed into paper is varying according to the relation $\phi = (5t^2 + 10t + 5)$ m Wb. Calculate the e.m.f. induced in the loop at $t = 5$ s.

Sol: Here $\phi = (5t^2 + 10t + 5)$ m Wb
$= (5t^2 + 10t + 5)\times10^{-3}$ Wb.

As $e = -\frac{d\phi}{dt} = -\frac{d}{dt}(5t^2 + 10t + 5)\times10^{-3}$ Wb/s

$$e = -(10t + 10)\times10^{-3}\ \text{Volt}$$

At $t = 5$ Sec.

$$e = -(10\times5 + 10)\times10^{-3}\ \text{Volt}$$

$$e = -0.06\ \text{Volt}.$$

Lenz's Law -

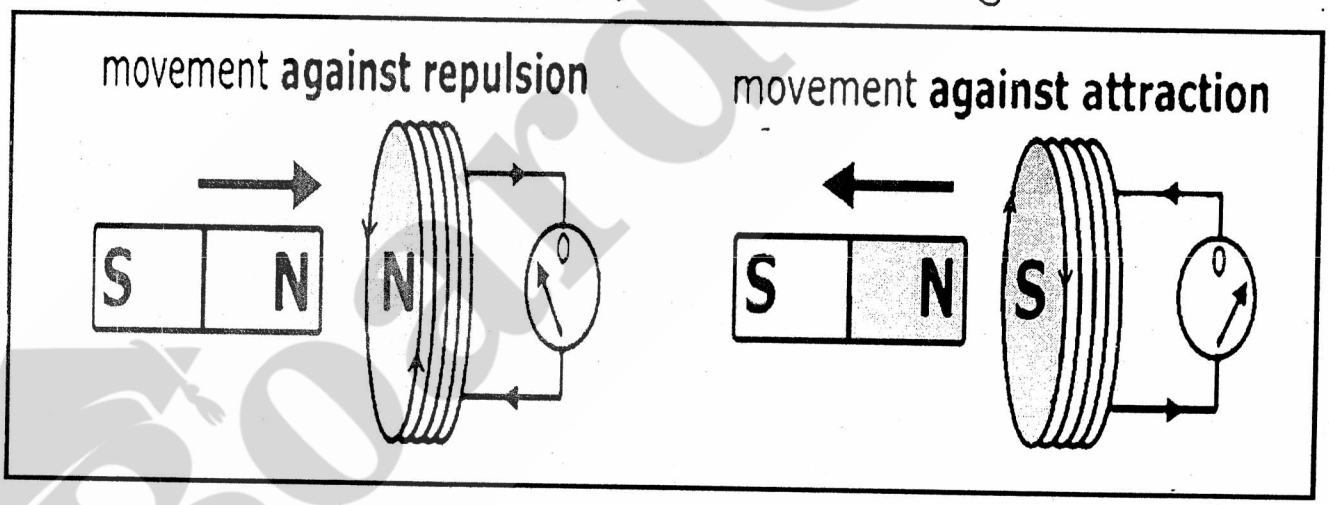
According to 'Lenz's Law', the polarity of the induced e.m.f. is such that it opposes the change in the magnetic flux responsible for its production.

This Law gives the direction of current induced in a circuit.

Explanation -

When north pole of a bar magnet is being pushed towards the coil, the amount of magnetic flux linked with the coil increases.

Current is induced in the coil in such a direction that it opposes the increase in flux. This is possible only when the current induced in the coil is in anticlockwise direction with respect to an observer on the side of the bar magnet.



Similarly, when north pole of the bar magnet is moved away from the coil the magnetic flux linked with the coil decreases. To oppose this decrease in magnetic flux, current induced in the coil is in clockwise direction so that its face towards the bar magnet becomes south pole.

This would result in an attractive force, which opposes the motion of the magnet and the corresponding decrease in magnetic flux.

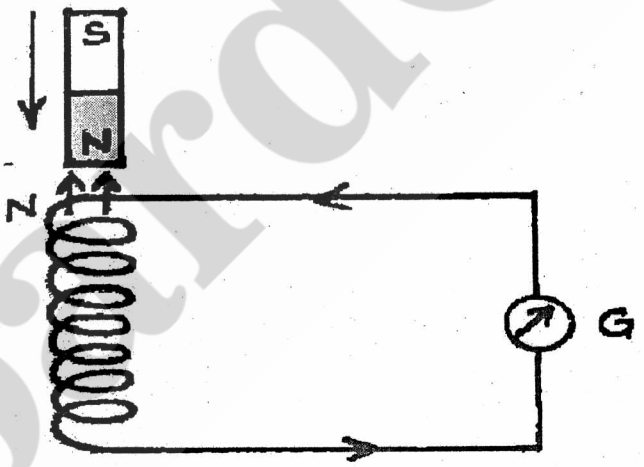
Lenz's Law & Energy Conservation

Lenz's Law is in accordance with the Law of conservation of energy.

Explanation -

In the experimental verification of Lenz's Law, when N-pole of magnet is moved towards the coil, the upper face of the coil acquires north polarity. Therefore work has to be done against the force of repulsion, in bringing the magnet closer to the coil.

Similarly when north pole of magnet is moved away south polarity develops on the upper face of the coil. Therefore work has to be done against the force of attraction, in taking the magnet away from the coil.



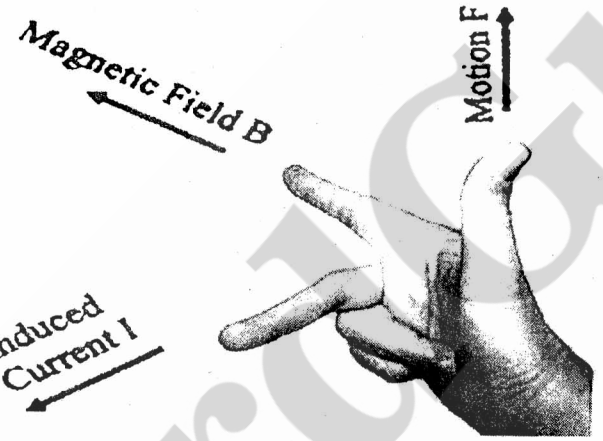
It is this mechanical work done in moving the magnet w.r.t. the coil that changes into electrical energy producing induced current. Thus energy is being transformed only.

When we do not move the magnet, work done is zero. Therefore, induced current is also not produced.

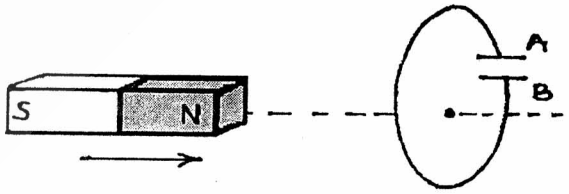
Fleming's Right Hand Rule

Fleming's right hand rule gives us the direction of induced e.m.f / current in a conductor moving in a magnetic field.

If we stretch the first finger, central finger and thumb of our right hand in mutually perpendicular directions such that first finger points along the direction of the field and thumb is along the direction of motion of the conductor, then the central finger would give us the direction of induced current.

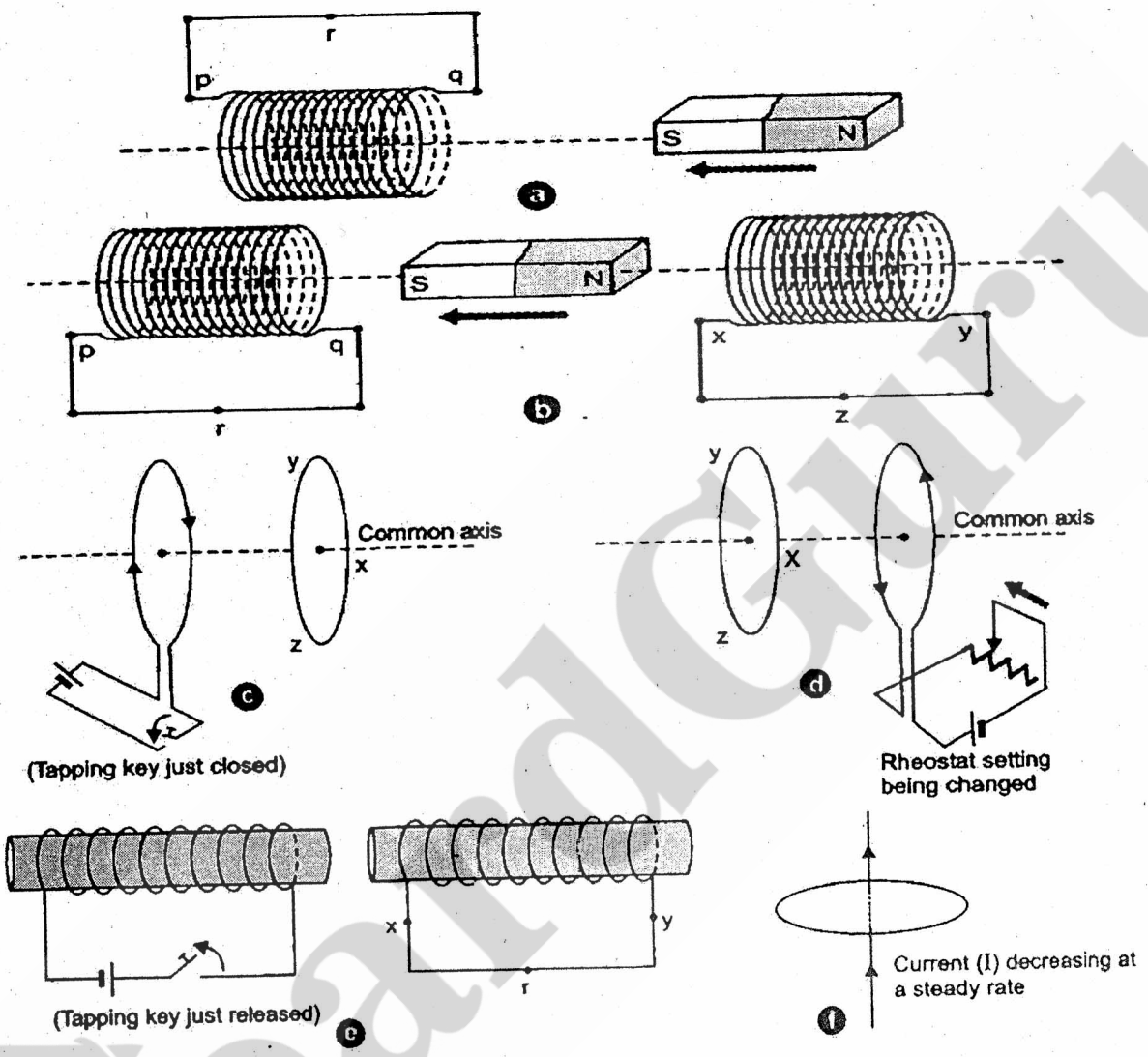


Q. CBSE 2010 In the given figure a bar magnet is quickly moved towards a conducting loop having a capacitor. Predict the polarity of the plates A and B of the capacitor.



Sol. Here the north pole is approaching the magnet so the induced current in the face of loop viewed from left side will flow in such a way that it will behave like north pole so, south pole developes in loop when viewed from right hand side of the loop. The flow of induced current is clockwise, hence A acquires positive polarity and B negative.

Q. Predict the direction of induced current in the situations described by the following figures -



Sol.

(a) South pole develops at q, so current induced must be clockwise at q.

(b) Coil pq in this case would develop S-pole at q and coil XY would also develop S pole at X. So the induced current in coil pq will be from q to p and induced current in coil XY will be from X to Y.

(c) Induced current in the right loop will be along XYZ.

(d) Induced current in the left loop will be along ZYX as seen from front.

(e) Induced current in the right coil is from X to Y.

(f) No current is induced because magnetic lines of force lie in the plane of the loop.

Q. A square loop of side 12 cm. with its sides parallel to x and y axes is moved with a velocity of 8 cm/s in the positive x-direction in an environment containing a magnetic field in the positive z direction. The field is neither uniform in space nor constant in time. It has a gradient of 10^{-3} T/cm. along the negative x-direction and it is decreasing in time at the rate of 10^{-3} T/s. Determine the direction and magnitude of the induced current in the loop if its resistance is 4.5 m Ω.

Sol. Given that.

$A = (12 \times 10^{-2})^2 = 144 \times 10^{-4}\ m^2$, Velocity $V = 8 \times 10^{-2}$ m/s

$\frac{dB}{dx} = 10^{-3}$ T/cm., $\frac{dB}{dt} = 10^{-3}$ T/s, $R = 4.5 \times 10^{-3}\ \Omega$

Rate of change of magnetic flux due to variation in B with time.

$$e_1 = A\left(\frac{dB}{dt}\right) = 144 \times 10^{-4} \times 10^{-3} = 1.44 \times 10^{-5}\ \text{Wb/s}.$$

Rate of change of magnetic flux due to motion of the loop in a non-uniform magnetic field.

$$e_2 = A\left(\frac{dB}{dx}\right)\left(\frac{dx}{dt}\right) = 144 \times 10^{-4} \times 10^{-3} \times 8 = 11.52 \times 10^{-5}\ \text{Wb/s}$$

As both the effects cause a decrease in magnetic flux along the positive z-direction, therefore they add up.

$\therefore$ Total induced emf $(e) = e_1 + e_2$

$$e = 1.44 \times 10^{-5} + 11.52 \times 10^{-5}$$

$$e = 12.96 \times 10^{-5}\ V$$

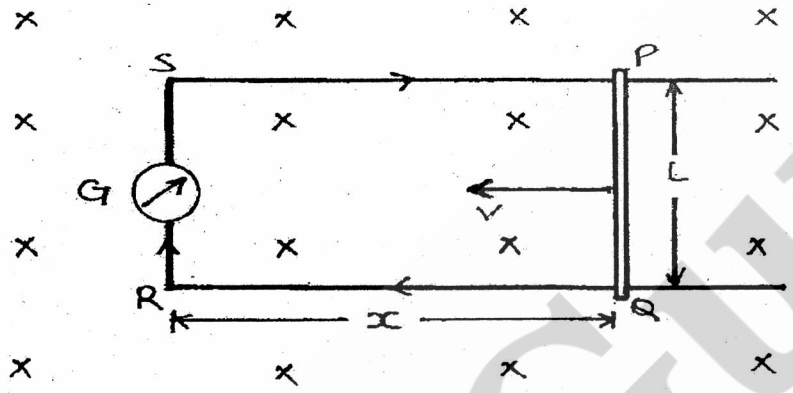
Induced Current $I = \frac{e}{R} = \frac{12.96 \times 10^{-5}}{4.5 \times 10^{-3}}$

$$I = 2.88 \times 10^{-2}\ A$$

The direction of induced current is such as to increase the flux through the loop along positive z-direction, hence current will be seen to anticlockwise.

Motional Electromotive Force

Fig. shows a rectangular conducting Loop PQRS in the plane of paper. The conductor PQ is free to move without any loss of energy due to friction etc. The magnetic field is perpendicular to the plane of paper and directed inwards.



Let the conductor PQ moved towards the left with a velocity V, due to which the area enclosed by the loop PQRS decreases. Therefore amount of magnetic flux linked with the loop decreases and an e.m.f. is induced in the loop.

If at any time, the length RQ = x and PQ = L, then magnetic flux Linked with the loop PQRS is.

$$\phi = BLx$$

Then the induced e.m.f. in the loop can be given by.....

$$e = \frac{-d\phi}{dt} = -\frac{d}{dt}(BLx)$$

$$e = BL\left(-\frac{dx}{dt}\right)$$

$$\boxed{e = BLV} \qquad \left(V = -\frac{dx}{dt}\right)$$

Here e is called Motional Electromotive Force.

If R is resistance of loop at a given instant, the induced current I at that instant can be given as-

$$\boxed{I = \frac{e}{R} = \frac{BLV}{R}}$$

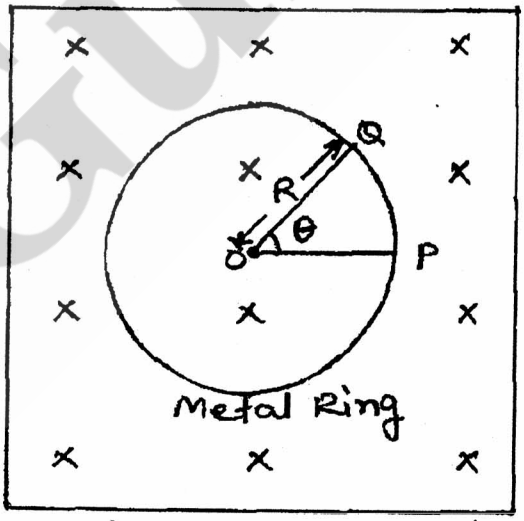
* The direction of induced current is given by Fleming's right hand rule or by Lenz's Law.

Q. A metallic rod of 1 m length is rotated with a frequency of 50 rev/s, with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius 1 m, about an axis passing through the centre and perpendicular to the plane of the ring. A constant uniform magnetic field of 1 T parallel to the axis is present everywhere. What is the e.m.f. between the centre and the metallic ring ?

Sol.

Here, radius of circle $R = 1$ m.
Frequency $\nu = 50$ rev/s , $B = 1$ T

Let the rotating metallic rod goes from OP to OQ in t sec. tracing $\angle POQ = \theta$



The potential difference across the rod = e.m.f. between Centre & Metallic Ring

$e = B \times$ rate of change of area of loop

$$e = B \times \frac{d}{dt}(\text{area of loop } POQ)$$

$$e = B \times \frac{d}{dt}\left(\pi R^2 \times \frac{\theta}{2\pi}\right) = \frac{1}{2} B R^2 \frac{d\theta}{dt}$$

$$e = \frac{1}{2} BR^2 (2\pi\nu) = \pi\nu BR^2$$

$$e = 3.14 \times 50 \times 1 \times (1)^2 = 157.1 \text{ V}$$

Q. A circular copper disc of radius 10 cm, rotates at a speed of 20π rad/s about an axis through its centre and perpendicular to the disc. A uniform magnetic field of 0.2 T acts perpendicular to the disc.
(i) Calculate potential difference developed between the axis of the disc and the rim.
(ii) What is the induced current if the resistance of disc is 2 Ω?

Sol. Here $r = 10\text{ cm} = 10^{-1}\text{ m}$. $\omega = 20\pi$ rad/s
$B = 0.2$ T, $R = 2\ \Omega$

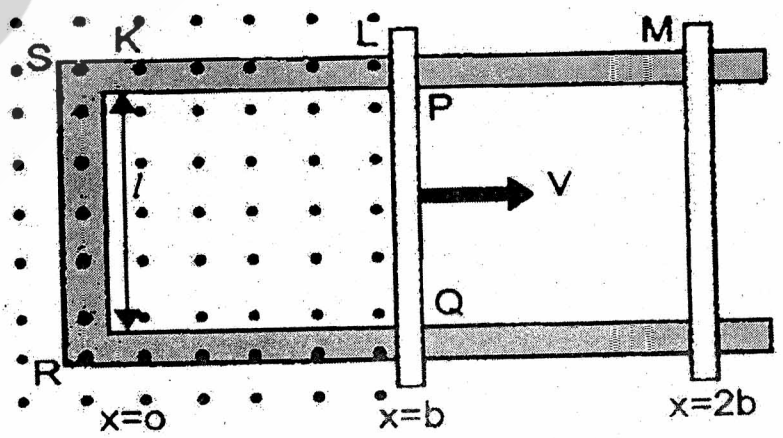
$$\text{So } e = \frac{1}{2} B\omega r^2$$

$$e = \frac{1}{2} \times 0.2 \times 20\pi (10^{-1})^2 = 0.0629 \text{ V}$$

$$i = \frac{e}{R} = \frac{0.0629}{2} = 0.0315 \text{ A}$$

Q. The arm PQ of the rectangular conductor is moved from $x = 0$ to the right side as shown in figure. The uniform magnetic field is perpendicular to the plane and extends from $x = 0$ to $x = b$ and is zero for $x > b$. Only the arm PQ possesses substantial resistance r. Consider the situation when the arm PQ is pulled outwards from $x = 0$ to $x = 2b$, and is then moved back to $x = 0$ with constant speed v. Obtain expression for the flux, the induced e.m.f., the necessary force to pull the arm and the power dissipated as Joule heat. Sketch the variation of these quantities with distance.

Sol.



Consider the forward motion from $x=0$ to $x=2b$. When arm PQ is at x, magnetic flux linked with the circuit PQRS is -

$$\phi = BLx \quad \text{for} \quad 0 \leq x < b$$

$$\phi = BLb \quad \text{for} \quad b \leq x \leq 2b$$

So the induced e.m.f. is -

$$e = -\frac{d\phi}{dt} = -BLV \quad \text{for} \quad 0 \leq x < b$$

$$e = -\frac{d\phi}{dt} = 0 \quad \text{for} \quad b \leq x \leq 2b$$

When the induced e.m.f is non-zero, the magnitude of current is $I = \frac{E}{r} = \frac{BLV}{r}$

Force required to keep the arm PQ in constant motion $= BIL$

$$F = B\left(\frac{BLV}{r}\right)L.$$

$$= \frac{B^2L^2V}{r}, \text{ for } 0 \leq x < b$$

$F =$ Zero, for $b \leq x \leq 2b$

The direction of the force on PQ is to the left

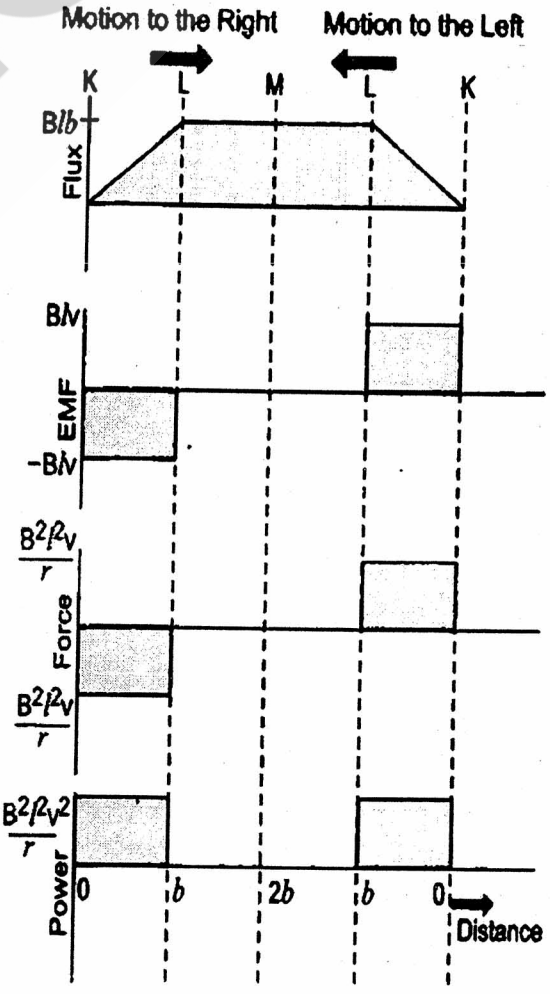
The Joule heating loss is -

$$P = I^2r = \frac{B^2L^2V^2}{r}$$

$$P = \frac{B^2L^2V^2}{r}, \text{ for } 0 \leq x < b$$

$$= 0, \text{ for } b \leq x \leq 2b$$

similar expressions are obtained for inward motion of PQ from $x=2b$ to $x=0$.



Energy Consideration in Motional E.M.F.

When a conductor of length L is moved with a velocity V in a perpendicular magnetic field B, then the motional e.m.f. produced in the conductor is

$$e = BLV$$

Let r be the resistance of movable arm PQ of the rectangular conductor. Here we assume that the resistance of remaining arm QR, RS and SP is negligible compared to r.

Current Induced in the loop $I = \frac{e}{r} = \frac{BLV}{r}$

The magnitude of force on the conductor PQ moving in the magnetic field is -

$$F = BIL = B\left(\frac{BLV}{r}\right)L = \frac{B^2L^2V}{r}$$

The direction of this force is opposite to the velocity of the conductor.

Power required to push the conductor is

$$P_M = F \times V = \frac{B^2L^2V}{r} \times V = \frac{B^2L^2V^2}{r}$$

As the conductor is pushed mechanically, the mechanical energy dissipated per second is given by

$$P_E = I^2r = \left(\frac{BLV}{r}\right)^2 = \frac{B^2L^2V^2}{r}$$

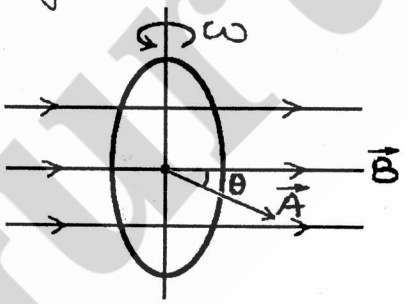
which is the same as the power required to push the conductor.

Hence, mechanical energy required to move the conductor PQ is converted into electrical energy first (i.e. the induced e.m.f.) and then to thermal energy.

Heat energy produced/Sec. $= \frac{B^2L^2V^2}{r}$

Q. Delhi 2008 A coil of number of turns N, area A is rotated at a constant angular speed ω, in a uniform magnetic field B and connected to a resistor R. Deduce the expression for -
(a) Maximum emf induced in the coil
(b) Power dissipation in the coil.

Sol. Let initially area vector $\vec{A}$ of coil makes an angle θ with the direction of magnetic field. Let the coil rotates by an angle θ with the magnetic field in time t.



$\therefore \quad \omega = \frac{\theta}{t}$

or $\quad \theta = \omega t$

$\therefore$ Magnetic flux linked with each turn of the rectangular coil is -

$$\phi = \vec{B}\cdot\vec{A} = BA\cos\theta = BA\cos\omega t$$

$$\therefore \quad \frac{d\phi}{dt} = -BA\omega\sin\omega t \qquad [\omega = 2\pi f]$$

For N turns of the coil by Faraday's Law -

$$e = -N\frac{d\phi}{dt} = NBA\omega\sin\omega t$$

or $\quad e = e_0 \sin\omega t$

where e_0 is maximum or peak value of emf induced in coil and given by -

$$\boxed{e_0 = NBA\omega = NBA(2\pi f)}$$

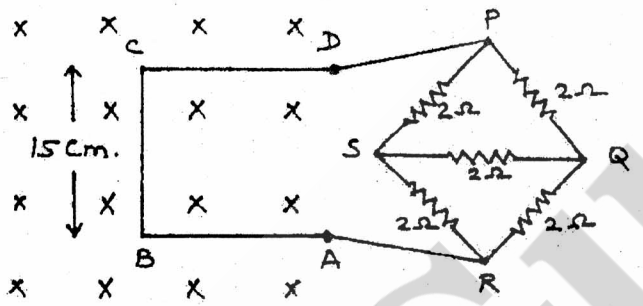
(b) Power dissipated $(P) = \frac{V_{rms}^2}{R} = \frac{e_{rms}^2}{R}$

where R is the resistance.

$$\because \quad e_{rms} = \frac{e_0}{\sqrt{2}} = \frac{NBA\omega}{\sqrt{2}}$$

$$\therefore \quad P = \frac{(NBA\omega/\sqrt{2})^2}{R} \quad \text{or} \quad \boxed{P = \frac{N^2B^2A^2\omega^2}{2R}}$$

Q. CBSE 2006 A Metallic square loop ABCD of size 15 cm. and resistance 1 Ω is moved at a uniform velocity of v m/s in a uniform magnetic field of 2 T, the field lines being normal to the plane of the paper. The loop is connected to an electrical network of resistors, each of resistance 2 Ω. Calculate the speed of the loop for which 2 mA current flows in the loop.



Sol: Given - $l = 15\text{ cm}$, $B = 2\text{ T}$, $R_{ABCD} = 1\ \Omega$, $I = 2\text{ mA}$

$\because$ PQRS is a balanced wheatstone bridge.

$\therefore$ SQ wire of 2 Ω resistance can be removed.

$\therefore$ The equivalent resistance across PR is given by-

$$R = \frac{4 \times 4}{4+4} = 2\ \Omega$$

Net resistance of the circuit $= 1 + 2 = 3\ \Omega$

$$\therefore \quad V = IR = 2 \times 10^{-3} \times 3 = 6 \times 10^{-3}\text{ Volt.}$$

But potential difference across conductor $= vBl$

So $vBl = 6 \times 10^{-3}$

$$v = \frac{6 \times 10^{-3}}{2 \times 0.15} = 2 \times 10^{-2}\text{ m/s.}$$

Eddy Currents

The eddy currents are basically the currents induced in the body of a conductor due to change in magnetic flux linked with the conductor.

For example when we move a metal plate out of a magnetic field, then the relative motion of the field and the conductor induces a current in the conductor.

The conduction electrons making up the induced current whirl about within the plate as if they were caught in an eddy (or whirlpool) of water. This is called the eddy current.

$$\text{Eddy Current } (i) = \frac{\text{induced e.m.f } (e)}{\text{resistance } (R)}$$

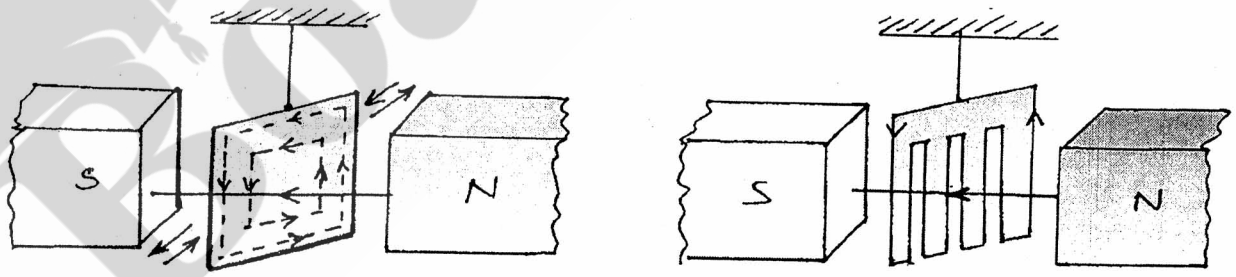
But we know that $e = -\frac{d\phi}{dt}$ so

$$i = -\frac{d\phi/dt}{R}$$

The direction of eddy current is given by Lenz's law or Fleming's right hand rule.

Experimental Demonstration

Suspend a flat metallic plate between pole pieces N and S of an electromagnet. Let it come to rest with its plane perpendicular to magnetic field.



When the magnetic field is off, the metallic plate disturbed from its equilibrium position and left, oscillates freely for a longer time.

But when the electromagnet is switched on, the vibrations of the plate are damped and the plate stops vibrating sooner. This is because of eddy currents developed in the vibrating plate.

Reason :-

In the normal position of rest of the plate, magnetic flux linked with the plate is maximum. When it is displaced towards any one extreme position, area of plate in the field decreases. Therefore magnetic flux linked with the plate decreases. Eddy currents develop in the plate which, according to Lenz's Law, oppose the decrease in flux and hence the motion of the plate towards extreme position.

Similarly when plate returns from extreme position to mean position, area of plate in the field increases, magnetic flux linked with the plate increases. Eddy currents are developed which oppose the increase in flux and hence the motion of the plate towards the mean position.

In either case, vibrations of the plate are opposed and hence damped.

How to reduce eddy currents -

When a plate with slots cut in it is made to oscillate in the magnetic field, the damping effect is there, but it is much smaller compared to the case when no slots were cut.

This means eddy currents are reduced. This is because magnetic moment of the induced current (which opposes the motion) depend upon the area enclosed by the currents.

$$\vec{M} = I\vec{A}$$

Hence to minimize the eddy currents, the metal core to be used in an appliance like dynamo, transformer, choke coil, motor etc. is taken in the form of thin steets. Each sheet is electrically insulated from the other by insulating varnish. Such a core is called a Laminated core.

Large resistance between the thin sheets confines the eddy currents to the individual sheets. Hence the eddy currents are reduced to a large extent.

Q. Mention any two useful applications of eddy currents. Delhi 2009

Sol. (a) Magnetic brake
(b) Magnetic Furnace / Induction cooker.

Q. A Jet plane is travelling west at the speed of 1800 Km/h. What is the voltage difference developed between the ends of the wing 25 m long if the earth's magnetic field at the location has a magnitude of 5×10^{-4} T and the dip angle is 30°? *2005

Sol. Given that - $V = 1800$ Km/h $= 500$ m/s.
$L = 25$ m, $B = 5 \times 10^{-4}$ T, $\delta = 30°$

So $e = B_v L V$ ($B_v \rightarrow$ Verticle Component of earth's field)

$e = (B \sin\delta) L V$

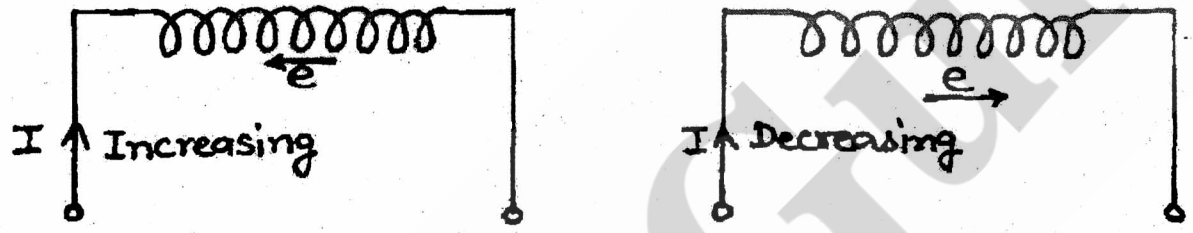
$= 5 \times 10^{-4} \times \sin 30° \times 25 \times 500$

$e = 3.1$ Volt

Self Induction -

It is the property of a coil by virtue of which the coil opposes any change in the strength of current flowing through it by inducing an e.m.f. in itself

When current (I) is increasing, the self induced e.m.f. (e) appears accross the coil in a direction such that it opposes the increase. Therefore it would be in a direction opposite to I.



when Current (I) is decreasing, the self induced e.m.f. (e) appears across the coil in a direction, such that it opposes the decrease. Therefore it would be in the direction of I.

Self Inductance -

Let the current I flowing through a coil at any time and ϕ is the amount of magnetic flux linked with all the turns of the coil at that time. Then it is found that

$$\phi \propto I \quad \text{or} \quad \boxed{\phi = LI}$$

Here L is a constant of proportionality and is called self inductance or coefficient of self induction of the coil.

* The value of L depends upon the numbers of turns, area of cross section and nature of material of the core on which the coil is wound.

If $I = 1$, $\phi = L \times 1$ or $\boxed{L = \phi}$

Self inductance of a coil is numerically equal to the amount of magnetic flux linked with the coil when unit current flows through the coil.

The e.m.f. induced in the coil is given by.

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(LI)$$

or $$\boxed{e = -L \times \frac{dI}{dt}}$$

* The SI unit of L is henry.

$$L = \frac{-e}{dI/dt} \quad \therefore \quad 1 \text{ henry} = \frac{1 \text{ Volt}}{1 \text{ ampere/sec.}}$$

The self inductance of a coil is said to be 1 henry when a current change at the rate of 1 ampere/sec through the coil induces an e.m.f. of 1 volt in the coil.

Q. (CBSE 2011) Current in a circuit falls steadily from 2 A to 0 A in 10 ms. If an average emf of 200 V is induced, Calculate the self inductance of the circuit.

Sol. Given that -

$$\Delta I = -2 \text{ A}, \quad \Delta t = 10 \times 10^{-3} \text{ s}, \quad V = 200 \text{ V}$$

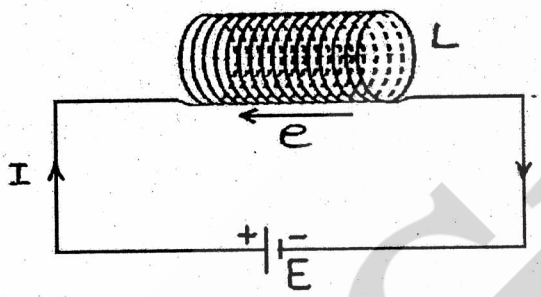
$\because$ Induced emf $e = -L\frac{\Delta I}{\Delta t}$

$$L = -e\frac{\Delta t}{\Delta I} = \frac{-200 \times 10 \times 10^{-3}}{-2} = 1 \text{ H}$$

$\therefore$ self Inductance $L = 1 \text{ H}$

Q. Delhi 2010 A source of emf E is used to establish a current I, through a coil of self inductance L. Show that the work done by the source of build up the current I is $\frac{1}{2}LI^2$.

Sol. The source of emf E establishes current in coil in opposition of induced emf (Lenz's Law) and hence does work for the same. This work done by sources stores in the form of magnetic energy in the coil.



Let I current flows through the coil of self inductance L at any instant t when rate of change of current in coil is dI/dt.

$\therefore$ Induced emf $e = -L\frac{dI}{dt}$

$\therefore$ Work done in establishing the current in small time interval dt is given by-

$$dw = P \cdot dt = -EI\,dt$$

$$dw = -\left(-L\frac{dI}{dt}\right) I\,dt$$

$$dw = LI\,dI$$

$\therefore$ Total work done in increasing the current from zero to I is -

$$W = \int_0^I LI\,dI = L\int_0^I I\,dI$$

$$W = L\left[\frac{I^2}{2}\right]_0^I$$

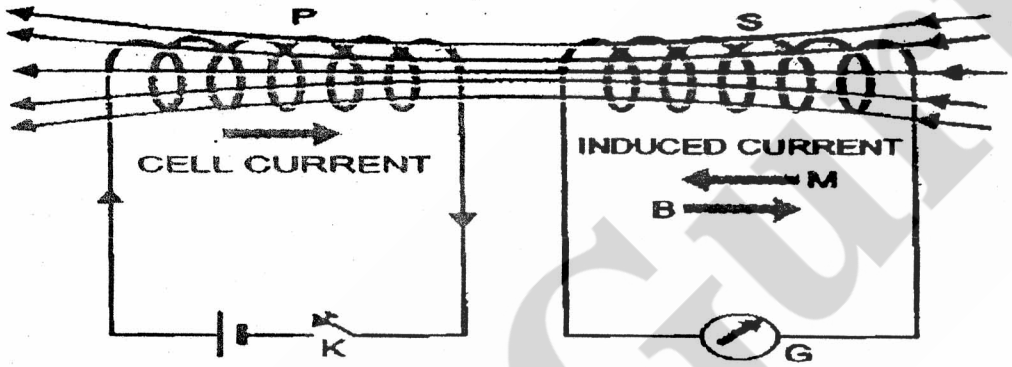
$$W = \frac{1}{2}L\left[I^2 - 0^2\right]$$

$$\boxed{W = \frac{1}{2}LI^2}$$

Mutual Inductance

It is the property of two coils by virtue of which each opposes any change in the strength of current flowing through the other by developing an induced e.m.f.

Let us take two coils P and S held closeby. P is connected to a cell through a key K. S is connected to a sensitive galvanometer G.



On pressing or releasing K, galvanometer shows a sudden temporary deflection. This is due to mutual induction as detailed below –

(i) On pressing Key K, current in P increases from zero to maximum value. It takes some time. During this time, current in P is increasing. Therefore magnetic flux linked with P is increasing. As S is closeby, magnetic flux associated with S also increases. An e.m.f. is induced in S. According to Lenz's Law, the induced current in S would oppose increase in current in P by flowing in a direction opposite to the cell current in P.

(ii) On releasing Key K, current in P decreases from maximum to zero value. It takes some time. During this time current in P is decreasing. Therefore magnetic flux linked with P is decreasing. As S is held closeby, magnetic flux associated with S also decreases. An e.m.f is induced in S.

According to lenz's Law induced current in S during break flows in the direction of the cell current in P so as to oppose the decrease in current in P.

Mutual Inductance

Let I is the strength of current in one coil and ϕ is the total amount of magnetic flux linked with all the turns of the neighbouring coil.
Then it is found that -

$$\phi_2 \propto I_1 \quad \text{or} \quad \boxed{\phi_2 = M I_1}$$

where M is a constant of proportionality and is called coefficient of mutual induction or mutual inductance of the two coils.

If $I_1 = 1$, then $\boxed{\phi_2 = M}$

Thus the mutual inductance of two coils is equal to the amount of magnetic flux linked with one coil when unit current flows through the neighbouring coil.

The induced e.m.f. in the neighbouring coil is given by -

$$e_2 = -\frac{d\phi_2}{dt} = -\frac{d}{dt}(MI_1)$$

$$\boxed{e_2 = -M\frac{dI_1}{dt}}$$

SI Unit of M is henry

If $dI_1/dt = 1$ then $e_2 = -M$

So the mutual inductance of two coils is said to be one Henry, when a current changes at the rate of one ampere/sec. in one coil induces an e.m.f. of one volt in the other coil.

Mutual Inductance depends on-

(i) Geometry of two coils (size of coils, their shape, number of turns, nature of material on which two coils are wound).

(ii) Distance between two coils.

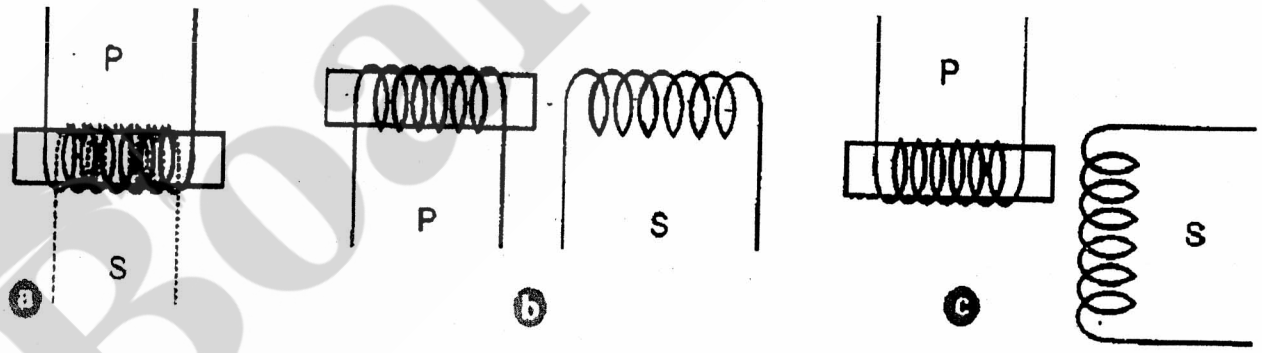
(iii) Relative placement of two coils (orientation of the two coils).

Coefficient of Coupling (K)

Coefficient of coupling (K) of two coils is a measure of the coupling between the two coils. It is given by-

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

where L_1 and L_2 are coefficients of self inductance of the two coils and M is coefficient of mutual inductance of the two coils.



In fig. (a) the value of K is maximum.

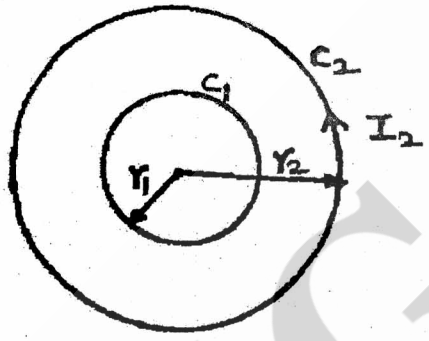
In fig. (b) the value of K is less than that in (a).

In fig. (c) the K is minimum.

BoardGuru

Q. CBSE 2009 2011 Two concentric circular coils C_1 and C_2 of radius r_1 and r_2 ($r_1 << r_2$) respectively are kept coaxially. If current is passed through C_2, then find an expression for mutual inductance between the coils.

Sol. In case of two concentric coils when current flows through one coil, the emf is induced in the other coil.



Let I_2 current passes through the coil C_2.

Magnetic field at centre due to current loop C_2

$$B_2 = \frac{\mu_0 N_2 I_2}{2r_2} \quad [N_2 = \text{number of turns in coil } C_2.]$$

$\therefore$ Total Magnetic flux linked with coil C_1

$$\phi_1 = N_1 B_2 A_1$$

$$\phi_1 = N_1 \left(\frac{\mu_0 N_2 I_2}{2r_2}\right)(\pi r_1^2)$$

But $\phi = M I_2$

so that $M I_2 = \frac{\mu_0 \pi N_1 N_2 r_1^2 I_2}{2r_2}$

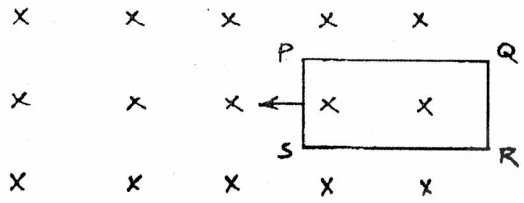
or

$$\boxed{M = \frac{\mu_0 \pi N_1 N_2 r_1^2}{2r_2}}$$

Very Important Questions

1 Mark Questions -

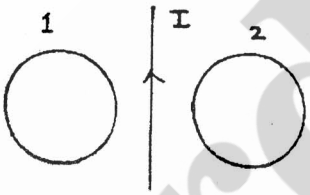
Q.1. The closed loop (PQRS) of wire is moved into a uniform magnetic field at right angles to the plane of the paper as shown in fig. Predict the direction of the induced current in the loop.



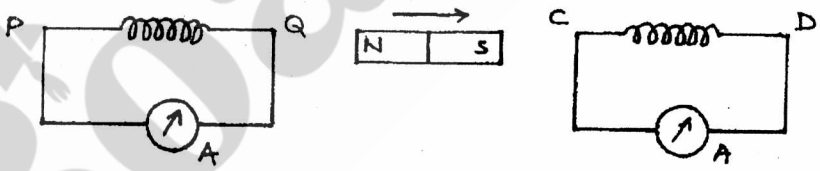
(CBSE 2012,07)

Q.2. Predict the direction of induced current in metal rings 1 and 2 when current I in the wire is continuously decreasing ?

(Delhi 2012,02)



Q.3. A bar magnet is moved in the direction indicated by the arrow between two coils PQ and CD. Predict the directions of induced current in each coil.



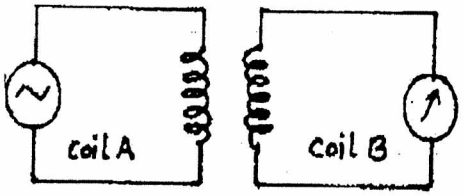
(RBSE 2012)

Q.4. The given circuit shows that when an AC passes through the coil A, the current starts flowing in the coil B.

(a) State the underlying principle involved.

(b) Mention two factors on which the current produced in the coil B depends.

(CBSE 2008, 03)



2 Marks Questions-

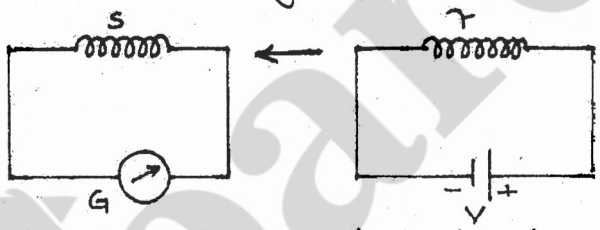
Q.1. A metallic rod of Length l is rotated with angular frequency of ω with one-end hinged at the centre and the other end at the circumference of a circular metallic ring of radius l, about an axis passing through the centre and perpendicular to the plane of the ring. A constant and uniform magnetic field B parallel to the axis is present everywhere. Deduce the expression for the emf between the centre and the metallic ring.

(Delhi 2012,08)

Q.2. Two identical loops, one of copper and the other of aluminium, are rotated with the same angular speed in the same magnetic field. Compare
(a) The induced emf
(b) The current produced in the two coils.

(CBSE 2010,04)

Q.3. (a) When primary coil P is moved towards the secondary coil S, the galvanometer shows some deflection. What can be done to have larger deflection in the galvanometer with the same battery?



(Delhi 2010,07)

(b) State the related law.

Q.4. State the Faraday's Law of electromagnetic induction.

(CBSE 2009,03)

Q.5. A conducting rod of length l is moved in a magnetic field $\vec{B}$ with velocity $\vec{v}$ such that the arrangement is mutually perpendicular. Prove that the emf induced in the rod is $e = Blv$.

(Delhi 2006,09)

Q.6. Define self-inductance and mutual inductance. Give its SI units.

(Delhi 2009,03)

Q.7. Mention any two useful applications of eddy currents.
(Delhi 2009,03)

Q.8. Define mutual inductance between two long coaxial solenoids. Find out the expression for the mutual inductance of inner solenoid of length l having the radius r_1 and the number of turns n_1 per unit length due to the second outer solenoid of same length and n_2 number of turns per unit length.
(Delhi 2012,08)

Q.9. Derive an expression for the self inductance of a long air cored solenoid of length l and number of turns N.
(Delhi 2008,05)

Q.10. How is the mutual inductance of a pair of coils affected when.
(a) Separation between the coils is increased ?
(b) The number of turns of each coil is increased?
(c) A thin iron sheet is placed between the two coils, other factors remaining the same ?
(Delhi 2006)

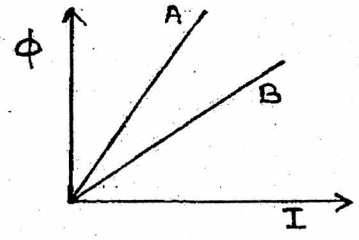
Q.11. What are eddy currents ? How are these minimized? mention two applications of eddy currents.
(CBSE 2006,02)

3 Marks Questions-

Q.1. State Lenz's Law. Give one example to illustrate this Law. "The Lenz's Law is a consequence of the principle of conservation of energy." Justify this statement.
(CBSE 2005,09)

Q.2. What are eddy currents. How are these produced In what sense are eddy currents considered undesirable in a transformer and how are these reduced in such a device?
(CBSE 2006)

Q.3. A plot of magnetic flux (ϕ) versus current (I) is shown in figure for two inductors A and B. Which of the two has larger value of self inductance? (CBSE 2004, 10)



Answer - $L_A > L_B$

(Hint - As $L = \frac{\phi}{I}$)

Q.4. A wire which is in N-S direction is dropped freely. Will any potential difference be induced across its ends? (CBSE 2011)

Answer - No, because neither horizontal component H nor vertical component V of earth's magnetic field is intercepted by the falling wire.

Q.5. Magnetic flux of 5 μ Weber is linked with a coil when a current of 1 mA flows through it. What is self inductance of the coil? (Pb. Board 2011)

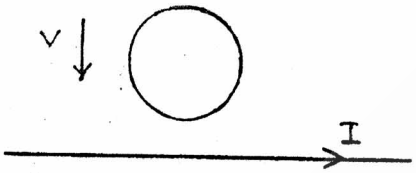
Answer - $L = 5 \times 10^{-3}$ H

Q.6. The flux, linked with a large circular coil, of radius R is 0.5×10^{-3} Wb. when a current of 0.5 A flows through a small neighbouring coil of radius r. Calculate the coefficient of mutual inductance for the given pair of coils.

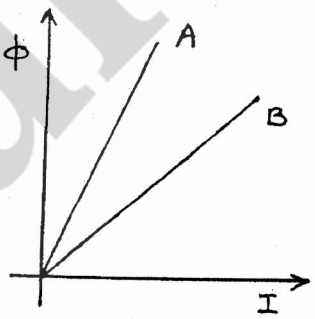
If the current through the small coil suddenly falls to zero, what would be its effect in the larger coil? (Delhi 2008)

Answer - $M = 10^{-3}$ H

Q.1. Pridict the direction of induced current in a metal ring when the ring is moved towards a straight conductor with constant speed V. The conductor is carrying current I in the direction shown in fig.

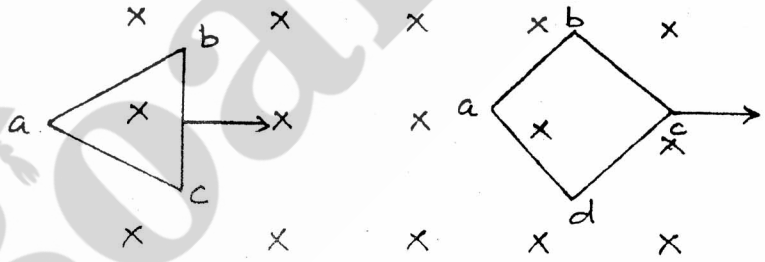


Q.2. A plot of magnetic flux (ϕ) versus current (I), is shown in the figure for two inductors A and B. Which of the two has large value of self-inductance ?



Ans. $L_A > L_B$

Q.3. Two different loops are moved in a uniform magnetic field pointing downward. The loops are moved in the directions shown by arrows. What is the direction of induced current in each loop

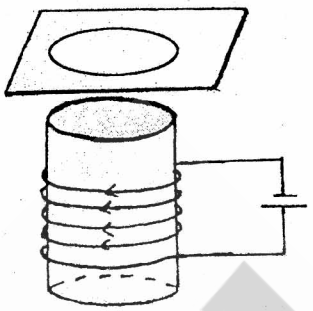


Ans.
abc → anticlockwise
abcd → clockwise

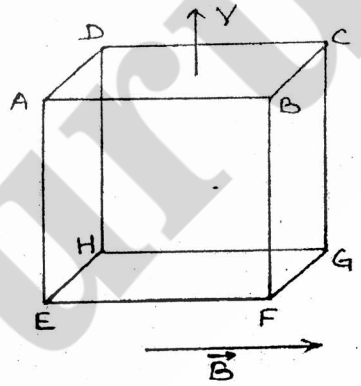
Q.4. A wire in the form of a tightly wound solenoid is connected to a DC source, and carries a current. If the coil is stretched so that there are gaps between successive elements of the spiral coil, will the current increase or decrease ? Explain.

Ans. Current will increase.

Q.5. Consider a metal ring kept on top of a fixed solenoid (say on a cardboard). The centre of the ring coincides with the axis of solenoid. If the current is suddenly switched on, the metal ring jumps up. Explain.



Q.6. Twelve wires of equal length 'L' are connected to form a skeleton cube which moves with a velocity V perpendicular to the magnetic field B. What will be the induced emf in each arm of the cube?



Q.7. A long solenoid with 15 turns per cm. has a small loop of area $2\ cm^2$ placed inside normal to the axis of the solenoid. The current carried by the solenoid changes steadily from 2A to 4A in 0.1 s, what is the induced emf in the loop while the current is changing?

Ans. $e = 7.5\ \mu V$

Q.8. The motion of copper plate is damped when it is allowed to oscillate between the two poles of a magnet. What is the cause of damping?

Q.9. Why does a metallic piece become very hot when it is surrounded by by a coil carrying high frequency alternating current?

Q.10. A horizontal straight wire 10 m long extending from east to west is falling with a speed of $5\ ms^{-1}$ at right angle to the horizontal component of earth's magnetic field equal to $0.3 \times 10^{-4}\ Wb/m^2$. Calculate

(a) Induced emf in the wire and its direction

(b) which end of the wire is at higher potential?

Ans. (a) $e = 1.5\ mV$ (west to east) (b) Eastern end

PHYSICS

Unit - 7. Alternating Current

Class - 12th

BoardGuru

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7. Alternating current

Most of the electric power generated and used in the world is in the form of alternating current (a.c.). This is because-

(i) Alternating voltage can easily and efficiently be converted from one value to other by means of transformers.

(ii) The Alternating current energy can be transmitted and distributed over long distances economically without much loss of energy.

Alternating Current (a.c.)

The magnitude of alternating current (a.c.) changes continuously with time and its direction is reversed periodically.

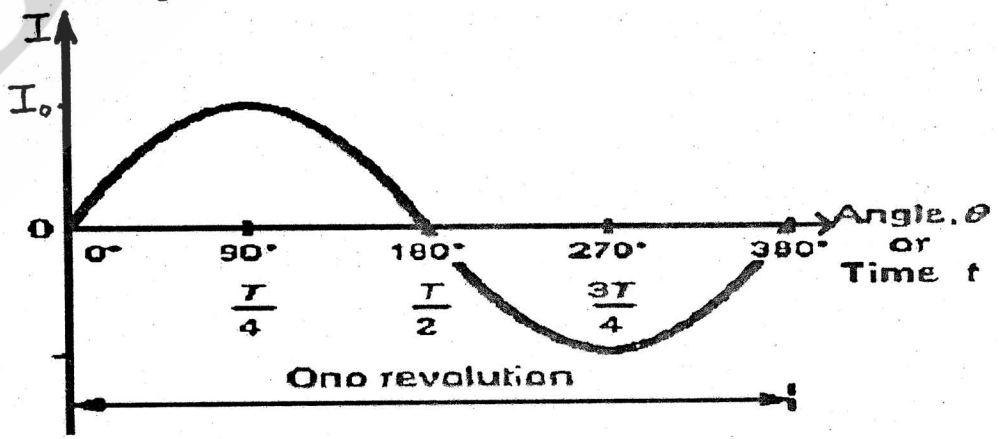
It is represented by -

$$I = I_0 \sin \omega t \quad \text{or} \quad I = I_0 \cos \omega t$$

Here I is instantaneous value of current and I_0 is the peak value or maximum value of a.c. ω is called angular frequency of a.c.

Where $\omega = \frac{2\pi}{T} = 2\pi f$ $\qquad \frac{1}{T} = f$ (frequency)

Where T is the time period of a.c. It is equal to the time taken by the a.c. to go through one complete cycle of variations.



Also f is the frequency of a.c. It is equal to the number of complete cycles of variations gone through by the a.c. in one second.

Similarly the alternating e.m.f. may be represented by-

$$E = E_0 \sin\omega t \quad \text{or} \quad E = E_0 \cos\omega t$$

In a.c. circuits, two additional circuit elements are used. These are inductor (L) and capacitor (C). Thus current and voltage in a.c. circuit are controlled by three circuit elements R, L and C.

* The voltage across a pure inductor L is given by-

$$V = L\frac{dI}{dt}$$

where dI/dt is rate of change of current.
The above equation shows that an inductor affects the voltage only when current I changes with time.

* For an ideal capacitor of capacitance C,

$$Q = CV$$

As $I = \frac{dQ}{dt} \Rightarrow I = \frac{d(CV)}{dt}$

or

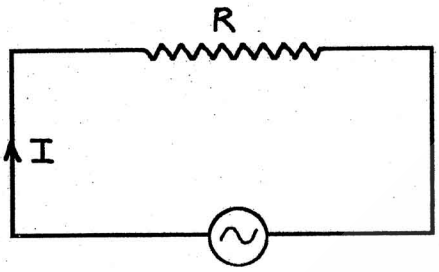
$$I = C\frac{dV}{dt}$$

where dV/dt is rate of change of potential.
The equation shows that a capacitor affects the current only when V changes with time

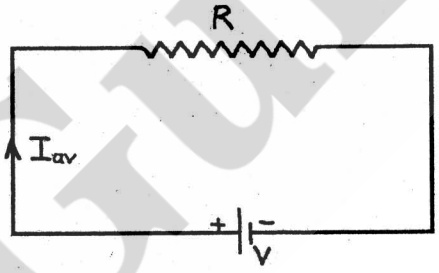
Mean or Average value of Alternating Current –

It is defined as that value of steady current which would send the same amount of charge through a circuit in the time of half cycle (i.e. $T/2$) as is sent by the a.c. through the same circuit, in the same time.

* The mean or average value of a.c. over one complete cycle is zero. So we can calculate the mean or average value of a.c. over any half cycle.



$I = I_0 \sin\omega t$



Let an alternating current be represented by –

$$\boxed{I = I_0 \sin\omega t}$$

If the strength of current is assumed to remain constant for a small time dt, then small amount of charge sent in a small time dt is –

$$dq = I\,dt$$

Let q be the total charge sent by a.c. in the first half cycle ($0 \to T/2$)

$$q = \int_0^{T/2} I\,dt \quad \text{or} \quad q = \int_0^{T/2} I_0 \sin\omega t\,dt$$

$$q = I_0 \left[-\frac{\cos\omega t}{\omega} \right]_0^{T/2} \qquad [\omega T = 2\pi]$$

$$= -\frac{I_0}{\omega} \left[\cos\omega\frac{T}{2} - \cos 0^\circ \right]$$

$$= -\frac{I_0}{\omega} \left[\cos\pi - \cos 0^\circ \right]$$

$$q = \frac{2I_0}{\omega} \quad \text{— (1)}$$

If I_{av} represents the mean or average value of a.c. over the 1st half cycle then -

$$q = I_{av} \times \frac{T}{2} \quad \text{— (2)}$$

from equation (1) and (2) we get -

$$I_{av} \times \frac{T}{2} = 2\frac{I_0}{\omega} = \frac{2I_0}{2\pi/T}$$

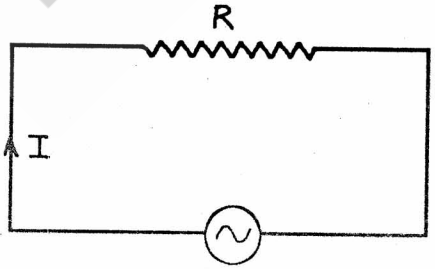
$$I_{av} = \frac{2I_0}{\pi} = 0.637 I_0$$

Hence mean or average value of a.c. over positive half cycle is 0.637 times the peak value of a.c., or we can say 63.7 % of the peak value.

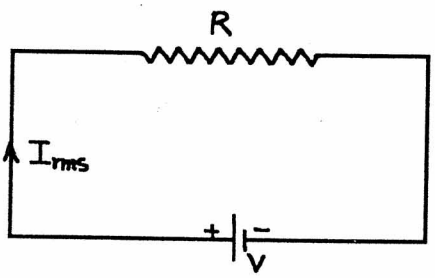
Note -
The ordinary d.c. ammeter and d.c. voltmeter cannot measure alternating current / voltage. They record zero reading, when used in ac. circuit, because average value of alternating current / voltage over a full cycle is zero.

Root Mean Square Value of Alternating Current

It is defined as that value of steady current, which would generate the same amount of heat in a given resistance in a given time, as is done by the a.c., when passed through the same resistance for the same time.



$I = I_0 \sin\omega t$



The r.m.s value is also called effective value of a.c. or virtual value of a.c. It is represented by I_{rms} or I_v.

Let an alternating current is represented by -

$$\boxed{I = I_0 \sin \omega t} \quad \text{— (1)}$$

Let this current flow through a resistance R. In a small time dt, the amount of heat produced in resistance R is -

$$dH = I^2 R\, dt \quad \text{— (2)}$$

In one complete cycle (time $0 \to T$), the total amount of heat produced in the resistance R is -

$$H = \int_0^T I^2 R\, dt$$

$$\text{or} \quad H = \int_0^T (I_0^2 \sin^2 \omega t) R\, dt$$

$$H = I_0^2 R \int_0^T \frac{(1 - \cos 2\omega t)}{2}\, dt$$

$$H = \frac{I_0^2 R}{2}\left[\int_0^T 1 \cdot dt - \int_0^T \cos 2\omega t\, dt\right]$$

$$H = \frac{I_0^2 R}{2}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$H = \frac{I_0^2 R}{2}\left[T - \frac{\sin 2\omega T}{2\omega}\right] \qquad (\because \omega T = 2\pi)$$

$$\text{So} \quad H = \frac{I_0^2 R T}{2} \quad \text{— (3)}$$

If r.m.s. value or virtual value of a.c. is represented by I_{rms}, then the amount of heat produced in the same resistance R in the same time T would be -

$$H = I_{rms}^2 R T \quad \text{— (4)}$$

So from eq. (3) and (4) -

$$I_{rms}^2 RT = \frac{I_0^2 RT}{2}$$

$$\boxed{I_{rms} = \frac{I_0}{\sqrt{2}} = 0.707\, I_0}$$

Hence the r.m.s. value or virtual value or effective value of a.c. is 0.707 times the peak value of a.c. or we can say 70.7 % of the peak value of a.c.

Note -

Alternating current / voltage are measured by a.c. ammeter / voltmeter respectively. These instruments are called hot wire instruments and they measure only virtual or r.m.s. values of alternating currents / voltages.

Q. CBSE 2012 A light bulb is rated 150 W for 220 V AC supply of 60 Hz. Calculate (a) the resistance of the bulb (b) the rms current through the bulb.

Sol. Given that - $P = 150\,W$, $V = 220\,V$

(a) Resistance of the bulb $R = \frac{V^2}{P}$

$$R = \frac{220 \times 220}{150} = 322.7\ \Omega$$

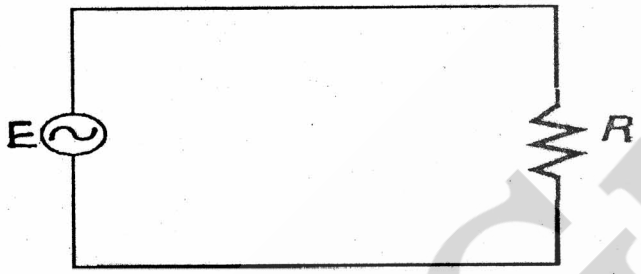
(b) $$I_{rms} = \frac{V_{rms}}{R} = \frac{220}{322.7} = 0.68\ A$$

A.C. Circuit Containing Resistance Only

Let a source of alternating e.m.f. is connected to a pure resistance R.

Suppose the alternating e.m.f. supplied is represented by -

$$E = E_0 \sin \omega t \quad \text{— (1)}$$



Let I be the current in the circuit at any instant t. The potential difference developed across R will be IR.

This must be equal to e.m.f. applied at that instant -

$$IR = E = E_0 \sin \omega t$$

or $$I = \frac{E_0}{R} \sin \omega t = I_0 \sin \omega t \quad \text{— (2)}$$

where $I_0 = E_0/R$ (Maximum value of current)

★ The behaviour of R in d.c. and a.c. circuits is the same. R can reduce a.c. as well as d.c. equally effectively

By comparing equation (1) and (2) we can see that in an a.c circuit containing R only, the voltage and current are in the same phase.

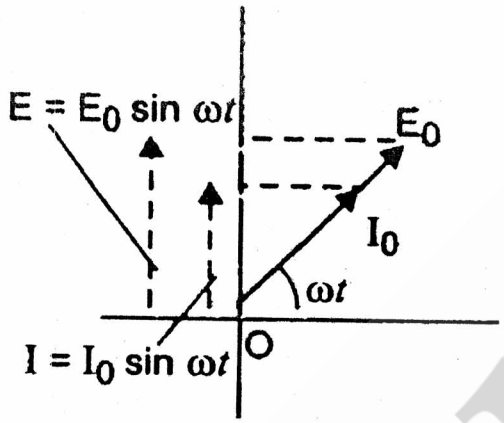
Phasor Diagram-

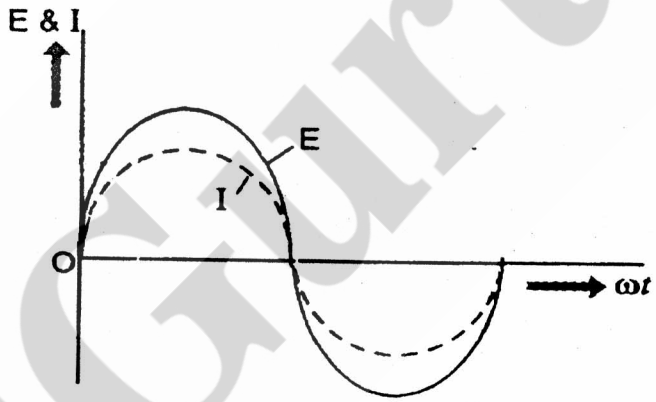
A rotating vector that represents a quantity varying sinusoidally with time is called a phasor.

(i) In phasor diagram, peak values of alternating current (I_0) and alternating e.m.f. (E_0) are represented by arrows called phasors.

(ii) They are inclined to horizontal axis at $\angle \omega t$ and rotate in the anticlockwise direction.

The length of the arrow represents the maximum value of the quantity i.e. I_0 and E_0.





(iii) The projection of the arrow on any axis represents the instantaneous value of the quantity.

(iv) The phasor difference between the two alternating quantities is represented by the angle between the two vectors $\vec{E_0}$ and $\vec{I_0}$.

(v) In the a.c. circuit containing R only, current and voltage are in the same phase i.e. the phase angle between alternating voltage and current through R is zero.

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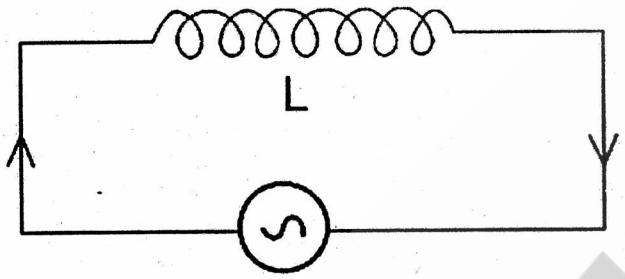
A.C. Circuit Containing Inductance only

Let a source of e.m.f. is connected to a circuit containing a pure inductance L only Suppose the alternating e.m.f. supplied is given by-

$$E = E_0 \sin \omega t$$



$E = E_0 \sin \omega t$

If dI/dt is the rate of change of current through L at any instant, then induced e.m.f. in the inductor at the same instant is-

$$e = -L \frac{dI}{dt}$$

The negative sign indicates that induced e.m.f. oppose the change of current.

To maintain the flow of current in this circuit, applied voltage must be equal and opposite to the induced voltage.

$$E = -\left(-L \frac{dI}{dt}\right) = E_0 \sin \omega t$$

$$\text{or} \quad dI = \frac{E_0}{L} \sin \omega t \, dt$$

By integrating both sides we get -

$$\int dI = \frac{E_0}{L} \int \sin \omega t \, dt$$

$$I = \frac{E_0}{L} \left(\frac{-\cos \omega t}{\omega}\right) = \frac{-E_0}{\omega L} \cos \omega t$$

$$I = -\frac{E_0}{\omega L} \sin\left(\frac{\pi}{2} - \omega t\right)$$

$$\text{or} \quad I = \frac{E_0}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

The current (I) will be maximum (I_0), when

$\sin(\omega t - \pi/2)$ = maximum = 1

So $$I_0 = \frac{E_0}{\omega L}$$

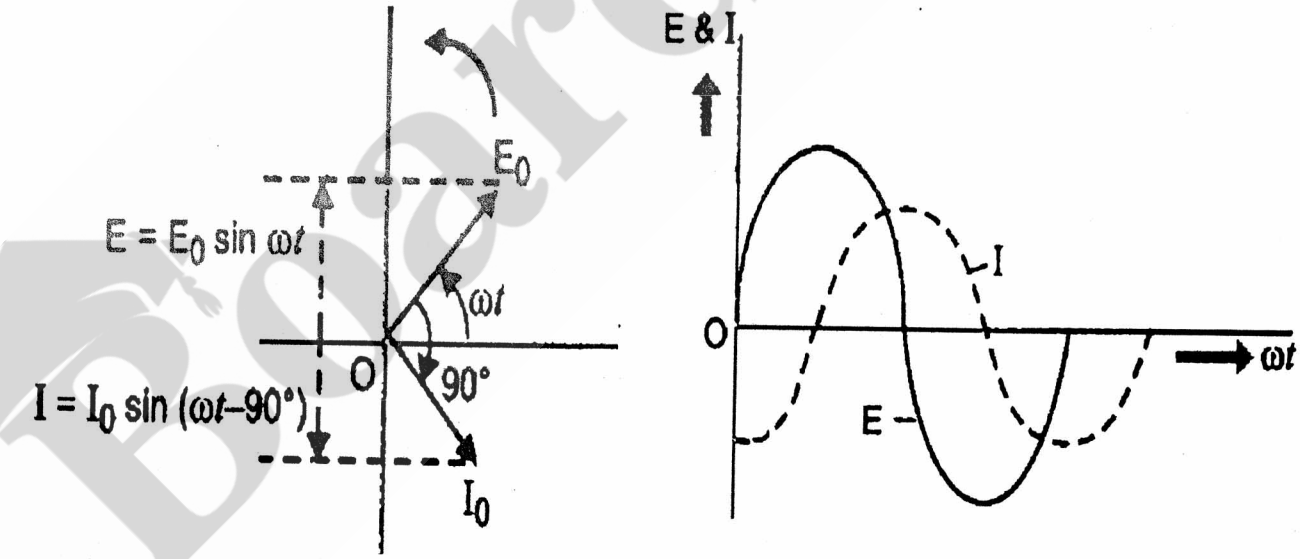
Hence $$I = I_0 \sin(\omega t - \pi/2)$$

So we find that in an a.c. circuit containing L only, alternating current I Lags behind the alternating voltage E by a phase angle of 90°. or we can say one fourth of a period.

Phasor Diagram -

In the phasor diagram of a.c. circuit containing L current Lags behind the e.m.f. by 90°, so the phasor representing I_0 is turned clockwise through 90° from the direction of E_0.

The projections of these phasors on y axis give instantaneous values E and I.



Here ωL represents the effective resistance offered by inductance L. This is called inductance reactance and is denoted by X_L.

So $$X_L = \omega L = 2\pi f L$$

$f \rightarrow$ frequency of a.c. supply

* In dc circuits $f = 0$ so $X_L = 0$
so that a pure inductance offers zero resistance to d.c. It means a pure inductor cannot reduce d.c.

SI unit of inductive reactance is ohm.(Ω)

$$X_L = \omega L = \frac{\text{henry}}{\text{sec.}} = \frac{\text{Volt}}{\text{sec.} \times \text{amp./se.}} = \text{ohm.}(\Omega)$$

Q. A 44 mH inductor is connected to 220 V, 50 Hz a.c. supply. Determine the r.m.s. value of current in the circuit.

Sol. Given that $L = 44\,mH = 44 \times 10^{-3}\,H$
$E_{rms} = 220\,V$, $f = 50\,Hz$

So $X_L = \omega L = 2\pi f L$
$X_L = 2 \times 3.14 \times 50 \times 44 \times 10^{-3} = 13.83$ ohm (Ω)

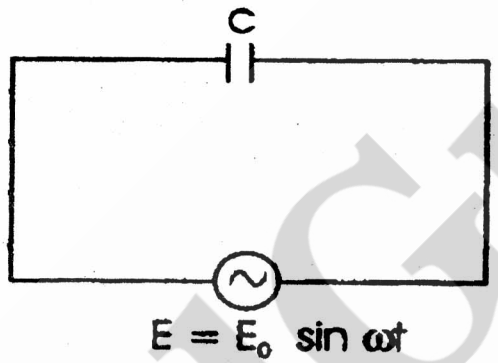
$$I_{rms} = \frac{E_{rms}}{X_L} = \frac{220}{13.83} = 15.9\,A$$

A.C. Circuit Containing Capacitance Only

Let a source of alternating e.m.f. is connected to a capacitor only of capacitance C. Suppose the alternating e.m.f. supplied is -

$$E = E_0 \sin \omega t$$

The current flowing in the circuit transfers charge to the plates of the capacitor. This produces a potential difference between the plates.



At any instant t, suppose q is the charge on the capacitor. So the potential difference across the plates of capacitor is

$$V = q/C$$

At every instant the potential difference V must be equal to the e.m.f. applied so -

$$V = E$$

$$\frac{q}{C} = E_0 \sin \omega t$$

Or

$$q = C E_0 \sin \omega t$$

If I is the instantaneous value of current in the circuit at instant t, then -

$$I = \frac{dq}{dt} = \frac{d}{dt}(C E_0 \sin \omega t)$$

$$I = C E_0 (\cos \omega t) \cdot \omega$$

$$I = \frac{E_0}{1/\omega C} \sin\left(\omega t + \frac{\pi}{2}\right)$$

The current will be maximum $I = I_0$, when $\sin\left(\omega t + \frac{\pi}{2}\right)$ = maximum = 1

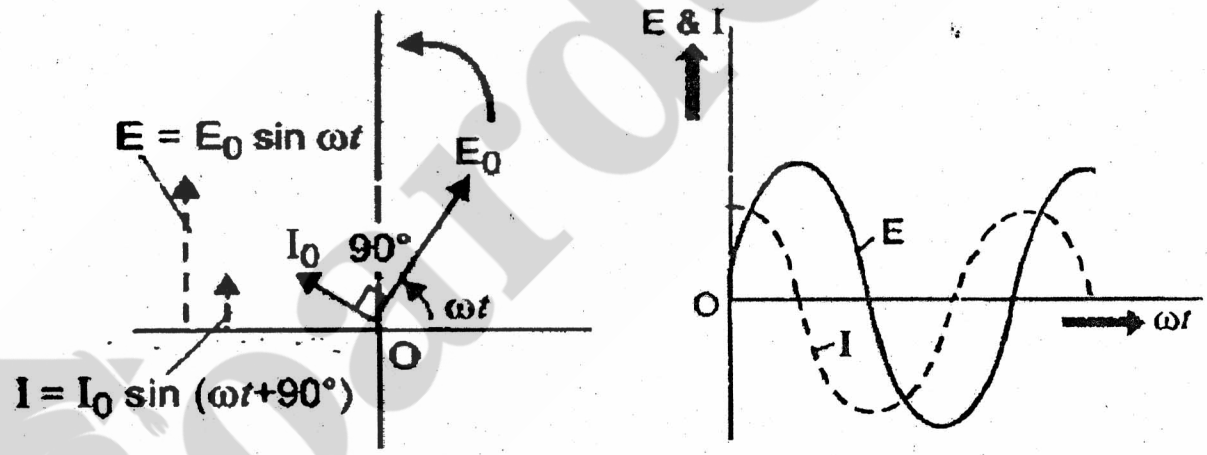
so $$I_0 = \frac{E_0}{1/\omega C}$$

and $$\boxed{I = I_0 \sin(\omega t + \pi/2)}$$

We find that in an a.c. circuit containing C only alternating current I Leads the alternating e.m.f. by a phase angle of 90°

Phasor Diagram-

The phasor I_0 is turned anticlockwise through 90° from the direction of phasor E_0. These projections on Y-axis gives the instantaneous values of E and I.



Here $1/\omega C$ represents effective resistance offered by the capacitor. This is called capacitive reactance and is denoted by X_C.

$$\boxed{X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}}$$

★ In dc circuit $f = 0$ so $X_C = \infty$
So a capacitor will block d.c.

★ SI unit of X_C is ohm. (Ω)

Note-

(i) R can reduce a.c. as well as d.c.
L cannot reduce d.c. It can reduce only a.c.
C blocks d.c. but allows a.c. to pass through.

(ii) Through R, alternating current is in phase with alternating voltage.
Through L, alternating current Lags behind the alternating voltage by a phase angle 90°.
Through C, alternating current leads the alternating voltage by a phase angle of 90°.

Q. CBSE 2009 A 15 μF capacitor is connected to 220 V, 50 Hz source, Find the peak current.

Sol. Here $C = 15\ \mu F = 15 \times 10^{-6}\ F$
$E_{rms} = 220\ V$, $f = 50\ Hz$.

$$\text{So } X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2 \times 3.14 \times 50 \times 15 \times 10^{-6}}$$

$$X_C = 212.1\ \Omega$$

$$I_{rms} = \frac{E_{rms}}{X_C} = \frac{220}{212.1} = 1.037\ A$$

$$\text{and } I_0 = \sqrt{2}\, I_{rms} = 1.414 \times 1.037$$

$$\text{So } I_0 = 1.47\ A$$

A.C. Circuit Containing Resistance, Inductance and Capacitance in Series (RLC Circuit)

Let R, L, C are connected in series to a source of alternating e.m.f. so the current at any instant through the three elements has the same amplitude and phase.

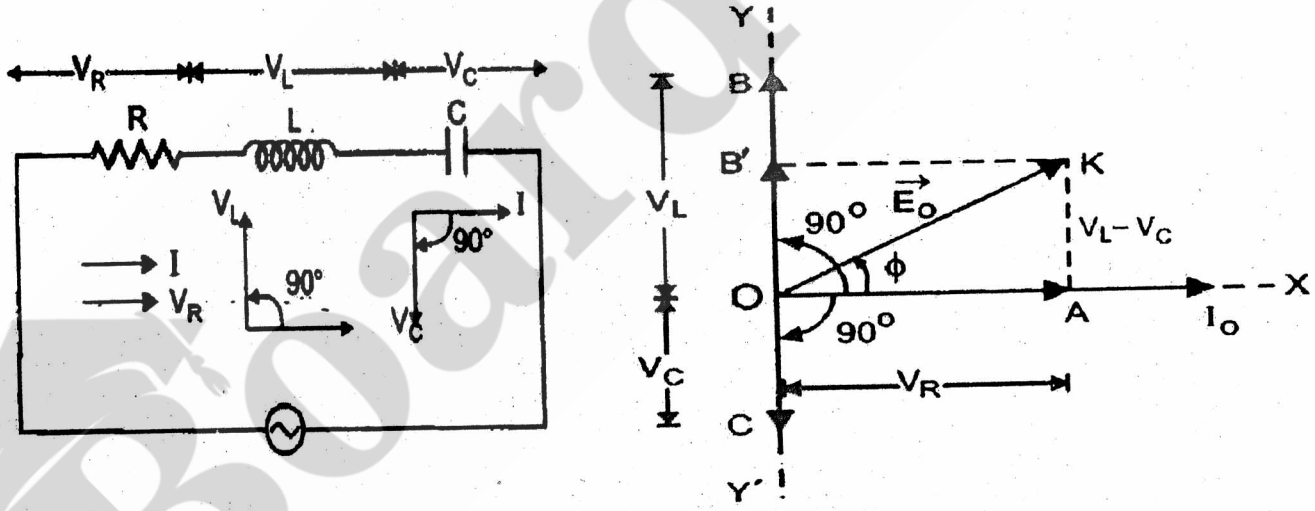
$$I = I_0 \sin \omega t$$

But the voltage across each element has a different phase relationship with the current.

(i) The maximum voltage across R is

$$V_R = I_0 R$$

In figure, current phasor I_0 is represented along OX. As V_R is in phase with current, so it is represented by the vector $\vec{OA}$, along OX.



(ii) The maximum voltage across L is $V_L = I_0 X_L$. As voltage across the inductor leads the current by 90°, so it is represented by $\vec{OB}$ along OY, 90° ahead of I_0.

(iii) The maximum value across C is $V_C = I_0 X_C$. As the voltage across the capacitor lags behind the alternating current by 90°, so it is represented by $\vec{OC}$ rotated clockwise through 90° from the direction of I_0. $\vec{OC}$ is along OY'.

As the voltage across L and C have a phase difference of 180°, so net reactive voltage is $(V_L - V_C)$, assuming that $V_L > V_C$. In figure it is represented by $\overrightarrow{OB'}$.

The resultant of $\overrightarrow{OA}$ and $\overrightarrow{OB'}$ is the diagonal $\overrightarrow{OK}$.

Hence the vector sum of $\vec{V}_R$, $\vec{V}_L$ and $\vec{V}_C$ is phasor $\vec{E}_0$ represented by $\overrightarrow{OK}$, making an angle ϕ with current phasor $\vec{I}_0$.

$$\text{So} \quad OK = \sqrt{OA^2 + OB^2}$$

$$\therefore \quad E_0 = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$E_0 = \sqrt{(I_0 R)^2 + (I_0 X_L - I_0 X_C)^2}$$

$$\text{or} \quad E_0 = I_0 \sqrt{R^2 + (X_L - X_C)^2}$$

The total effective resistance of RLC circuit is called Impedance of the circuit. It is represented by Z, where,

$$\boxed{Z = \frac{E_0}{I_0} = \sqrt{R^2 + (X_L - X_C)^2}}$$

* So in an a.c. circuit containing R, L, C, the voltage leads the current by a phase angle ϕ, where

$$\tan\phi = \frac{AK}{OA} = \frac{OB'}{OA} = \frac{V_L - V_C}{VR} = \frac{I_0 X_L - I_0 X_C}{I_0 R}$$

$$\text{or} \quad \boxed{\tan\phi = \frac{X_L - X_C}{R}}$$

So the alternating e.m.f. in an RLC circuit is represented by -

$$\boxed{E = E_0 \sin(\omega t + \phi)}$$

There may be three cases -

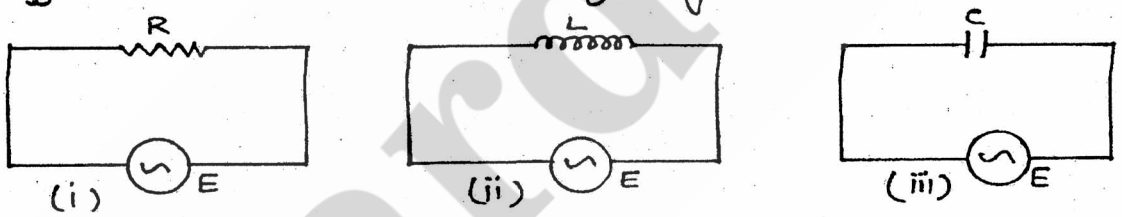
(i) When $X_L = X_C$, then $\tan\phi = 0$

So $\boxed{\phi = 0^\circ}$

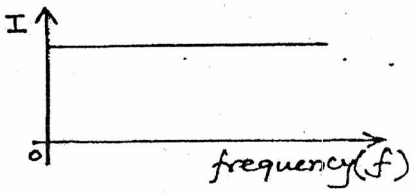
(ii) When $X_L > X_C$, then $\tan\phi$ is positive.
So ϕ is positive.
Hence voltage leads the current by a phase angle ϕ.

(iii) When $X_L < X_C$, then $\tan\phi$ is negative.
So ϕ is negative.
Hence voltage Lags behind the current by a phase angle ϕ.

Q. Delhi 2011 Three electrical circuits having AC sources of variable frequency are shown in figures. Initially the current flowing in each of these is same. If the frequency of the applied AC source is increased how will the current flowing in these circuits be affected ? Give reason for your answer.



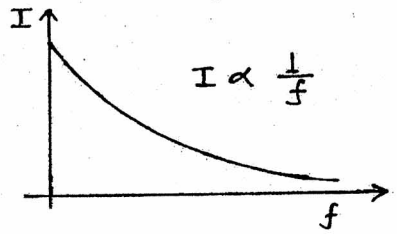
Sol. (i) There will not be any effect in the current on changing the frequency of AC source.



(ii) With the increase of freq. of AC source, inductive reactance increases as -

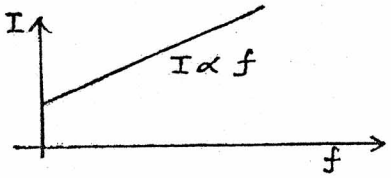
$$I_{rms} = \frac{V_{rms}}{X_L} = \frac{V_{rms}}{2\pi f L}$$

so current decreases with the increase of frequency.



(iii) The capacitive reactance decreases with increase in frequency so the current will increase.

$$I_{rms} = \frac{V_{rms}}{X_C} = \frac{V_{rms}}{1/2\pi f C}$$



Q. CBSE 2010 A resistor of 400 Ω, an inductor of $\frac{5}{\pi}$ H and a capacitor of $\frac{50}{\pi}$ μF are connected in series across a source of alternating voltage of 140 Sin 100 πt V. Find the voltage (rms) across the resistor, the inductor and the capacitor. Is the algebraic sum of these voltages more than the source voltages? If yes, resolve the paradox.

Sol. Applied Voltage $V = 140 \sin 100\pi t$

$C = \frac{50}{\pi}$ μF, $C = \frac{5}{\pi}$ H, $R = 400\ \Omega$

Comparing with $V = V_0 \sin \omega t$

$V_0 = 140$ V, $\omega = 100\pi$

Inductive reactance $X_L = \omega L$

$$X_L = 100\pi \times \frac{5}{\pi} = 500\ \Omega$$

Capacitive reactance $X_C = 1/\omega C$

$$X_C = \frac{1}{100\pi \times \frac{50}{\pi} \times 10^{-6}} = 200\ \Omega$$

Impedance of the circuit $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$Z = \sqrt{(400)^2 + (500-200)^2} = 500\ \Omega$$

Maximum current in the circuit

$$I_0 = \frac{V_0}{Z} = \frac{140}{500}$$

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{140}{500 \times \sqrt{2}} = 0.2\ A$$

Vrms across resistor $V_R = I_{rms} R$

$$V_R = 0.2 \times 400 = 80\ V$$

Vrms across inductor $V_L = I_{rms} X_L$

$$V_L = 0.2 \times 500 = 100\ V$$

Vrms across Capacitor $V_C = I_{rms} X_C$

$$V_C = 0.2 \times 200 = 40\ V$$

* Here $V = V_R + V_L + V_C$

Because V_L and V_C and V_R are not in same phase instead $V = \sqrt{V_R^2 + (V_L - V_C)^2}$

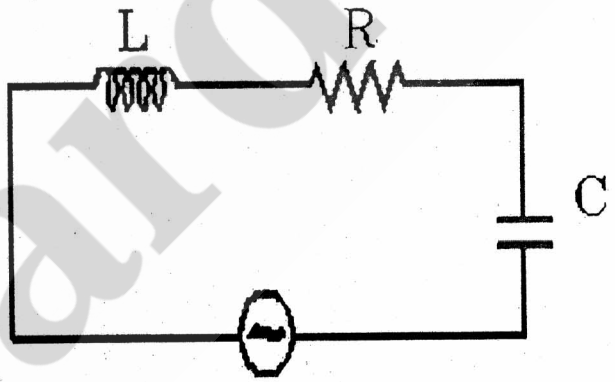
Electrical Resonance -

The phenomenon of resonance is common among systems that have a tendency to oscillate at a particular frequency. This frequency is called the natural frequency of oscillation of the system.

If such a system is driven by an energy source, whose frequency is equal to the natural frequency of the system, then the amplitude of oscillations becomes large and resonance is said to occur.

Series Resonance Circuit

A circuit in which inductance L, capacitance C and resistance R are connected in series, and the circuit admits maximum current corresponding to a given frequency of a.c., is called series resonance circuit.



At very low frequencies, inductive reactance $X_L = \omega L$ is negligible but capacitive reactance $X_C = 1/\omega C$ is very high.

As the frequency of alternating e.m.f. applied to the circuit is increased, X_L goes on increasing and X_C goes on decreasing.

For a particular value of ω (say ω_r)

$$X_L = X_C$$

$$\omega_r L = \frac{1}{\omega_r C}$$

or $$\boxed{\omega_r = \frac{1}{\sqrt{LC}}}$$

and $$2\pi f_r = \frac{1}{\sqrt{LC}}$$

So $$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$

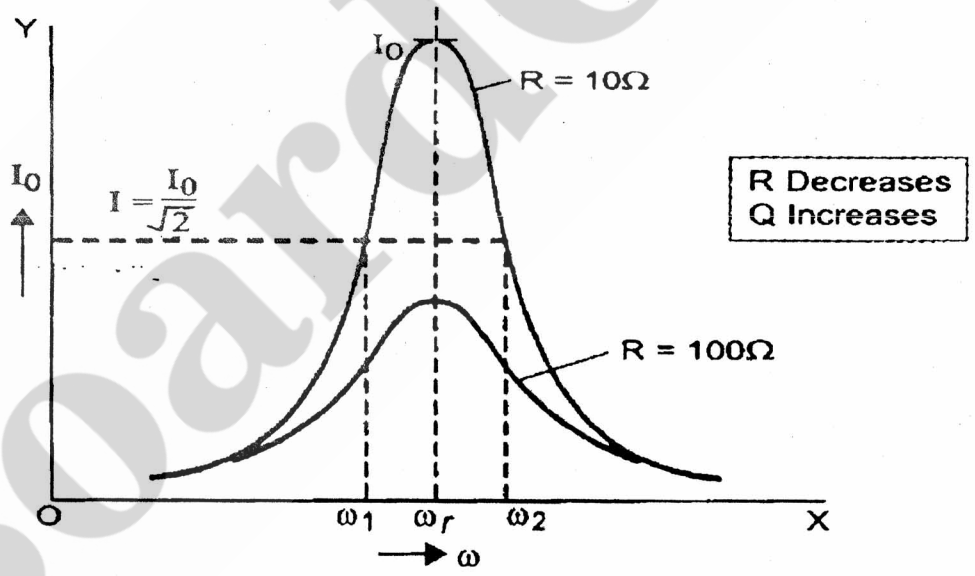
At this particular frequency f_r as $X_L = X_C$ so

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{R^2 + 0} = R \quad (\text{minimum})$$

So Here impedance of RLC circuit is minimum and hence the current $I_0 = \frac{E_0}{Z} = \frac{E_0}{R}$ becomes maximum.

This frequency is called series resonance frequency.



The variation of circuit peak current with the changing frequency of applied voltage is shown in figure.

(i) It is clear that for frequencies greater than or less than ω_r the values of peak current are less than the maximum value (I_0)

(ii) at $\omega = \omega_r$ the value of peak current is maximum (I_0).

Applications -

Series resonance circuit is used in radio and T.V. receiver sets.

The antenna of a radio/T.V. intercepts signals from many broadcasting stations. To receive one particular radio station/T.V. channel, we tune our receiver set by changing the capacitance of a capacitor in the tuning circuit of the set such that resonance frequency of the circuit becomes equal to the frequency of the disired station.

Therefore the resonance occurs and the amplitude of current with the frequency of the signal from the desired station becomes maximum and it is received in our set.

Q-Factor of Resonance Circuit

The characteristic of a series resonant circuit is determined by the Q factor or Quality factor of the circuit.
It defines sharpness of tuning at resonance.

The Q-factor of a series resonant circuit is defined as the ratio of the voltage developed across the inductance or capacitance at resonance to the impressed voltage, which is the voltage applied across R.

$$Q = \frac{\text{Voltage across L or C}}{\text{applied voltage (=voltage across R)}}$$

$$Q = \frac{V_L}{V_R} \quad \text{or} \quad Q = \frac{V_C}{V_R}$$

$$Q = \frac{(\omega_r L)I}{RI} = \frac{\omega_r L}{R}$$

using $\omega_r = \frac{1}{\sqrt{LC}}$,

$$Q = \frac{L}{R}\frac{1}{\sqrt{LC}}$$

$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

$$Q = \frac{(1/\omega_r C)I}{RI} = \frac{1}{RC\omega_r}$$

using $\omega_r = \frac{1}{\sqrt{LC}}$,

$$Q = \frac{1}{RC}\sqrt{LC}$$

$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

* As R is increased, Q factor of the circuit decreases.
* The values of Q usually vary from 10 to 100.
* The electronic circuits with high Q values would respond to a very narrow range of frequencies and sharper is the resonance.

Q. CBSE 2007 In a series L-C-R circuit, the voltages across an inductor, a capacitor and a resistor are 30V, 30V and 60V respectively. What is the phase difference between the applied voltage and the current in the circuit?

Sol. Given that - $V_L = 30V$, $V_C = 30V$, $V_R = 60V$

Here we can see that $V_L = V_C$, Hence the circuit is in resonance, so there will be no phase difference between the applied voltage and current.

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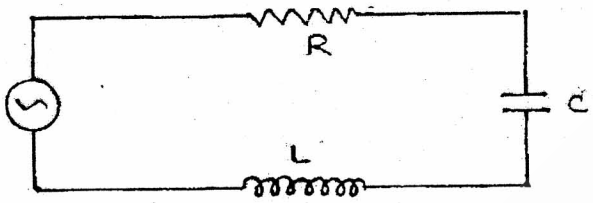
Q. Delhi 2012 The figure shows a series L-C-R circuit with L = 10 H, C = 40 μF, R = 60 Ω, connected to a variable frequency 240 V source, calculate-



(a) The angular frequency of the source which drives the circuit at resonance.
(b) The current at the resonating frequency.
(c) the rms potential drop across the inductor at resonance.

Sol. Given that- L = 10 H, C = 40 μF, R = 60 Ω
V_{rms} = 240 V

(a) Resonating angular frequency

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10 \times 40 \times 10^{-6}}}$$

$$\omega_r = 50 \text{ rad/s.}$$

(b) Current at resonating frequency

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{R} \quad (\because \text{at resonance } Z = R)$$

$$I_{rms} = \frac{240}{60} = 4 \text{ A}$$

(c) Inductive reactance $X_L = \omega L$
At resonance $X_L = \omega_r L$

$$X_L = 50 \times 10 = 500 \ \Omega$$

Potential drop across inductor

$$V_{rms} = I_{rms} \times X_L$$
$$= 4 \times 500$$
$$V_{rms} = 2000 \text{ V.}$$

Q. CBSE 2008 When a given coil is connected to a 200 V DC source, 2A current flows and when the same coil is connected to 200 V, 50 Hz AC source, only 1A current flows in the circuit.

(a) Explain why the current decreases in the later case?

(b) Calculate the self inductance of the coil used.

Sol. (a) In DC circuit –

$$X_L = 2\pi f L = 0 \qquad (\because f = 0)$$

∴ The impedance of the coil $Z = \sqrt{R^2 + X_L^2}$

In DC, $Z = R$ as $X_L = 0$

But in AC, $Z = \sqrt{R^2 + X_L^2} > R$

because X_L have certain value in AC.

As $I \propto \frac{1}{Z}$ hence in DC, more current flows than AC.

(b) In DC circuit –

$$R = \frac{V}{I} = \frac{200}{2} = 100\ \Omega$$

In AC circuit –

$$Z = \frac{V}{I} = \frac{200}{1} = 200\ \Omega$$

As $R^2 + X_L^2 = Z^2$

So $X_L^2 = Z^2 - R^2$

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{(200)^2 - (100)^2} = 100\sqrt{3}\ \Omega$$

$$\Rightarrow X_L = 2\pi f L = 100\sqrt{3}$$

∴ Inductance $L = \frac{100\sqrt{3}}{2\pi f}$

$$L = \frac{100\sqrt{3}}{2 \times 3.14 \times 50} = 0.55\ H$$

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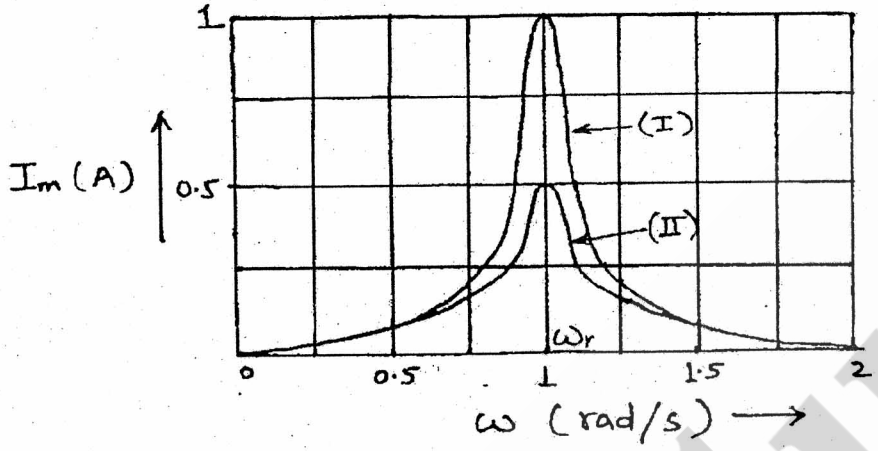
Q. Delhi 2009 The graphs shown here, depict the variation of current Irms angular frequency (ω) for two different series L-C-R circuits.



observe the graphs carefully.

(i) State the relation between L and C values of the two circuits, when the current in the two circuits is maximum.

(ii) Indicate the circuit for which
(a) power factor is higher
(b) Quality factor (Q) is larger. Give reasons for each case

Sol. (i) Since resonant angular frequency ωr is same for both the graphs -

$$\therefore \quad (\omega_r)_1 = (\omega_r)_2$$

$$\frac{1}{\sqrt{L_1 C_1}} = \frac{1}{\sqrt{L_2 C_2}} \Rightarrow \frac{L_1}{L_2} = \frac{C_2}{C_1}$$

(ii) (a) In graph (I), current I_{m_1} is greater than graph (II) Current I_{m_2}.

i.e. $I_{m_1} > I_{m_2} \Rightarrow R_1 < R_2$

Power factor of circuit represented by graph (II) has **higher** power factor as $\cos\phi \propto R$.

(b) $\because Q = \frac{1}{R}\sqrt{\frac{L}{C}}$ for two graphs

$Q \propto \frac{1}{R}$ $\quad \because R_1 < R_2$

$\therefore Q_1 > Q_2$

Quality factor of graph (I) is higher than the graph (II).

Q. Delhi 2008 An inductor 200 mH, capacitor 500 μF and a resistor 10 Ω are connected in series with a 100 V, variable frequency AC source. Calculate

(a) the frequency at which the power factor of the circuit is unity.

(b) Current amplitude at this frequency

(c) Q-factor.

Sol. (a) When power factor $\cos\phi = 1$, hence $\phi = 0$ and the L-C-R circuit is in resonance.

$\therefore$ Resonating frequency $\omega_0 = \frac{1}{\sqrt{LC}}$

$$\omega_0 = \frac{1}{\sqrt{200\times10^{-3}\times 500\times10^{-6}}} = 100\ s^{-1}$$

(b) $I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{R}$ ($\because$ at resonance $Z=R$)

$$I_{rms} = \frac{100}{10} = 10\ A$$

Amplitude of maximum current $I_0 = \sqrt{2}\, I_{rms}$

$$I_0 = 10\sqrt{2}\ A$$

(c) Q-factor $= \frac{1}{R}\sqrt{\frac{L}{C}}$

$$= \frac{1}{10}\sqrt{\frac{200\times10^{-3}}{500\times10^{-6}}} = 20$$

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Average Power in RLC Circuit

Let the alternating e.m.f. applied to an RLC circuit is –

$$E = E_0 \sin\omega t$$

If alternating current developed Lags behind the applied e.m.f. by a phase angle ϕ, then –

$$I = I_0 \sin(\omega t - \phi)$$

Power at instant t, $\frac{dW}{dt} = EI$

so $\frac{dW}{dt} = E_0 \sin\omega t \times I_0 \sin(\omega t - \phi)$

$= E_0 I_0 \sin\omega t (\sin\omega t \cos\phi - \cos\omega t \sin\phi)$

$= E_0 I_0 \sin^2\omega t \cos\phi - E_0 I_0 \sin\omega t \cos\omega t \sin\phi$

$= E_0 I_0 \sin^2\omega t \cos\phi - \frac{E_0 I_0}{2} \sin 2\omega t \sin\phi$

If this instantaneous power is assumed to remain constant for a small time dt, then small amount of work done in this time is –

$$dW = \left(E_0 I_0 \sin^2\omega t \cos\phi - \frac{E_0 I_0}{2} \sin 2\omega t \sin\phi\right) dt$$

Total work done over a complete cycle is –

$$W = \int_0^T E_0 I_0 \sin^2\omega t \cos\phi \, dt - \int_0^T \frac{E_0 I_0}{2} \sin 2\omega t \sin\phi \, dt$$

$$W = E_0 I_0 \cos\phi \int_0^T \sin^2\omega t \, dt - \frac{E_0 I_0}{2} \sin\phi \int_0^T \sin 2\omega t \, dt$$

As $\int_0^T \sin^2\omega t \, dt = \frac{I}{2}$ and $\int_0^T \sin\omega t \, dt = 0$

$\therefore \quad W = E_0 I_0 \cos\phi \times \frac{I}{2}$

$\therefore$ Average power in the RLC circuit over a complete cycle is – $P_{av} = \frac{W}{T}$

So $$P_{av} = \frac{E_0 I_0 \cos\phi}{T} \times \frac{T}{2}$$

$$P_{av} = \frac{E_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}} \cos\phi$$

$$\boxed{P_{av} = E_{rms}\ I_{rms} \cos\phi}$$

Hence average power over a complete cycle in an inductive circuit is the product of r.m.s. e.m.f. and r.m.s. current and cosine of the phase angle between the voltage and current.

* The above relation is applicable to all a.c. circuits.
Cosϕ and Z will have appropriate values for different circuits.

(i) Only R circuit -
$Z = R$ and $\cos\phi = 1$

(ii) Only L circuit -
$Z = X_L$ and $\cos\phi = 0$

(iii) Only C circuit -
$Z = X_C$ and $\cos\phi = 0$

(iv) In RLC circuit -
$Z = \sqrt{R^2 + (X_L - X_C)^2}$ and $\cos\phi = \frac{R}{Z}$

(v) In RL circuit -
$Z = \sqrt{R^2 + X_L^2}$ and $\cos\phi = \frac{R}{Z}$

(vi) In RC circuit -
$Z = \sqrt{R^2 + X_C^2}$ and $\cos\phi = \frac{R}{Z}$

(vii) In LC circuit -
$Z = \sqrt{(X_L - X_C)^2} = X_L - X_C$, $\cos\phi = 0$, and $\phi = 90^\circ$

Power Factor of an A.C. Circuit-

We Know that the average power in RLC Circuit is given by -

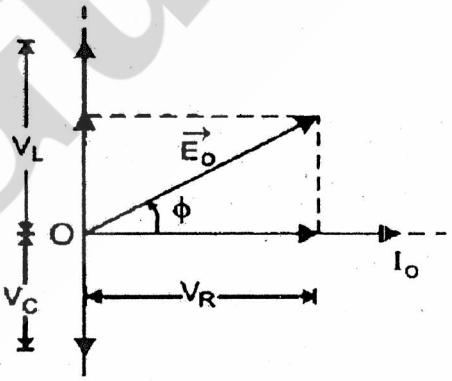
$$P = E_{rms}\, I_{rms} \cos\phi$$

Here P is called true power, ($E_{rms}\, I_{rms}$) is called apparent power or virtual power and $\cos\phi$ is called power factor of the circuit.

$$\text{Power Factor}(\cos\phi) = \frac{\text{True Power}(P)}{\text{apparent Power}}$$

From diagram -

$$\cos\phi = \frac{V_R}{E_0} = \frac{I_0 R}{I_0 Z} = \frac{R}{Z}$$



$$\text{Power factor} = \cos\phi = \frac{R}{Z}$$

(i) In a non-inductive circuit, $X_L = X_C$

$$\therefore \text{Power factor} = \cos\phi = \frac{R}{\sqrt{R^2}} = \frac{R}{R} = 1$$

$$\phi = 0^\circ$$

This is the maximum value of power factor.

(ii) In a Pure inductor or an ideal capacitor

$$\phi = 90^\circ$$

$$\therefore \text{Power Factor} = \cos\phi = \cos 90^\circ = 0$$

So average power consumed in a pure inductor or ideal capacitor

$$P = E_{rms}\, I_{rms} \cos\phi = 0$$

In actual practice we do not have ideal inductor or ideal capacitor - Therefore, there does occur some dissipation of energy.

Q. CBSE 2008 An inductor and a bulb are connected in series to an AC source of 12V, 50 Hz. 2A current flows in the circuit and the phase angle between voltage and current is $\pi/4$ rad. Calculate the impedance and inductance of the circuit.

Sol. Given that - $V_{rms} = 12V$, $f = 50\ Hz$

$\phi = \pi/4$, $I_{rms} = 2A$

$\therefore$ Impedance $Z = \frac{V_{rms}}{I_{rms}} = \frac{12}{2} = 6\ \Omega$

Also, power factor $\cos\phi = \frac{R}{Z}$

$R = Z\cos\phi$

$R = 6 \times \cos\frac{\pi}{4} = 6 \times \frac{1}{\sqrt{2}} = 3\sqrt{2}\ \Omega$

In L-R circuit $Z^2 = R^2 + X_L^2$

So $X_L^2 = Z^2 - R^2$

$X_L = \sqrt{(6)^2 - (3\sqrt{2})^2} = 3\sqrt{2}\ \Omega$

But $X_L = 2\pi f L = 3\sqrt{2}$

so $L = \frac{3\sqrt{2}}{2\pi f} = \frac{3\sqrt{2}}{2 \times 3.14 \times 50} = 13.5\ mH$

Q. CBSE 2006 A capacitor and a resistor are connected in series with an AC source. If the potential difference across C, R are 120V, 90V respectively and if the rms current of the circuit is 3A, calculate the (a) Impedance, (b) Power factor of the circuit.

Sol. Given that - $V_C = 120V$, $V_R = 90V$, I_{rms} 3A

(a) In C-R series AC circuit

$V_{rms} = \sqrt{V_R^2 + V_C^2}$

$V_{rms} = \sqrt{(120)^2 + (90)^2} = 150\ V$

$I_{rms} = \frac{V_{rms}}{Z}$ So $Z = \frac{V_{rms}}{I_{rms}}$

$$I_{rms} = \frac{V_{rms}}{Z} \quad \text{So} \quad Z = \frac{V_{rms}}{I_{rms}}$$

$$Z = \frac{150}{3} = 50\ \Omega$$

(b) $V_R = R \Rightarrow R = \frac{V_R}{I_{rms}}$

$$R = \frac{90}{3} = 30\ \Omega$$

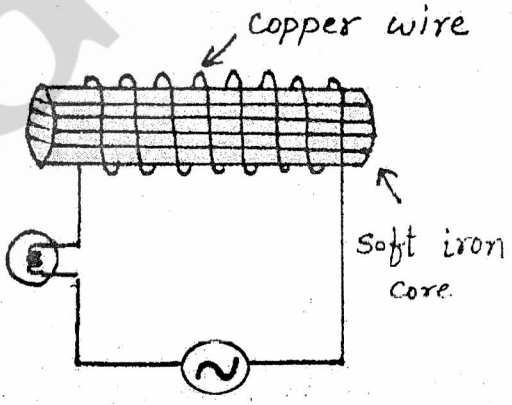
$\therefore$ Power factor $(\cos\phi) = \frac{R}{Z}$

$$\text{Power factor} = \frac{30}{50} = 0.6$$

CHOKE COIL -

Choke coil is simply a coil having a large inductance but a small resistance.

Construction - Choke coil is made by a thick copper wire to reduce its resistance. It is wounded on a plated soft iron core to increase its self inductance.



Principle -

At most places, the household electric power is supplied at 220 V, 50 Hz. If such a source is directly connected to a mercury tube, the tube will be damaged.

To reduce the current, a choke coil is connected in series with the tube.

Power Consumed in an A.C. circuit is $P = E_{rms} I_{rms} \cos\phi$

For pure inductor $\phi = \frac{\pi}{2}$ so that $P = 0$

Hence the advantage of using choke coil to reduce the voltage is that an inductor does not consume power. Hence we do not lose electric energy in the form of heat.

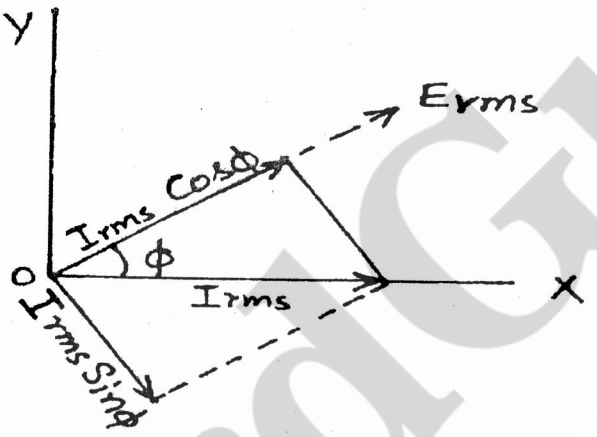
Wattless Current or Idle Current

The current which consumes no power for its maintenance in the circuit is called wattless current.

We know that average power over a complete cycle in an inductive circuit is -

$$P = E_{rms} I_{rms} \cos\phi$$

In a particular circuit, suppose E_{rms} leads I_{rms} by phase angle ϕ, as shown in figure.



I_{rms} Can be resolved into two rectangular components as -

$I_{rms} \cos\phi$ along E_{rms}

and $I_{rms} \sin\phi$ perpendicular to E_{rms}.

Thus I_{rms} is the vector sum of two perpendicular components $I_{rms}\cos\phi$ and $I_{rms}\sin\phi$.

As phase angle between E_{rms} and $I_{rms}\cos\phi$ is zero.

∴ average power consumed per cycle in the circuit due to component $I_{rms}\cos\phi$ is -

$$P_{av} = E_{rms}(I_{rms}\cos\phi)\cos 0^\circ$$

or

$$P_{av} = E_{rms} I_{rms} \cos\phi$$

Again as phase angle between E_{rms} and $I_{rms}\sin\phi$ is $\pi/2$, so average power consumed per cycle in the circuit due to component $I_{rms}\sin\phi$ is -

$$P'_{av} = E_{rms}(I_{rms}\sin\phi)\cos\frac{\pi}{2} = 0$$

$$P'_{av} = 0$$

Thus, the average power consumed / cycle in the a.c. circuit is wholly due to component $I_{rms}\cos\phi$ of the virtual current.

The component $I_{rms}\sin\phi$ makes no contribution to the consumption of power in the a.c. circuit.

That is why this component $I_{rms}\sin\phi$ is called the idle component or **wattless** component of a.c. and the component $I_{rms}\cos\phi$ is called **wattfull** component of a.c.

Q. A radio can tune over the frequency range of a portion of MW broadcast band (800 KHz to 1200 KHz). If its LC circuit has an effective inductance of 200 μH, then what must be the range of its variable capacitor?

Sol Given that –

$f_1 = 800\ KHz = 8\times10^5\ Hz$

$f_2 = 1200\ KHz = 12\times10^5\ Hz$

$L = 200\ \mu H = 200\times10^{-6}\ H$

Capacity range between C_1 and C_2 –

From $f_1 = \dfrac{1}{2\pi\sqrt{LC_1}}$

So $C_1 = \dfrac{1}{4\pi^2 L f_1^2} = \dfrac{1}{4\times(3.14)^2\times 2\times10^{-4}\times(8\times10^5)^2}$

$C_1 = 197.7\times10^{-12}\ F = 197.7\ PF$

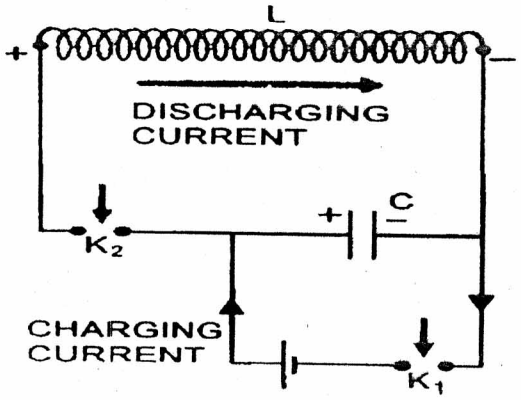
Similarly, $C_2 = \dfrac{1}{4\pi^2 L f_2^2}$

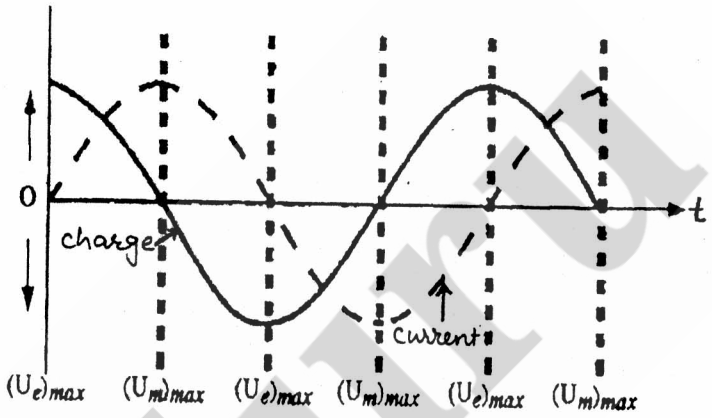
$C_2 = \dfrac{1}{4\times(3.14)^2\times 2\times10^{-4}\ (12\times10^5)^2}$

$C_2 = 87.8\times10^{-12}\ F = 87.8\ PF$

LC Oscillations

A capacitor C is connected to an inductor L through Key K_2 and a cell is connected to C through Key K_1.





When Key K_1 is on, the cell charges the capacitor to a potential $V = q/C$, where q is the charge on capacitor plates at any instant and V is the voltage across the plates at the same instant.

Some energy from the cell is stored in the dielectric medium between the plates of capacitor in the form of electrostatic energy, which is –

$$U_E = \frac{q^2}{2C}$$

Now off the Key K_1 and when Key K_2 is on then the charged capacitor is connected to L and starts discharging through L.

An induced e.m.f. developes in the circuit which opposes the growth of current in L and hence delays it.

When capacitor is completely discharged, the energy stored in the capacitor appears in the form of magnetic field energy around L, which is.

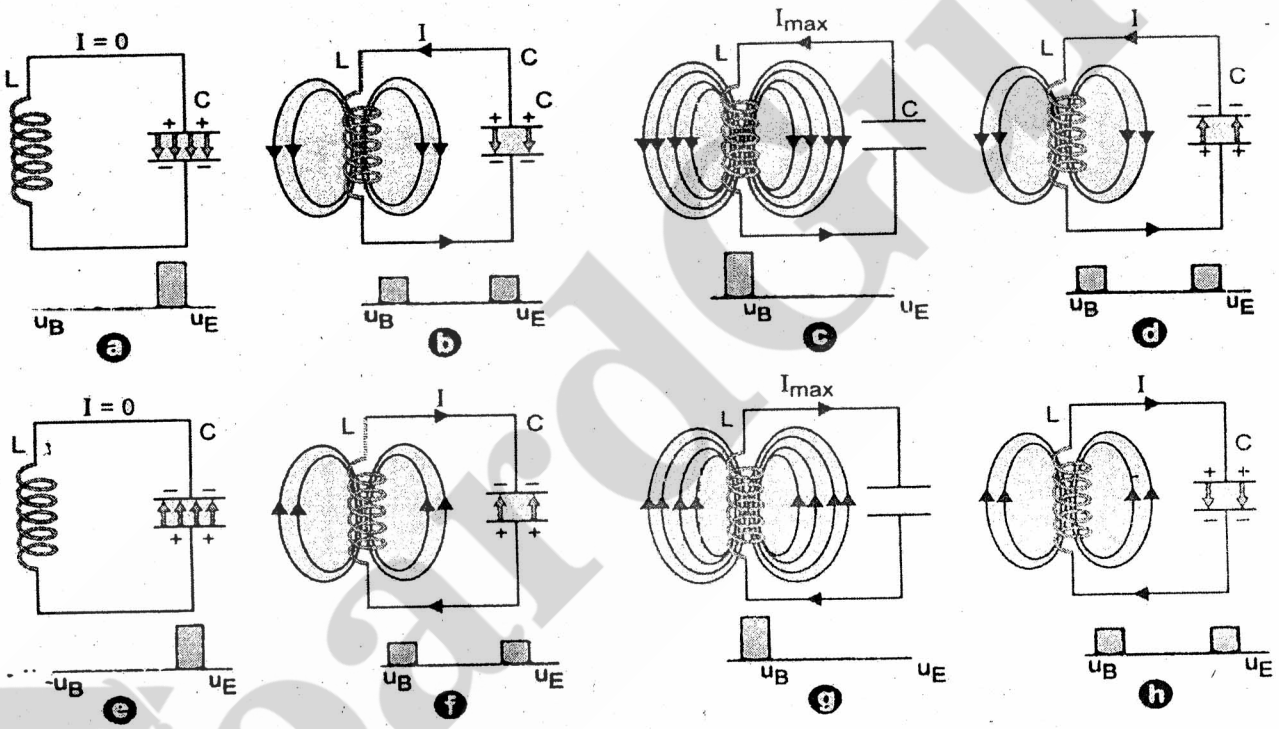
$$U_B = \frac{1}{2} LI^2$$

As soon as discharging of capacitor is complete, current stops and magnetic flux linked with L starts collapsing. Therefore an induced e.m.f. developes which starts recharging the capacitor in the opposite direction.

The recharging is also opposed and hence delayed. When capacitor is recharged completely, the magnetic field energy around L reappears in the form of electric field energy between the plates. The entire process is repeated.

Thus energy taken once from the cell and given to the capacitor keeps on oscillating between C and L.

The eight stages in a single cycle of oscillation of LC circuit are shown in figure.



Undamped oscillations -

If the circuit has no resistance then no loss of energy would occur. The oscillations produced will be of constant amplitude. These are called undamped oscillations.

Damped Oscillations -

In actual practice, there occur some losses of energy. So the amplitude of oscillations goes on decreasing. These are called damped oscillations.

A.C. Generator or A.C. Dynamo

An a.c. generator / dynamo is a machine which produces alternating current energy from mechanical energy.

It is most important applications of the phenomenon of electromagnetic induction. The generator was designed originally by a Yugoslav scientist, Nikola Tesla.

Principle -

An a.c. generator / dynamo is based on the phenomena of electromagnetic induction. Whenever the amount of magnetic flux linked with a coil changes, an e.m.f. is induced in the coil.
The direction of current induced is given by Fleming's right hand rule.

Construction -

The essential parts of an a.c. generator are-

(i) Armature -

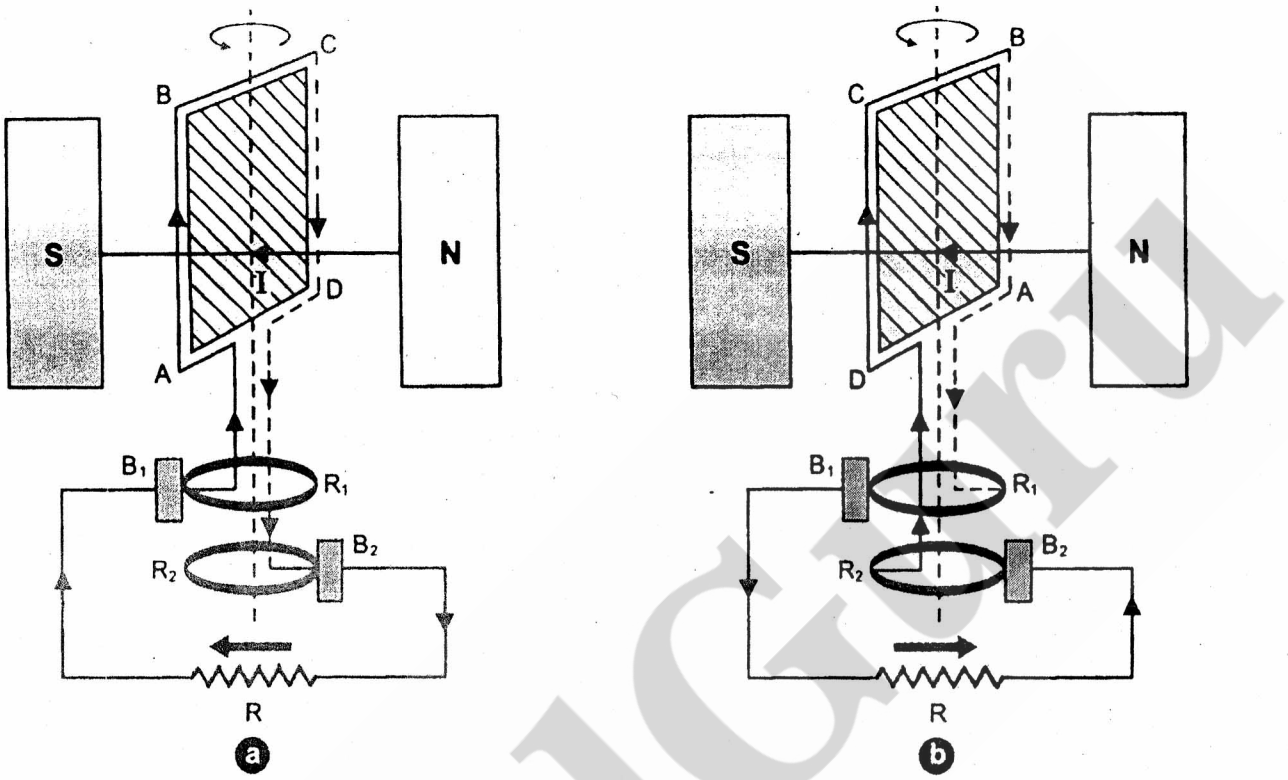
ABCD is a rectangular armature coil. It consist of a large number of turns of insulated copper wire wound over a laminated soft iron core. The coil can be rotated about the central axis.

(ii) Field Magnets -

N and S are the pole pieces of a strong electromagnet in which the armature coil is rotated. Axis of rotation is perpendicular to the magnetic field lines.

(iii) Slip Rings -

R_1 and R_2 are two hollow metallic rings, to which two ends of armature coil are connected. These rings rotate with the rotation of the coil.



(iv) **Brushes** - B_1 and B_2 are two flexible metal plates or carbon rods. They are fixed and are kept in light contact with R_1 and R_2 respectively. The purpose of brushes is to pass on current from the armature coil to the external Load resistance R.

Theory & Working

As the armature coil is rotated in the magnetic field, angle θ between the field and normal to the coil changes continuously. Therefore magnetic flux linked with the coil changes. And an e.m.f. is induced in the coil.

To start with, Suppose the coil is perpendicular to the plane of the paper in which magnetic field is applied, with AB at front and CD at the back. The amount of flux linked with coil in this position is maximum.

As the coil is rotated anticlockwise, AB moves inwards and CD moves outwards. The amount of magnetic flux linked with the coil changes.

According to Fleming's right hand rule, current induced in AB is from A to B and in CD, it is from C to D. In the external circuit, current flows from B_2 to B_1.

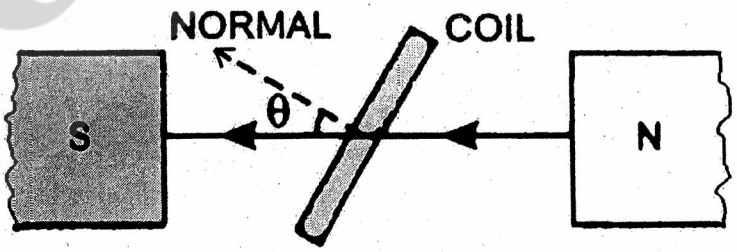
After half the rotation of the coil, AB is at the back and CD is at the front (figure(b)). Therefore on rotating further, AB moves outwards and CD moves inwards.

The current induced in AB is from B to A and in CD, it is from D to C. Through external circuit, current flows from B_1 to B_2.

Induced current in the external circuit changes direction after every half rotation of the coil. Hence the current induced in coil is alternating in nature.

Magnitude of induced e.m.f. & Induced Current

Let N = number of turns in the coil
A = area enclosed by each turn of the coil
$\vec{B}$ = strength of magnetic field
θ = angle which normal to the coil makes with $\vec{B}$ at any time t.



So the magnetic flux linked with the coil in this position is-

$$\phi = N(\vec{B}\cdot\vec{A}) = NBA\cos\theta$$

or $\phi = NBA\cos\omega t$ —(1)

where ω is angular velocity of the coil.

As the coil is rotated, θ changes, therefore magnetic flux ϕ linked with the coil changes and hence an e.m.f. is induced in the coil.

At this instant t, if e is the e.m.f. induced in the coil, then -

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(NBA\cos\omega t)$$

$$e = -NBA\frac{d}{dt}(\cos\omega t) = -NBA(-\sin\omega t)\omega$$

or $e = NBA\omega\sin\omega t$ — (2)

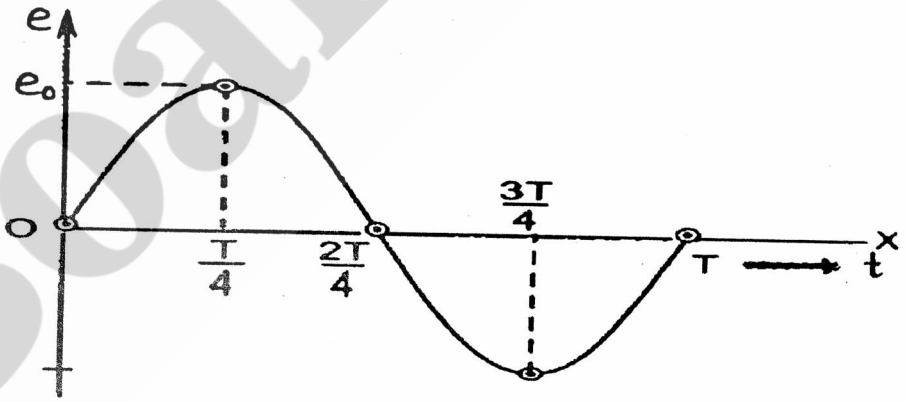
The induced e.m.f. will be maximum, when

$\sin\omega t$ = maximum = 1

$$e_{max} = e_0 = NBA\omega \times 1 \quad —(3)$$

by equation (2) we get -

$$\boxed{e = e_0 \sin\omega t}$$



The current supplied by the a.c. generator is also sinusoidal. It is given by -

$$I = \frac{e}{R} = \frac{e_0}{R}\sin\omega t \qquad \boxed{I = I_0 \sin\omega t}$$

where $I_0 = \frac{e_0}{R}$ = maximum value of current.

Transformer

A transformer is an electrical device which is used for changing the a.c. voltages.

A transformer which increases the a.c. voltages is called step up transformer and a transformer which decreases the a.c. voltages is called a step down transformer.

Principle -

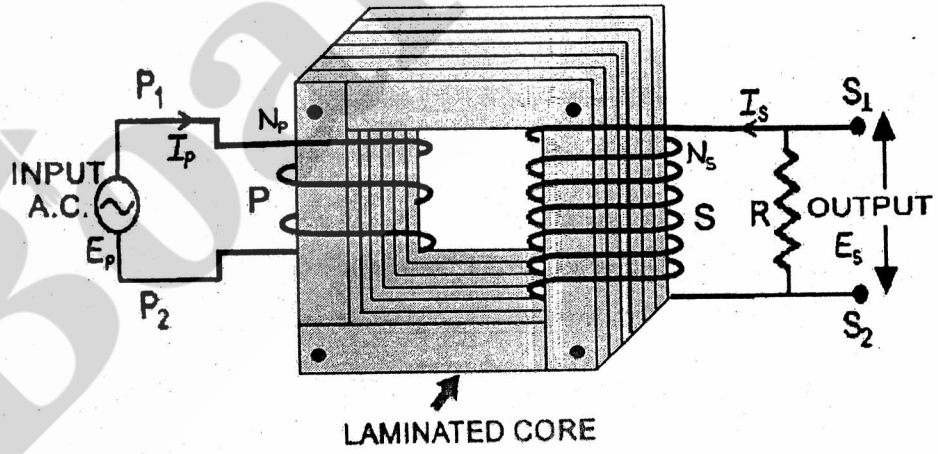
A transformer is based on the principle of mutual induction i.e. whenever the amount of magnetic flux linked with a coil changes, an e.m.f. is induced in the neighbouring coil.

Construction -

A transformer consist of a soft iron core made of laminated sheets, well insulated from each other.

Two coils P_1P_2 (Primary Coil) and S_1S_2 (secondary coil) are wound on the same core, but are well insulated from each other. Note that both the coils are also insulated from the core.

The source of alternating e.m.f. (to be transformed) is connected to the primary coil P_1P_2 and a load resistance R is connected to the secondary coil S_1S_2.



Theory & Working -

Let the alternating e.m.f. supplied by the a.c. source connected to primary is -

$$E_P = E_0 \sin \omega t$$

The alternating primary current induces an alternating magnetic flux ϕ_B in the iron core. Because the core extends through the turns of secondary, so the induced flux also extends through the secondary winding.

According to the Faraday's Law of electromagnetic induction, the induced e.m.f. per turn (E_{turn}) is same for both primary & secondary winding.

$$E_{turn} = \frac{d\phi_B}{dt} = \frac{E_P}{N_P} = \frac{E_S}{N_S}$$

Here n_p; n_s represents the total number of turns in the primary and secondary coils respectively.

$$\boxed{\frac{E_S}{E_P} = \frac{N_S}{N_P}}$$

As we assume that no energy is lost along the way, conservation of energy requires that -

$$\boxed{\text{Input Power} = \text{Output Power}}$$

$$I_P E_P = I_S E_S \quad \therefore \quad \boxed{\frac{I_P}{I_S} = \frac{E_S}{E_P}}$$

In general -

$$\boxed{\frac{E_S}{E_P} = \frac{I_P}{I_S} = \frac{N_S}{N_P} = K}$$

Here K = transformation ratio

Note -

If $N_S > N_P$ then $E_S > E_P$ then the transformer is called **step up transformer.**

If $N_S < N_P$ then $E_S < E_P$ then the transformer is called a **step down transformer.**

Efficiency of transformer

It is defined as the ratio of output power to the input power.

$$\eta = \frac{\text{output Power}}{\text{Input Power}} \times 100\ \% = \frac{E_s I_s}{E_p I_p} \times 100\ \%$$

Note - The transformer cannot work on d.c. A transformer changes a.c. voltages / currents. It does not affect the frequency of a.c. When a.c. voltage is raised n times, the corresponding alternating current reduces to 1/n times.

Energy Losses in a transformer

(i) Copper Loss - It is the energy loss in the form of heat in the copper coils of a transformer. This is due to Joule heating of conducting wires. These are minimised using thick wires.

(ii) Iron Loss - It is the energy loss in the form of heat in the iron core of the transformer. This is due to formation of eddy currents in iron core. It is minimised by taking Laminated cores.

(iii) Leakage of magnetic flux -

Due to this the rate of change of magnetic flux linked with each turn of S_1S_2 is less than the rate of change of magnetic flux linked with each turn of P_1P_2. It can be reduced by winding the primary and secondary coils one over the other.

(iv) Hysteresis Loss - This is the loss of energy due to repeated magnetisation and demagnetisation of the iron core when a.c. is fed to it. The loss is kept to a minimum by using a magnetic material which has a low hysteresis loss.

(v) **Magnetostriction** - It is humming noise of a transformer. Therefore, output power in the best transformer may be roughly 90% of the input power.

Q. CBSE 2007 Calculate the current drawn by the primary of a transformer which step-down 200 V to 20 V to operate a device of resistance 20 Ω. Assuming the efficiency of the transformer to be 80%.

Sol.

$$\text{Efficiency of transformer} = \frac{\text{Output Power}}{\text{Input Power}} \times 100$$

$$80 = \frac{V_S I_S}{V_P I_P} \times 100$$

$$4 V_P I_P = 5 V_S I_S$$

$$4 V_P I_P = 5 V_S \times \left(\frac{V_S}{R_S}\right) \qquad \left[I_S = \frac{V_S}{R_S}\right]$$

$$4 \times 200 \times I_P = 5 \times 20 \times \frac{20}{20}$$

$$I_P = \frac{1}{8} \text{ A}$$

Q. CBSE 2010 A step-down transformer operated on a 2.5 KV line. It supplies a load with 20 A. The ratio of the primary winding to the secondary is 10 : 1. If the transformer is 90% efficient, then calculate - (a) the power output (b) the voltage (c) the current in the secondary.

Sol.

Input Voltage $V_P = 2.5 \times 10^3$ V

Input Current $I_P = 20$ A

Also $\frac{N_P}{N_S} = \frac{10}{1}$

$$\text{Percentage efficiency} = \frac{\text{Output Power}}{\text{Input Power}} \times 100$$

$$90 = \frac{\text{output Power}}{V_P I_P} \times 100$$

(a) $$\text{Output Power} = \frac{90}{100} \times (2.5 \times 10^3 \times 20)$$

$$= 4.5 \times 10^4 \text{ W}$$

(b) $$\because \frac{V_S}{V_P} = \frac{N_S}{N_P} \Rightarrow V_S = \frac{N_S}{N_P} \times V_P$$

$$V_S = \frac{1}{10} \times 2.5 \times 10^3 = 250 \text{ V}$$

(c) $$V_S I_S = 4.5 \times 10^4 \text{ W}$$

$$\therefore \text{ Current } I_S = \frac{4.5 \times 10^4}{250} = 180 \text{ A}$$

Very Important Questions

1 Mark Questions -

Q.1. The current flowing through a pure inductance 2mH is $I = 15 \cos 300t$ A. What is the (a) rms (b) average value of current for a complete cycle? **(CBSE 2011)**

Answer - (a) $I_{rms} = 15/\sqrt{2}$ A (b) $I_{av} = 0$

Q.2. Define the "rms value of the current". How is it related to the peak value? **(CBSE 2010)**

Q.3. How much average power, over a complete cycle, does an AC source supply to a capacitor? **(CBSE 2009)**

Answer - $P_{av} = V_{rms} \times I_{rms} \cos\phi$

For capacitor $\phi = 90^\circ$ so $P_{av} = 0$

Q.4. An AC current, $I = I_0 \sin \omega t$, produces a certain heat H, in a resistor R, over a time $T = 2\pi/\omega$. Write the value of the DC current that would produce the same heat, in the same resistor, in the same time. **(RBSE 2009)**

Q.5. The instantaneous current and voltage of an AC circuit are given by $I = 10 \sin 300t$ A and $V = 200 \sin 300t$ V. What is the power dissipation in the circuit? **(CBSE 2008)**

Answer - $P_{av} = 1000$ W

Q.6 Define the term "wattless current".

Q.7. Mention the two characteristic properties of the material suitable for making core of a transformer.

Q.8. Write any two factors responsible for energy losses in actual transformers.

2 Marks Questions -

Q.1. Distinguish between the terms 'average value' and 'rms value' of an alternating current. The instantaneous current from an AC source is $I = 5 \sin 314t$ A. What are the average and rms values of the current.

Answer- $I_{rms} = 5/\sqrt{2}$, $I_{av} = 10/\pi$. (Delhi 2003,07)

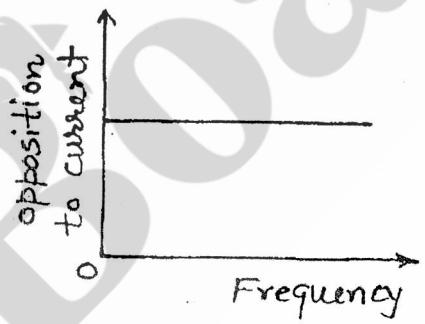
Q.2. An alternating voltage given by $V = 140 \sin 314t$ is connected across a pure resistor of 50 Ω. Find
(a) the frequency of the source
(b) the rms current through the resistor.

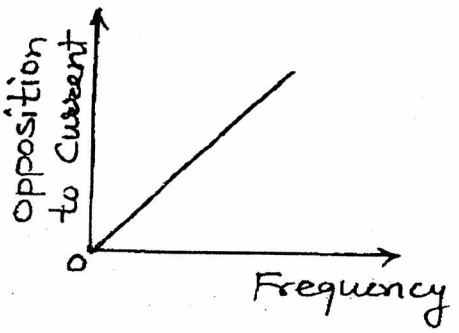
Answer- ν or $f = 50$ Hz , $I_{rms} = 1.9$ A (CBSE 2004,09)

Q.3. An alternating voltage $V = V_m \sin \omega t$ applied to a series L-C-R circuit drives a current given by $I = I_m \sin(\omega t + \phi)$. Deduce an expression for the average power dissipated over a cycle.

(CBSE 2009, 11)

Q.4. (a) The graph (I) and (II) represent the variation of the opposition offered by the circuit element to the flow of alternating current with frequency of the applied emf. Identify the circuit element corresponding to each graph.





(b) Write the expression for the impedance offered by the series combination of the above two elements connected across the AC sources. Which will be ahead in phase in this circuit, voltage or current?

(CBSE 2011)

Q.5. An AC source of voltage $V = V_m \sin\omega t$ is applied across a series L-C-R circuit. Draw the phasor diagram for this circuit when the.
(a) Capacitive impedance exceeds the inductive impedance.
(b) inductive impedance exceeds the inductive impedance.
(CBSE 2008)

Q.6. An AC voltage of 100 V, 20 Hz is connected across a 20 Ω resistor and 2 mH inductor in series. Calculate (a) Impedance of the circuit (b) rms current in the circuit.
(CBSE 2007)

Answer -(a) $Z = \sqrt{R^2 + X_L^2}$ (b) $I_{rms} \cong 5$ A.

Q.7. An inductor 200 mH, a capacitor C and a resistor 10 Ω are connected in series with a 100 V, 50 Hz AC source. If the current and voltage are in phase with each other, calculate the capacitance of the capacitor.

Answer - C = 50 μF (CBSE 2006)

Q.8. State the underlying principle of a transformer How is the large scale transmission of electric energy over long distance done with the use of transformers?
(CBSE 2012)

Q.9. Can a transformer be used to step up or step down a DC voltage? Justify your answer.
(CBSE 2004, 11)

Q.10. Mention various energy losses in a transformer.
(CBSE 2011)

Q.11. Calculate the quality factor of a series L-C-R circuit with L = 2 H, C = 2 μF and R = 10 Ω. Mention the significance of quality factor in L-C-R circuit.
(CBSE 2004, 12)

3 Marks Questions -

Q.1. Explain the term 'Inductive reactance'. Show graphically the variation of inductive reactance with freqdency of the applied alternating voltage.
An AC voltage $E = E_0 \sin \omega t$ is applied across a pure inductor of inductance L. Show mathematically that the current flowing through it legs behind the applied voltage by a phase angle of $\pi/2$.
(CBSE 2007, 10)

Q.2. In a series L-C-R circuit, define the quality factor Q at resonance. Illustrate its significance by giveng one example.
Show that power dissipated at resonance in L-C-R circuit is maximum.
(RBSE 2007)

Q.3. A series L-C-R circuit is connected to an AC source. Using the phasor diagram, derive the expression for the impedance of the circuit. plot a graph to show the variation of current with frequency of the source, explaining the nature of its variation.
(CBSE 2012)

Q.4. A series L-C-R circuit is connected to a 220V variable frequency AC supply. If L = 20 mH, $C = (800/\pi^2)$ MF and $R = 110\Omega$.
(a) Find the frequency of the source, for which average power absorbed by the circuit is max.
(b) Calculate the value of maximum current amplitude.
Answer - (a) $f = 125$ Hz (b) $I_0 = 2\sqrt{2}$ A **(Delhi 2010)**

Q.5. A coil of inductance 0.5 H and resistance 100 Ω is connected to a 200V, 50 Hz AC source, calculate.
(a) the maximum current in the coil
(b) time lag between voltage maximum and current maximum.
(CBSE 2008)

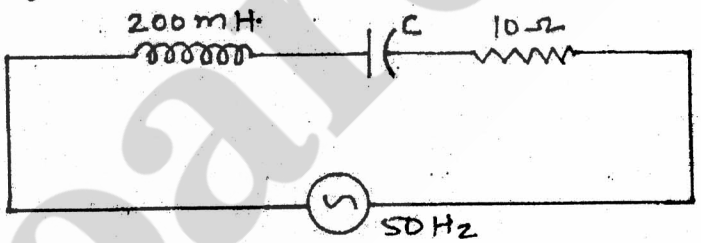
Answer - (a) $I_0 = \sqrt{2}\, I_{rms} = \sqrt{2}\, E_{rms}/Z$
$I_0 = 0.55$ A

Q.6. A 100 MF capacitor is in series with a 40Ω resistor and is connected to a 100 V, 50 Hz AC source. Calculate the following-
(a) Maximum current in the circuit
(b) Time lag between current maximum and voltage maximum.
(CBSE 2003, 08)

Answer – (a) $I_0 = 2.76$ A (b) Time Lag = $\frac{1}{200}$ s

Q.7. What does the term 'phasors' means in AC circuit analysis? An AC source of voltage $V = V_m \sin \omega t$ is applied across a pure inductor of inductance L. Obtain an expression for the current I, flowing in the circuit. Also draw the
(a) phasor diagram
(b) Graphs of V and I versus ωt for this circuit.
(Delhi 2008)

Q.8. In the given circuit, calculate (a) the capacitance C of the capacitor, if the power factor of the circuit is unity and (b) Also calculate the Q-factor of the circuit.
(CBSE 2006)



Q.9. Draw a labelled diagram of an AC generator. Explain briefly its principle and working.
(Delhi 2007)

Q.10. Explain with the help of the labelled diagram the principle of working of a step-up tansformer. Why can not such a device be used to step-up DC voltage?
(Delhi 2004, 07)

5 Marks Questions -

Q.1. What do you understand by sharpness of resonance in a series L-C-R circuit? Derive an expression for Q-factor of the circuit.

Derive an expression for the impedance of a series L-C-R circuit connected to an AC supply of variable frequency.

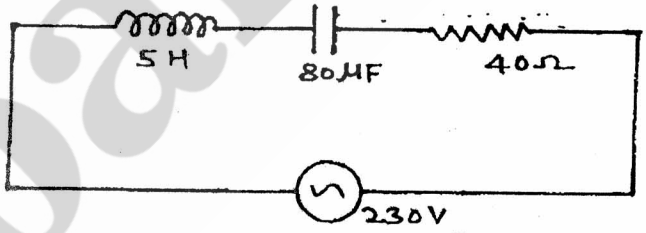
(Delhi 2011)

Q.2. An AC source generating a voltage $V = V_m \sin \omega t$ is connected to a capacitor of capacitance C. Find the expression for the current I, flowing through it. plot a graph of V and I versus ωt to show that the current is $\pi/2$ ahead to a voltage.

A resistor of 200 Ω and a capacitor of 15 MF are connected in series to a 200 V, 50 Hz AC source. Calculate the current in the circuit and the rms voltage across the resistor and the capacitor. Is the algebraic sum of these voltages more than the source voltage? If yes, resolve the paradox.

(CBSE 2004, 08)

Q.3. The given circuit diagram shows a series L-C-R circuit connected to a variable frequency 230 V source.



(Delhi 2006)

(a) Determine the source frequency which drives the circuit in resonance.

(b) Obtain the impedance of the circuit and the amplitude of current at the resonating frequency.

(c) Determine the rms potential drops across the three elements of the circuit.

(d) How do you explain the observation that the algebraic sum of the voltages across the three elements obtained in (c) is greater than the supplied voltage?

Q.4. State the working of AC generator with the help of a labelled diagram. The coil of an AC generator having N turns, each of area A, is rotated with a constant angular velocity ω.

Deduce the expression for the alternating emf generated in the coil. What is the source of energy generation in this device.

(CBSE 2005, 11)

Q.5. (a) A step-up transformer converts a low input voltage into a high output voltage. Does it violate law of conservation of energy ? Explain.

(b) Write any two sources of energy loss in a transformer.

(CBSE 2003, 11)

Q.6. The primary coil of an ideal step-up transformer has 100 turns and the transformation ratio is also 100. The input voltage and the power are 220 V and 1100 W respectively. Calculate

(a) number of turns in the secondary
(b) The current in the primary
(c) Voltage across the secondary (Delhi 2006)
(d) the current in the secondary
(e) Power in the secondary.

BoardGuru

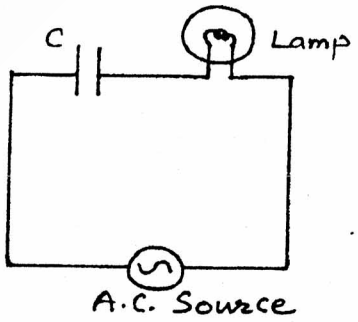
Q.1. A Lamp is connected in series with a capacitor. Predict your observation for dc and ac connections. What happens in each case if the capacitance is reduced ?

Q.2. Why is choke coil needed in use of fluorescent tubes with ac mains ? Why can we not use an ordinary resistor instead of choke coil ?

Q.3. What is the power dissipated in an ac circuit in an ac circuit in which voltage and current are given by $V = 230 \sin(\omega t + \frac{\pi}{2})$ and $I = 10 \sin \omega t$

Ans. Zero

Q.4. An electrical lamp connected in series with a capacitor and an ac source is glowing with of certain brightness. How does the brightness of the lamp change on reducing the (i) Capacitance (ii) Frequency ?



Ans. (i) Reduced (ii) Reduced

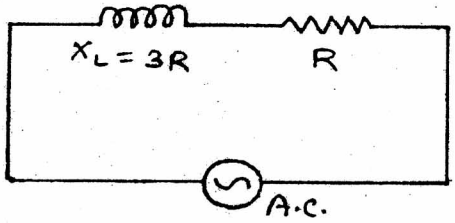
Q.5. A light bulb is rated 100W for 220V ac. supply of 50 Hz. Calculate (a) the resistance of the bulb (b) the rms current through the bulb

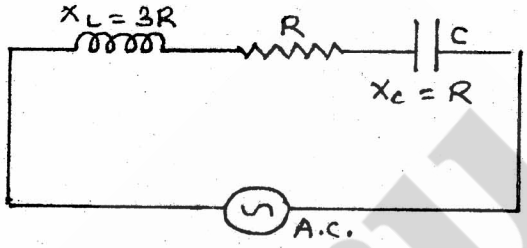
Ans (a) 484Ω (b) 0.45A

Q.6 How much current is drawn by the primary coil of a transformer which steps down 220V to 22V to operate a device with an impedance of 220Ω.

Ans. $I_p = 0.01$ A

Q.7. Two electrical circuits A and B are given below. Calculate the ratio of power factor of circuit B to the power factor of Circuit A.





Ans. $\frac{(\cos\phi)_B}{(\cos\phi)_A} = \sqrt{2}$

Q.8. In India domestic power supply is at 220 V, 50 Hz; while in USA it is 110 V, 50 Hz. Give one advantage and one disadvantage of 220 V supply over 110 V supply.

Q.9. A step-up transformer converts a low voltage into high voltage. Does it not violet the principle of conservation of energy ? Explain.

Q.10. A power transmission line needs input power 2300V to a step down transformer with its primary windings having 4000 turns. What should be the number of turns in the secondary windings in order to get output power at 230 V ?

Ans. 400 turns

Q.11. Calculate the Quality factor of a series LCR circuit with $L = 2H$, $C = 2\mu F$ and $R = 10\Omega$. Mension the significance of quality factor in LCR Circuit.

Ans. $Q = 100$

Q.12. A series LCR circuit is connected to an ac source (200V, 50 Hz). The voltages across the resistor, capacitor and inductor are respectively 200V, 250 V and 250 V. If the value of the resistance is 40Ω, calculate the current in the circuit.

Ans. $I_{rms} = 5A$